

An Asymptotic Formula Involving Primes

Problem 08-005, by PAUL S. BRUCKMAN¹ (Sointula, BC, Canada).

For all $x > 0$, let

$$\pi(x) = \sum_{p \leq x} 1$$

denote the number of positive integers $\leq x$ that are prime. Prove the asymptotic relation

$$(*) \quad \pi(N^{n+1}) \sim \sum_{p \leq N} p^n \quad (N \rightarrow \infty)$$

for fixed $n \geq 0$. Note that for $n = 0$, the relation $(*)$ reduces to the definition of the counting function for primes, except that the asymptotic relation is replaced by equality.

Status. The proposer has a solution. Other solutions are welcome.

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