A Fourier–Laplace Transform

Problem 06-008, *by* A. E. Merzon[1] (University of Michoacán of San Nicolás de Hidalgo, México).

It can be convenient to use the Method of the Complex Characteristics [1], [2] for construction of the rigorous mathematical theory of the nonstationary dispersion on wedges [3] as well as for solution of some other problems, which are reduced to Boundary Value problems in angles for the elliptic equations of the second order (see, e.g., [4]). One of the most important elements of the method is the so-called Connection Equation between the Fourier–Laplace transforms of the four Cauchy data (two Dirichlet data and two Neumann data) of the solution on the sides of an angle [3]. This Connection Equation is the direct consequence of the Paley-Wiener Theorem for the angles less than π, while it is nontrivial affirmation for the angles more than π. To obtain this equation in the nonconvex case it is necessary to construct a function of the Cauchy kernel type

$$K(w) = \frac{F(w)}{w - w_0}, \quad w \in \mathbb{C},$$

in some strip between two "parallel" curves in $\mathbb{C}$. Moreover, the function F must be, roughly speaking, the Fourier–Laplace transform of some function of the Schwartz class S with support in the first quadrant, with an exponential decrease and having no zeros in the strip.

The proposed problem is dedicated to construction of some one-variable function of this type.

Problem. Find a function f from the Schwartz space $S(\mathbb{R})$ of the rapidly decreasing smooth functions with the support in $\overline{\mathbb{R}^+} := \{x \in \mathbb{R} : x \geq 0\}$, whose Fourier–Laplace transform

$$\tilde{f}(z) := \int_0^\infty e^{izx} f(x) dx$$

has no zeros in $\overline{\mathbb{C}^+} := \{z \in \mathbb{C} : \ \mathrm{Im}\ z \geq 0\}$.

Status. The proposer has a solution. Additional solutions are welcome.

[1]University of Michoacán of San Nicolás de Hidalgo, Ciudad Universitaria, 58090, Morelia, Michoacán, México (anatoli@ifm.umich.mx). This work was supported by Project 46769, CONACYT, México; SNI, México; Project MA-4.12, CIC of UMSNH, México.

REFERENCES

[1] A. I. Komech, *Elliptic boundary value problems on manifolds with piecewise smooth boundary*, Math. USSR-Sb., 21 (1973), pp. 91–135.

[2] A. Komech, A. Merzon, and P. Zhevandrov, *A method of complex characteristics for elliptic problems in angles and its applications*, Amer. Math. Soc. Transl. Ser. (2), 206 (2002), pp. 125–159.

[3] A. I. Komech, N. J. Mauser, A. E. Merzon, *On Sommerfeld representation and uniqueness in diffraction by wedges*, Math. Methods Appl. Sci., 28 (2005), pp. 147–183.

[4] A. E. Merzon, *On Ursell's problem*, in Proceedings of the Third International Conference on Mathematical and Numerical Aspects of Wave Propagation, Gary Cohen, ed., SIAM – INRIA, Philadelphia, 1995, pp. 613–623.