Uniqueness of Ellipsoidal Solutions to a Geometric Problem

Problem 05-004, *by* Lucio R. Berrone (CONICET, Departamento de Matemática, Facultad de Ciencias Exactas, Ing. y Agrim., Universidad Nacional de Rosario, Argentina).

A sphere $B \subset \mathbb{R}^n$ *centered at the origin* satisfies the following property:

$$|B \cap (x + \varepsilon B)| = M(\varepsilon), \; x \in \partial B, \, \varepsilon > 0, \tag{1}$$

where $|\cdot|$ denotes the Lebesgue measure and M is a real function of ε. In other words, property (1) says that the measure of the intersection of B with its homothetic image εB translated to a boundary point $x \in \partial B$ does not depend on the particular choice of the boundary point x (but only on the homothety scale ε). When their centers coincide with the homothety center (both coinciding with the origin of coordinates), ellipsoidal domains in $\mathbb{R}^n$ enjoy the same property, as can be quickly deduced from (1) by suitably rescaling the coordinate axis. The problem consists of deciding whether domains $B \subset \mathbb{R}^n$ different from ellipsoidal domains exist or not satisfying property (1) when "centered" at a certain interior point $O \in B^\circ$ (the center of homothety).

Status. This problem is open.