

## Apply It.

### The math behind... Pleasant and Unpleasant Music



#### Technical terms used:

Fourier theory, Fourier transform, wave equation, sine and cosine equation, frequency

#### Uses and applications:

By understanding the mathematics behind pleasant and unpleasant music, one can compose sounds for specific psychological purposes, clean and compressing MP3 files, or artificially create sounds of certain instruments.

#### How it works:

When several notes are played together, the ratio of their frequencies determines whether or not the sounds will be pleasant to the sound. Fourier theory shows that a sound produced by a note on an instrument with certain frequencies can be decomposed as a sum of sine and cosine waves with various frequencies and amplitudes. Through inspecting the graphs of these wave equations (wave patterns), composers can create music that effects listeners in specific ways.

Pleasant sounds, or consonances, can be formed by the notes whose wave patterns intersect in a relatively small integer ratio. For example, at the opening of Beethoven's *Moonlight Sonata*, the notes A (440Hz), F# (370Hz), and D (294Hz) played together create the infamous captivating sounds. The wave pattern of these notes intersect with the integer period ratio of 6:5:4. On the other hand, unpleasant sounds, or dissonances, can be formed by notes whose wave patterns are largely out of sync and rarely, if at all, intersect. This occurs with the notes B and C.

Using a Fourier transform, the sounds from different types of instruments, such as string, wind, drum, and horn, can be converted into wave equations. This collection of waves can then be manipulated with relative ease, allowing for recording compression and suppression of noise. This also allows us to combine pure sine and cosine tones to artificially reproduce the diverse sounds of instruments. This is often used in the creation of modern music.

#### Interesting facts:

Bach used dissonances to communicate religious ideas. At the end of his sacred oratorio, *St. Matthew Passion*, he portrays the agony of Christ's crucifixion, and at the opening aria of Cantata BWV 54, *Widerstehe doch der Sünde*, he illustrates the horror of the curse upon sin and desperate resistance of the Christian to the powers of evil.

Classical music geniuses can 'see' wave patterns beyond mathematical approaches, which enables them to create emotionally twisting masterpieces by combining consonances and dissonances in ways that cannot yet be accomplished by artificial intelligence. This opens various possibilities for deeper studies of the application of mathematics in musical composition.

#### Reference:

D. J. Benson, *Music: a Mathematical Offering* (Cambridge University Press, Nov 2006), ch 1-4.

A. Gupta, *Fourier Transform and Its Application in Cell Phones*, International Journal of Scientific and Research Publications. 3, Issue 1 (January 2013)

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