

IP1**Algebraic Statistics for Social Network Models: Recent Results and Challenges**

In collaboration with Petrovic and Rinaldo, I have developed algebraic statistical tools for the study of some dyadic random graph models, including Markov bases, that have important implications for the existence of maximum likelihood estimation and other statistical problems. These tools do not extend in a simple fashion to more complex models in the class of exponential random graph models. In this presentation, I explain why there are difficulties as we move away from dyadic models and I describe some of the challenges for algebraic statistics in this area of research.

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IP2**Complexity of the Separation of Variables and Related Observations**

Abstract not available at time of publication.

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IP3**Calculus and Constructible Sheaves**

This talk describes an ingenious integral calculus based on Euler characteristic, stemming from work on constructible sheaves due to MacPherson and Kashiwara in the 1970s, and connecting back further to classical integral geometry. The talk will emphasize (1) its novel utility in data management, particularly in aggregation of redundant data and inverse problems over sensor networks; and (2) how issues of numerical computation, inspired by applications, leads to fascinating connections with Morse theory and computational topology.

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IP4**The Geometry of Cell Structures in Foams and Metals**

Networks of crystals in a metal and networks of bubbles in a foam are both examples cell complexes that arise in nature. Both systems evolve over time according to mathematical equations. We believe that, for generic initial conditions, they evolve towards a statistically universal state. The topology and geometry of this universal state is great interest in applications. This talk will present both mathematical results and computer investigations on these systems in two and three space dimensions.

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IP5**Advances in Elliptic-Curve Cryptography**

The first part of this talk presents results on attacking elliptic-curve cryptography, in particular an ongoing effort to break the Certicom challenge ECC2K-130 and a detailed study on the correct use of the negation map in the Pollard-rho method. The second part presents a signature scheme which on a 390 USD mass-market quad-core 2.4GHz Intel Westmere (Xeon E5620) CPU can create 108000 signatures per second and verify 71000 signatures per second on an elliptic curve at a 2^{128} security level. Public keys are 32 bytes, and signatures are 64 bytes. These performance figures include strong defenses against software side-channel attacks: there is no data flow from secret keys to array indices, and there is no data flow from secret keys to branch conditions.

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IP6**B-splines: A Fundamental Tool for Analysis and Computation**

Starting from their inception in approximation theory 60 years ago, the use of B-splines has blossomed in diverse areas in science, engineering and electronic arts. These include animation, computational geometry, computer-aided design, computer-aided manufacturing, computer graphics, control theory, geometric design, image analysis, medical visualization, optimization, partial differential equations, robotics, and statistics. More recently B-splines have been assuming a fundamental role in the isogeometric analysis, an ambitious effort to unify shape representation and engineering analysis. Honoring its interesting historical development, this talk gives a mathematical introduction to splines and B-splines in one and several space dimensions and also considers various generalizations.

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IP7**Symbolic-numeric Algorithms for Computing the Singular Solution of Polynomial Systems**

In this talk we will describe some recent progress in symbolic-numeric algorithms for computing the singular solution of polynomial systems. Using the local dual structure of the isolated singular solution with limited accuracy, we present modifications of Newton's method to restore quadratic convergence. In particular, when the corank of the Jacobian matrix at the singular solution is one, based on the regularized Newton iteration and the computation of partial differential conditions satisfied approximately at the singular solution, we develop a new approach to compute a proper direction and step size of the Newton iteration to ensure the quadratic convergence. Finally, we will show recent results on computing verified error bounds for singular solutions of polynomial systems based on Rump's verification method.

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IP8 Multiview Geometry

The study of two-dimensional images of three-dimensional scenes is a foundational subject for computer vision, known as multiview geometry. We present recent work with Chris Aholt and Rekha Thomas on the polynomials defining images taken by n cameras. Our varieties are threefolds that vary in a family of dimension $11n-15$ when the cameras are moving. We use toric geometry and multigraded Hilbert schemes to characterize degenerations of camera positions.

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CP1 Discrete Bounded-curvature Paths and Path Planning

In 1957 Dubins classified shortest bounded curvature paths in the plane into two classes. We generalize this result to a new class of discrete polygonal paths. The properties and construction of discrete Dubins paths are discussed. Properties of discrete bounded-curvature motion are analysed and applied to prove properties of smooth bounded-curvature paths. In particular, the classification of discrete Dubins paths is used to obtain Dubins seminal result as a limiting case. Related to these problems the question of defining curvature for a polygonal path is addressed. Potential research questions, such as generalizing Cauchy's arm lemma, are discussed.

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CP1 On Middle Universal Weak and Cross Inverse Property Loops With Equal Length of Inverse Cycles

This study presents an isotopism under which the weak inverse property (WIP) is isotopic invariant in loops. It is shown that under this isotopism, whenever n is a positive even integer, a finite WIPL has an inverse cycle of length n if and only if its isotope is a finite WIPL with an inverse cycle of length n . Explanations and procedures are given on how these results can be used to apply CIPLs to cryptography.

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CP1 Extracting Topological Properties Using Manifold Learning Techniques

High-dimensional, unordered data, is often difficult to analyze or visualize. In such cases, manifold learning techniques - like PCA, Isomap, Hessian LLE - are quite helpful and efficient in extracting the low-dimensional representation of the data. When used individually, these methods just give the low-dimensional representation of the data. But owing to the complementary nature of the limitations of these methods, performing a hierarchy of tests and comparing the outputs can reveal useful topological features like the presence of holes and (degree of) non-linearity of the data. We develop and showcase such a hierarchical strategy on various material science data to provide insight into process-property-structure relationships.

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CP1 Lower Bounds on Stochastic Games

Shapley's *discounted stochastic games* are classical models of game theory describing two-player zero-sum games of potentially infinite duration. We show, based on a generalization of Eisenstein criterion, that there exists a game with N positions, m actions for each player in each position, such that its value is an algebraic number of degree m^{N-1} . This strengthens a result of Etessami and Yannakakis who considered the case of $m = 2$ and proved a $2^{\Omega(N)}$ lower bound.

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MS1 Symmetric Determinantal Representations of Polynomials

In a seminal paper (STOC 1979), Valiant expressed the polynomial computed by an arithmetic formula as the determinant of a matrix whose entries are constants or variables. This result was then extended by Malod and Toda to weakly-skew circuits. We are interested here in expressing formulas and weakly-skew circuits by determinants of symmetric matrices. In my talk, I will sketch some of these

constructions for fields of characteristic different from 2. In characteristic 2, the picture is quite different: First we can prove an impossibility result, showing that some polynomials have no symmetric determinantal representation. On the other hand, we answer a question by Bürgisser about the VNP-completeness of the partial permanent. This talk is based on joint works with E. L. Kaltofen, P. Koiran and N. Portier, and to a smaller extent with T. Monteil and S. Thomassé.

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MS1

Solving the Generalized MinRank Problem with Groebner Bases

Let \mathbf{K} be a field and M be a $p \times k$ matrix with entries in $\mathbf{K}[X_1, \dots, X_n]$ of degree D and $0 < r < \min(p, k)$. The generalized MinRank problem consists in finding the algebraic points in \mathbf{C}^n at which M has rank r . This is a generalization of the classical eigenvalue problem in Linear Algebra. It has applications in several areas such as cryptology and real geometry. The algebraic nature of the problem leads us to study the use Gröbner bases techniques to solve it. In this framework, solving the generalized MinRank problem is reduced to solve the polynomial system defined by the vanishing of all $(r + 1, r + 1)$ -minors of M . Such a polynomial system is highly structured. In the 0-dimensional case, we provide sharp complexity results on solving such a structured polynomial system with Gröbner bases when the characteristic of \mathbf{K} is 0 or when it is large enough, and when the entries of M are generic. The used ingredients rely on algebraic properties of determinantal ideals and classical tools such as Hilbert series to analyze the complexity of Gröbner bases computations.

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MS1

On the Role of Compactness in Algebraic Complexity Theory

In various situations in the study of the computational complexity of algebraic problems, it is convenient to restrict to a compact setting. For instance, real and complex analogs of Toda's Theorem of discrete complexity theory in the Blum-Shub-Smale model are shown for a compact version of the polynomial time hierarchy. We study the first level of this compact polynomial hierarchy over the reals (compact NP) and look for complete problems in it. The problem of deciding whether a real homogeneous polynomial has a non-trivial zero is a natural candidate. However, it is surprisingly difficult to prove this hardness result, since a decision procedure of a closed set does in general not yield a closed description of it. We report on partial results and approaches for this problem. In particular, we show NP-completeness in a restricted model of computa-

tion which by definition accepts closed sets only. Also, we explore which problems in NP can be reduced to the compact feasibility problem using an effective finiteness Theorem involving Lojasiewicz' inequality.

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MS1

On the Complexity of Computing with Zero-dimensional Triangular Sets

We study the complexity of some fundamental operations for triangular sets in dimension zero. Using Las-Vegas algorithms, we prove that one can perform such operations as change of order, equiprojectable decomposition, or quasi-inverse computation with a cost that is essentially that of *modular composition*. Over an abstract field, this leads to a subquadratic cost (w.r.t. the degree of the underlying algebraic set). Over a finite field, in a boolean RAM model, we obtain a quasi-linear running time using Kedlaya and Umans' algorithm for modular composition. Conversely, we also show how to reduce the problem of modular composition to change of order for triangular sets, so that all these problems are essentially equivalent.

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MS1

On the Complexity of Solving Quadratic Boolean Systems

A fundamental problem in computer science is to find all the common zeroes of m quadratic polynomials in n unknowns over the boolean field $\text{GF}(2)$. In particular, the cryptanalysis of several modern ciphers reduces to this problem. Up to now, the best complexity bound was obtained by exhaustive search in $O(\log(n)2^n)$. We give an algorithm that reduces the problem of solving boolean quadratic systems to a combination of exhaustive search and sparse linear algebra. Under precise algebraic assumptions, we show that when $m = n$, the complexity of the deterministic variant of this algorithm is upper bounded by $O(2^{0.841n})$, while a probabilistic Las Vegas variant has expected complexity $O(2^{0.792n})$. Experiments on random systems show that the algebraic assumptions are satisfied with probability close to 1.

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MS2

Secant Varieties of Segre-Veronese Varieties

In general, the secant varieties of Segre-Veronese varieties can be difficult to describe explicitly. In many cases, the dimensions are not even known. I will explain the following theorem: for r at most 5, the r th secant variety of $P^2 \times P^n$ embedded by $\mathcal{O}(\infty, \epsilon)$ is defined by the minors of the flattening and the Pfaffian of the exterior flattening.

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MS2

Tensor Complexes

The most fundamental complexes of free modules over a commutative ring are the Koszul complex, which is constructed from a vector (i.e., a 1-tensor), and the Eagon–Northcott and the Buchsbaum–Rim complexes, which are constructed from a matrix (i.e., a 2-tensor). I will discuss a multilinear generalization of these complexes, which we construct from an arbitrary higher tensor. Our construction provides detailed new examples of minimal free resolutions, as well as a unifying view on several well-known examples.

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MS2

Determinantal Equations for Secant Varieties

We show that if a smooth variety X is re-embedded by a sufficiently large Veronese embedding of its original embedding then, set theoretically, the equations of the r th secant variety of X , are just the equations defining the r th

secant variety of the Veronese embedding of original projective space and the linear equations. This reduces the question of finding the equations of the secant variety of a (sufficiently amply embedded) variety to the finding of the equations of the secant variety of a Veronese embedding of a projective space. Time permitting, context and other results will be mentioned. This is joint work with J. Buczynski and J.M. Landsberg.

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MS2

Border Bases, Hilbert Scheme and Tensor Decomposition

The commutativity is a natural property that we expect in the context of algebraic geometry. Surprisingly, this simple property is also enough to characterize the solution of several problems. In this talk, we will show how it is intrinsically related to the description of the Hilbert scheme of points and to the tensor decomposition problem. We will analyze the correspondence between these two problems and detailed different characterisation of ranks of a tensor. Revisiting an approach of J.J.Sylvester for the decomposition of a binary forms, we will describe an algorithm to decompose general tensors, based on techniques related to border basis computation and truncated moment problems. Some examples will illustrate the approach.

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MS2

Decomposition of Tensors of Small Rank

We discuss some algorithms to decompose a tensor in $A \otimes B \otimes C$ as a sum of decomposable tensors. The technique is connected to the equations of the secant varieties, which are known only when the rank is small. The results are better in the symmetric case. Some sufficient conditions which guarantee the uniqueness of the decomposition will be presented.

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MS3

Polynomials and Optimization in Computer Vision

Multiview geometry is the study of planar images of points in space, an essential basis for computer vision. I will describe the algebraic objects in this field. These include the multiview variety associated to a fixed set of cameras – along with an explicit universal Groebner basis for its ideal – and also a Hilbert scheme parametrizing all such multiview ideals. This data feeds into the polynomial optimization problem of triangulating a point from noisy image observations.

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MS3

Lossless Representation of Graphs using Distributions

Complex structures such as composite objects or networks are often represented by weighted graphs. Since many signal processing and analysis techniques require the input to take the form of a vector, it is desirable to find ways to represent such weighted graphs as vectors. For reasons that will be explained in the talk, it is important to make sure that any invariance of the structure under a relabelings of the node carry over to the vector representation scheme chosen. In this talk, I will discuss how one can use distributions of certain functions of the weights of the graph to represent the "vast majority" of graphs without losing any information about the structure of the graph.

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MS3

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MS3

A Categorical Approach to Agent Networks

In this research, we continue the work of Lawvere and Giry in developing a categorical approach to conditional probabilities. Specifically, we investigate categorical properties of the Kleisli category of the Giry monad with the aim of constructing relations. Our goal is to then use this categorical framework to model sensor networks in an abstract, relational and probabilistic way.

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MS3

Lower Bounds for the Gromov-Hausdorff distance Using Persistent Topology

Viewing shapes as metric spaces (or metric measure spaces) permits dealing with different problems in object matching under deformations. The Gromov-Hausdorff distance provides one possible measure of dissimilarity between shapes, but its computation leads to NP-hard problems. We will discuss a family of lower-bounds for the GH distance that

involve the comparison of certain invariants arising from persistent topology constructions. The computation of these lower bounds can be done in polynomial time.

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MS4

Toric Ideals of Hypergraph Models

This talk is about models parametrized by edges of a hypergraph. Such models are toric by definition, and the goal is to obtain a Markov basis for the model. Equivalently, we are interested in the defining ideal of the edge subring of a hypergraph. The ideal of the edge subring of a *graph* gives a Markov bases for the p1 model for random graphs (networks). The algebraic and combinatorial constructions used for these toric ideals give insight into the geometry of the model, existence of maximum likelihood estimators, and a better way to generate Markov moves. The ideal theory of graphs has a rich history, including some results that are well-known in algebraic statistics. A more complicated family of algebraic models motivates the generalization of this construction to hypergraphs. The talk will outline the motivation and the main results.

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MS4

The Polytope of Degree Sequences and Maximum Likelihood Estimation

In this talk, we show how the polytope of degree sequences can be used to characterize nonexistence of the maximum likelihood estimator (MLE) in two statistical models: the beta model for random undirected graphs with fixed degree sequences and the Rasch model from item response theory. We derive new asymptotic results about the existence of the MLE in the random graph case and describe numerically feasible ways for checking for existence of the MLE and for computing the associated facial sets.

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MS4

An Algebraic Statistics Approach to the Ecological Inference Problems

We present an algebraic computational framework that handles special cases of missing data problems. We focus

on the ecological inference problem where we use aggregate (ecological) data to infer discrete individual-level relationships of interest when individual-level data are unavailable. The proposed method can handle multi-dimensional contingency tables, deterministically respects bounds and can incorporate information from many different sources. We illustrate how the analysis can be done with the recently developed R-4ti2 package.

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MS4

Estimating the Number of Binary Multi-way Tables via Sequential Importance Sampling

In 2005, Chen et al introduced a sequential importance sampling (SIS) procedure to analyze binary two-way tables with given fixed marginal sums (row and column sums) via the conditional Poisson (CP) distribution. They showed that compared with Markov chain Monte Carlo (MCMC)-based approaches, their importance sampling method is more efficient in terms of running time and it also provides an easy and accurate estimate of the total number of contingency tables with fixed marginal sums. In 2007, then, Chen showed also that we can generalize it to binary two-way tables with structural zeros. In this talk we will review their results and then we will show that we can extend their result to binary multi-way (d -way, $d \geq 2$) contingency tables under the no d -way interaction model, i.e., with fixed $d - 1$ marginal sums.

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MS5

Computing Vanishing Ideals with the Help of Multivariate Polynomial Interpolation

In this paper, the problem of computing vanishing ideals is investigated with the help of multivariate polynomial interpolation. We generate certain subsets from the given point set whose associated interpolation bases are obtained subsequently in theory, with which the processes of known algorithms for vanishing ideals can be accelerated to some extent, according to the sizes of the subsets. Algorithms are implemented and experimental timings are given.

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MS5

The Role of Algebraic Geometry in Geometric Modeling

Algebraic Geometry and Geometric Modeling both deal with curves and surfaces generated by polynomial equations. Algebraic Geometry studies theory, algorithms, and computation; Geometry Modeling investigates applications

of polynomial, piecewise polynomial, and rational curves and surfaces to industrial design and manufacturing. The purpose of this talk is to elucidate the role that Algebraic Geometry plays in Geometric Modeling. We will discuss several topics of current research, including elimination theory, syzygies, and the analysis of singularities.

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MS5

Wachspress Varieties

We will examine an algebraic variety parametrized by barycentric coordinates of a polygon. We will make use of the combinatorics of the polygon to understand the geometry of this variety. In addition, we will observe that this variety can be seen as the blowup of certain points in the projective plane.

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MS5

Splines, Spectral Sequences, and Polyhedral Complexes

Piecewise polynomial functions on a polyhedral complex (splines) are fundamental objects in mathematics, with applications ranging from approximation theory and numerical analysis to algebraic geometry, where they appear as the equivariant Chow ring of a toric variety. In joint work with T. McDonald [Advances in Applied Math, 2009], we found an (asymptotic) version of the Alfeld-Schumaker dimension formula for planar splines. I will also describe recent work [Trans. A.M.S, to appear] which uses the Cartan-Eilenberg spectral sequence to obtain results on splines on polyhedral complexes of dimension greater than two.

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MS5

Multivariate Interpolation and Algebraic Geometry

I will describe connections between multivariate interpolation and algebraic geometry and present two counterexamples to problems in the field. First is to a conjecture of Carl de Boor regarding the existence of certain error formula for multivariate interpolation. Second is to a conjecture of Tomas Sauer regarding restrictions of interpolation projector to polynomials of lesser degree.

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MS6

Persistence Based Signatures for Metric Spaces

We introduce a family of signatures for compact metric spaces, possibly endowed with real valued functions, based on the persistence diagrams of suitable filtrations built on top of these spaces. We prove the stability of these signatures under Gromov-Hausdorff perturbations of the spaces. We illustrate their use through an application in shape classification.

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MS6

Rigid Rips Complexes and Topological Data Analysis

In recent years, persistence diagrams have become an integral part of topological data analysis, providing a tool to measure the scale of topological features. From the theoretical point of view, persistence diagrams can be seen as effectively computable invariants of metric spaces. A classical way to construct a persistence diagram is by way of the so-called Rips complexes. In this paper we define a 1-parameter family of simplicial complexes which we call *rigid Rips complexes*. This family includes the regular Rips complex at infinity and yields a 1-parameter family of persistence diagrams - the data which we call *rigid persistence*. In addition to measuring the scale of topological features, this family provides a tool to analyze the evolution of these features when the rigidity changes. While rigid persistence contains the regular persistence, we give an example showing that the reverse of that statement is not true, and that rigid persistence contains strictly more information than the regular one. (joint with E.-M. Feichtner, J. Lehmann)

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MS6

Topology, Geometry and Statistics: Merging Methods for Data Analysis

Dimension reduction and shape description for scientific data sets are difficult problems, ones that continue to grow in importance within the statistical, mathematical and computer science communities. Powerful new methods of Topological Data Analysis and Diffusion Geometry have emerged in the last 10 years, and these have added significantly to the data analysis toolbox. In this talk we will give an overview of these methods and describe some early efforts to make them work together with Statistics. In particular we will discuss how random walk methods give distance functions that enhance topological features, and how one goes about defining the mean and variance of a collection of persistence diagrams.

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MS6

Using Persistent Homology to Compute Probabilistic Failure of a Sensor Network

We consider the question of sensor network coverage for a

2-dimensional domain. We seek to compute the probability that a set of sensors fails to cover given only non-metric, local (who is talking to whom) information and a probability distribution of failure of each node. This builds off the work of de Silva and Ghrist who analyzed this problem in the deterministic situation. We first show that the problem as stated is $\#P$ -complete, and thus fast algorithms likely do not exist unless $P=NP$. We then give a deterministic algorithm which is feasible in the case of a small set of sensors or at least a set of sensors which is not very dense, and discuss other methods to approximate the probability of failure. These algorithms build on the theory of topological persistence.

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MS6

Topology of Spaces of Micro-images and Applications

One of the most celebrated success stories in topological data analysis is perhaps that of the Klein bottle, as a model space for relevant 3×3 patches coming from natural images. I will explore in this talk how the Klein bottle model can be extended to include other meaningful patches, and describe an application to texture discrimination.

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MS7

Riemann Theta Functions: An Applied Introduction

Riemann's theta function is the fundamental building block for function theory on compact Riemann surfaces and their Jacobians. In this talk I will present an incomplete overview of different applications where theta function play an important role.

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MS7

Tropical Curves, Tropical Theta Functions and Integrable Systems

Tropical geometry is the combinatorial algebraic geometry over the min-plus algebra. For the last dozen years, tropical geometry has been rapidly developed and widely applied not only in complex algebraic geometry, but also in mathematical physics. The aim of this talk is to introduce an application of tropical curve theory to an integrable piecewise-linear map which originates from the Toda lattice equation. First we recall basic notions of tropical curves, tropical Jacobian and tropical theta functions by following the work of Mikhalkin and Zharkov in 2006. Second, a tropical version of Fay's trisecant identity is formulated. As a special case of this identity, we finally obtain a general solution to the piecewise-linear map. (This talk is based on the collaborative works with T. Takenawa.)

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MS7

Algebraic Curves and Riemann Surfaces in Matlab

We present a fully numerical approach to Riemann surfaces defined via plane algebraic curves. The code in Matlab computes for a given algebraic equation in two variables the branch points and singularities, the holomorphic differentials and a base of the homology. The monodromy group for the surface is determined via analytic continuation of the roots of the algebraic equation on a set of contours forming the generators of the fundamental group. The periods of the holomorphic differentials are computed with Gauss-Legendre integration along these contours. The Abel map is obtained in a similar way. The performance of the code is illustrated for many examples. As an application we study quasi-periodic solutions to certain integrable partial differential equations.

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MS7

Thetanulls of Algebraic Curves of Genus 3 (A Preliminary Report)

In this talk we will discuss thetanulls of genus 3 hyperelliptic curves. The case of genus 3 hyperelliptic curves has been studied by Shaska/Wijesiri and relations among them have been determined for cyclic curves of small genus. In this talk the more general case for genus 3 will be discussed.

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MS7

Determinantal representations, Cauchy kernels, and theta functions

Determinantal representations of plane algebraic curves were the object of numerous studies during the last century and a half. They became important for optimization in the last decade since representing a convex set as a feasibility set for semidefinite programming boils down to constructing a positive symmetric determinantal representations (for certain multiples of the Zariski closure of the boundary). One can obtain explicit formulae for determinantal representations using theta functions. These formulae are closely related to the famous trisecant identity of Fay. Similarly to Fay's identity, these formulae can be easily derived tracing the poles and residues of various expressions involving Cauchy kernels for line bundles on a compact Riemann surface. Here the Cauchy kernel is a section in two arguments and is holomorphic except for a simple pole with residue one along the diagonal; it can be easily written down using theta functions (and the prime form). The purpose of the talk is to describe the various interrelations between the different objects: determinantal representations, theta functions, and Cauchy kernels. Many of the results are based on joint work with Daniel Alpay, Joe Ball, Bill Helton, and Dmitry Kerner.

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MS8

Closed Form Solutions of Difference Equations

In this talk we present an algorithm that finds closed form solutions for homogeneous linear recurrence equations. The key idea is transforming an input operator L_{inp} to an operator L_g with known solutions. The main problem of this idea is how to find a solved equation L_g to which L_{inp} can be reduced. To solve this problem, we use local data of a difference operator, that is invariant under the transformation.

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MS8

On the Structure of Compatible Rational Functions

A finite number of rational functions are compatible if they satisfy the compatibility conditions of a first-order linear functional system involving differential, shift and q -shift operators. We present a theorem that describes the structure of compatible rational functions. The theorem enables us to decompose a solution of such a system as a product of a rational function, several symbolic powers, a hyperexponential function, a hypergeometric term, and a q -hypergeometric term. We outline an algorithm for computing this product, and present an application.

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MS8

The $Q=0$ Expansion of Q -Holonomic Sequences

I will discuss recent results on the degree, leading term and $q=0$ expansion of a q -holonomic sequence of polynomials. The results are of interest in Knot Theory and Quantum Topology, as well as the physics of fivebranes.

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MS8

Restricted Lattice Walks and Creative Telescoping

We start with a combinatorial problem: determining the generating functions of certain restricted lattice walks. This can be done with support from computer algebra. The most efficient techniques are however not 'rigorous' (joint work with A. Bostan; FPSAC 2009). It is possible to make them rigorous by evaluating certain definite integrals via Zeilberger's method of creative telescoping. But the computational cost for doing so is quite high (ongoing joint work with A. Bostan, F. Chyzak, M. van Hoeij). This

leads to the general question whether creative telescoping can be done in a more efficient way. In the end, we sketch an idea for minimizing the cost for creative telescoping by using a Cylindrical Decomposition computation as a pre-processing step (ongoing joint work with S. Chen).

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MS8

Computing Belyi Maps

Let $L(y)=0$ be a Heun equation with no Liouvillian solutions. We consider the question of deciding if there exists a rational function f for which L can be solved in terms of $2F1(a,b;c;f)$. Here a,b,c are constants, and $2F1$ is the hypergeometric function. Computationally, the most non-trivial case occurs when f is a Belyi map. In a joint work with R. Vidunas, we have computed a complete table with all Belyi maps f that can occur in this context. It contains $48 + 366$ Belyi maps (48 parametric cases and 366 non-parametric cases). The degrees in this table reach up to degree 60, finding these Belyi maps and verifying completeness required new techniques.

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MS9

Report on the Recent Progress on the Study of Secant Defectivity of Segre-Veronese Varieties

In this talk, I report the recent progress on the secant defectivity of Segre-Veronese varieties. The primary goal of the talk is to present the conjecturally complete list of defective secant varieties of Segre-Veronese varieties with two factors. If time permits, I will discuss defective secants to Segre-Veronese varieties with three or more factors. This talk is based on joint work with Chiara Brambilla.

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MS9

Kruskal's Uniqueness Inequality is Sharp

Kruskal proved that a tensor in $V_1 \otimes V_2 \otimes \cdots \otimes V_m$ of rank r has a unique decomposition as a sum of r pure tensors if a certain inequality is satisfied. We will show the uniqueness fails if the inequality is weakened.

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MS9

A Proof of the Set-theoretic Version of a Salmon Conjecture

We show that the irreducible variety of $4 \times 4 \times 4$ complex

valued tensors of border rank 4 at most is given by polynomial equations of degree 5, (the Strassen commutative conditions), of degree 6, (the Landsberg-Manivel polynomials), and of degree 9, (the symmetrization conditions).

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MS9

Tensor Decomposition and Dimensions of Secant Varieties of Segre Varieties

I will discuss joint work with M.V. Catalisano (Genoa) and A. Gimigliano (Bologna) giving the dimensions of ALL the secant varieties of the Segre Varieties $P^1 \times \cdots \times P^1$. This solves the problem of finding the number of summands in the decomposition of a general tensor in $V \otimes \cdots \otimes V$ (any number of factors) when $\dim V = 2$. Similar results for the cases when V has higher dimension will also be mentioned.

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MS9

Representations with Finitely Many Orbits

In this talk I will report on the joint work with Witold Kraskiewicz on singularities of orbit closures in the representations of reductive groups with finitely many orbits. We are interested in Cohen-Macaulay, Gorenstein and rational singularities properties as well as in defining ideals of the orbit closures.

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MS10

Computing Explicit Optimal Value Functions by a Symbolic-numeric Cylindrical Algebraic Decomposition

We consider to solve parametric optimization problems. Our purpose is to compute optimal values as a function of the parameters, which is called an optimal value function. This is very useful for solving reactive/online optimizations, dynamic programming, hierarchical optimizations and so on. Thus constructing explicit optimal value functions is very important and has many applications in engineering. We present an efficient method, which is based on a specialized cylindrical algebraic decomposition using symbolic-numeric computation, to construct optimal value functions explicitly for polynomial parametric optimization problems. Moreover, we also show several practical application examples in control.

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MS10

Cylindrical Algebraic Decomposition without Aug-

mented Projection

Cylindrical algebraic decomposition is a fundamental tool in computational real algebraic geometry. It takes a set of multivariate polynomials and decomposes the real space into cylindrically arranged cells which can be described by boolean expressions of polynomial equations and inequalities. It begins by repeated projection (elimination of variables) and follows by repeated lifting. During the lifting, the descriptions of cells are constructed using Thoms lemma. This necessitates the so-called augmented projection (the derivatives are also considered). It often makes the projection time-consuming and in turn increases the number of cells. In this talk, we show how to avoid augmented projection by describing cells using Sturm-Habicht theorem (instead of Thoms lemma).

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MS10**Composing Hybrid Automata**

We address the problem concerning the decidability of certain hybrid automata that may be built hierarchically as is the case in many exemplar systems, be it natural or engineered. Parallel composition can be considered a fundamental tool in such constructions. Somewhat surprisingly, this operation does not always preserve the decidability of reachability problem i.e., even if we prove the decidability of reachability over component automata, we cannot guarantee the decidability over their parallel composition. Despite these limitations, it is still possible to provide a reduction for the reachability problem over parallel composition of first-order constant reset automata (FOCoRe) to the satisfiability of a particular linear Diophantine system. Moreover, by proving that such satisfiability problem is decidable for systems with semi-algebraic coefficients, this paper presents an interesting class of hybrid automata for which the reachability problem of parallel composition is decidable. The resulting hybrid automata appear in systems biological modeling, and hence could be applied when one is interested in understanding a complex biological system composed of smaller self-organizing systems. Joint work with Alberto Casagrande, Pietro Corvaja, and Carla Piazza, expanding on some earlier preliminary results.

Bud Mishra

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MS10**The Critical Point Method into Practice : State of the Art and Perspectives**

The Critical Point Method has been designed initially to provide an algorithmic alternative to Cylindrical Algebraic Decomposition for computing sample points in semi-algebraic sets or for performing quantifier elimination over the reals. One feature is that it provides a framework to obtain algorithms running in asymptotically optimal degree bounds. In this talk, I will present a new algorithm for computing sample points in semi-algebraic sets defined by equations and inequalities when the set of equations satisfies some regularity assumptions. Practical experiments show that it outperforms previously known algorithms on some examples. Some preliminary results on its complexity

will be given also.

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MS10**A Divide-and-conquer Algorithm for Roadmap Computation**

We consider the problem of constructing roadmaps of real algebraic sets. This problem was introduced by Canny to answer connectivity questions and solve motion planning problems. Given s polynomials with rational coefficients, of degree D in n variables, Canny's algorithm has a Monte Carlo cost of $s^n \log(s) D^{O(n^2)}$ operations in \mathbf{Q} . An improvement by Basu, Pollack and Roy has a deterministic cost $s^{O(n)} D^{O(n^2)}$, for the more general problem of computing roadmaps of a semi-algebraic set. We recently obtained a probabilistic algorithm of cost $(nD)^{O(n^{1.5})}$ for the problem of computing a roadmap of a compact and smooth hypersurface of degree D . In this talk, we show how to improve this algorithm using divide-and-conquer techniques, and extend it to compact and smooth algebraic sets. This requires us to introduce, and control the degree of, multi-homogeneous systems corresponding to the iterated introduction of Lagrange multipliers.

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MS11**Extensions and Applications of Parametric Alignment**

We review extensions and applications of parametric sequence alignment, including recently published and submitted works. Parametric alignment (or RNA folding, etc.) gives rise to sum-product algorithms over semirings of polytopes. Generalizing parametric alignment to the k-best alignment setting leads to k-set polytopes and related objects. General results about lattice polytopes give best-known output complexity and running time bounds for several extensions of parametric alignment and related problems.

Peter Huggins

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MS11**Algebraic Models in Systems Biology**

Progress in systems biology relies on the use of mathematical and statistical models for system level studies of biological processes. Several different modeling frameworks have been used successfully, including traditional differential equations based models, a variety of stochastic models, agent-based models, and Boolean networks, to name some common ones. This talk will focus on several types of discrete models, and will describe a common mathematical approach to their comparison and analysis, which relies

on computational algebraic geometry. A software package, Analysis of Dynamic Algebraic Models (ADAM) will be presented.

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MS11

A Semi-algebraic Description of the of the General Markov Model on Phylogenetic Trees

The phylogenetic variety for the general Markov model (GM) on a phylogenetic tree is the closure of the image of a parameterization map over the complex numbers of this k-state substitution model. Since the variety arises from a closure operation over the complex numbers, there are points in the variety that satisfy the defining polynomials, but do not correspond to real stochastic parameters for the model. Therefore a semi-algebraic description of the image is needed. We begin with the case of 3-dimensional tensors and 2 states and describe several approaches for giving the semi-algebraic conditions that a three dimensional tensor is in the image of the real parameterization map for the GM model. Then we discuss generalizations of this description to more leaves and more states.

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MS11

Reverse Engineering of Regulatory Networks Using Algebraic Geometry

Discrete models have been used successfully in modeling biological processes such as gene regulatory networks. When certain regulation mechanisms are unknown it is important to be able to identify the best model with the available data. In this context, reverse engineering of finite dynamical systems from partial information is an important problem. In this talk we will present a framework and algorithm to reverse engineer the possible wiring diagrams of a finite dynamical system from data. The algorithm consists on using algebraic sets to encode all possible wiring diagrams, and choose those that are minimal using the irreducible components.

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MS11

Why Do We Care about Inequalities in (Algebraic) Statistics?

Graphical models with hidden nodes usually have a complicated description involving inequality constraints. A standard approach in algebraic statistics is to ignore inequal-

ities which allows to use classical algebraic tools without referring to the real algebraic geometry. This led for example to the method of phylogenetic invariants. Only recently some effort has been put to understand the full semi-algebraic structure of graphical models with hidden data. In my talk I will make a small step back trying to explain why do we care about inequality constraints. I will show how they may improve our understanding of statistical inference. My analysis is very simplistic and focuses on the simplest naive Bayes model. However, the main ideas will generalize to more complicated models.

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MS12

Algebraic Geometry for Analysis of Microscope Images

This talk will describe potential applications for algebraic geometry in the analysis of microscope images.

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MS12

Signal Registration via Polynomial System Solution Method and Image Recognition via Moment Method

We will introduce a new method for solving over-constrained polynomial systems of equations that can be applied to register a query object in 1D or 2D onto a corresponding template. Also we will introduce the "shape dictionary", which is set up by extracting a set of moment features from the new moments we define and can be applied in image recognition.

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MS12

Finding the Non-reconstructible Locus

Given the unordered collection of the pairwise distances of a finite point configuration in affine space, Boutin and Kemper proved that one can recover the configuration up to Euclidean motion from these distances if the configuration is generic. In special cases, non-equivalent configurations may give the same distance set, but these non-reconstructible cases are contained in an algebraic hypersurface as Boutin and Kemper show also.

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MS12**Object-image Correspondence Under Projections**

We provide criteria for deciding whether a given planar curve is an image of a given spatial curve, obtained by a parallel or central projection with unknown parameters. These criteria reduce the projection problem to a certain modification of the equivalence problem of planar curves under affine and projective transformations. The latter problem can be addressed using Cartan's moving frame method. The computational advantage of the method presented here, in comparison to algorithms based on a straightforward solution, lies in a significant reduction of a number of real parameters that has to be eliminated in order to establish existence or non-existence of a projection that maps a given spatial curve to a given planar curve. The same technique can be applied to solve object image correspondence for finite lists of points.

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MS12**Invariant Histograms and Signatures for Object Recognition and Symmetry Detection in Images**

I will survey recent developments in the use of group-invariant histograms, using distances, areas, etc., and signatures, using differential invariants, joint invariants, invariant numerical approximations, etc., for object recognition and symmetry detection in images.

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MS13**Title Not Available at Time of Publication**

Abstract not available at time of publication.

Martin Avendano
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MS13**Bounding the Sum of Square Roots via Lattice Reduction**

Let k and n be positive integers. Define $R(n, k)$ to be the minimum positive value of

$$|e_1\sqrt{s_1} + e_2\sqrt{s_2} + \cdots + e_k\sqrt{s_k} - t|$$

where s_1, s_2, \dots, s_k are positive integers no larger than n , t is an integer and $e_i \in \{1, 0, -1\}$ for all $1 \leq i \leq k$. It is important in computational geometry to determine a good lower and upper bound of $R(n, k)$. In this talk we will present an algorithm to find lower bounds based on lattice reduction algorithms. It produces lower bounds much better than the root separation technique does.

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MS13**Chamber Cones and Fast Solving over Local Fields**

Abstract not available at time of publication.

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MS13**Symbolic-numeric Methods for Near-singular Systems**

In this talk we will discuss recent developments on handling multivariate polynomial systems which are near singular ones, using hybrid symbolic-numeric methods. First we will consider structure preserving iterative methods which find the distance to- and the nearest element on certain discriminant varieties. Second, we will consider methods that turn ill-conditioned systems (with clusters of roots) into well-conditioned ones such that the roots of the new system are at the center of gravity of the clusters.

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MS14**Morse Theory in Topological Data Analysis**

We introduce a method for analyzing high-dimensional data. Our approach is inspired by Morse theory and uses the nudged elastic band method from computational chemistry. As output, we produce an increasing sequence of cell complexes modeling the dense regions of the data. We test the method on data sets arising in social networks, in image processing, and in microarray analysis, and we obtain small cell complexes revealing informative topological structure.

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MS14**Expansion of Random Simplicial Complexes**

Expander graphs are widely studied in theoretical computer science and discrete mathematics and have now found a host of applications. There has been recent interest in defining and exhibiting higher-dimensional analogues of expander graphs, by Gromov and others. I will discuss recent joint work with Dotterrer, where we show that many kinds of random cell complexes satisfy a certain "coboundary expansion" property which is a generalization of edge expansion of graphs to higher dimensions. Time permitting, I will also discuss recent work with Babson, Hoffman, and Paquette on the threshold for Kazhdan's Property T in the fundamental groups of random complexes, and how

this ties in with the expansion picture.

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Dominic Dotterrer

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MS14

Alexander Duality for Functions

This work contributes to the point calculus of persistent homology by extending Alexander duality to real-valued functions. Given a perfect Morse function $f : S^{n+1} \rightarrow [0, 1]$ and a decomposition $S^{n+1} = U \cup V$ such that $M = U \cap V$ is an n -manifold, we prove elementary relationships between the persistence diagrams of f restricted to U , to V , and to M .

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MS14

The Optimality of the Interleaving Distance on Multidimensional Persistence Modules

Building on an idea of Chazal et al. [1], we define and study the interleaving distance, a pseudometric on multidimensional persistence modules generalizing the bottleneck distance. We present several results about the interleaving distance. Our main result is that when the underlying field is \mathbb{Q} or $\mathbb{Z}/p\mathbb{Z}$ for a prime p , the interleaving distance is (in a certain sense) optimal. This result is new even for 1-D persistence. As a byproduct of our results, we obtain a converse to the algebraic stability theorem of [1]. This answers a question posed in that paper.

Reference

[1] Chazal, Cohen-Steiner, Glisse, Guibas, and Oudot. Proximity of persistence modules and their diagrams.

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MS14

Algebraic Well Groups

Given a mapping $f : X \rightarrow M$ from a topological space to an orientable manifold and U a subset of M , we identify a subgroup of the homology of $f^{-1}(U)$ that is stable to homotopic perturbations of the mapping.

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MS15

Theta Functions, Curves and Monopoles

The modern approach to integrability proceeds via a Riemann surface, the spectral curve, and solutions of the integrable system may be built from theta (and allied) functions on the curve. In many applications this curve is specified by transcendental constraints in terms of periods and implementing these requires a good understanding of the curve, for example its homology and period matrix. Computational algebraic geometry facilitates this. In some cases, including the construction of magnetic monopoles, physical symmetries are inherited by the spectral curve and there may be consequent simplifications of both the function theory and transcendental constraints. We shall look at this interplay of ideas.

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MS15

Singularities of Theta Divisors

Abstract: We will survey the current state of knowledge in complex algebraic geometry of the structure of the singularities of the theta divisor of a principally polarized abelian variety. We will discuss the dimension of the singular locus of the theta divisor, the multiplicity of points on it, the rank of the tangent cone, and other recent results and the remaining classical open questions.

Samuel Grushevsky

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MS15

Some Computational Problems Using Riemann Theta Functions in Sage

Computational tools in algebraic geometry are useful for generating new conjectures and providing a means for solving problems in applications such as optimization and integrable systems. Recent features in Sage for performing computations with Riemann theta functions and algebraic curves provide steps towards solving a large class of these problems. In this talk we will discuss current and future developments in Sage for computational algebraic geometry and examine two applications in particular: generating genus two and three solutions to the Kadomtsev–Petviashvili equation and computing determinantal representations of homogenous plane curves.

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MS16

Asymptotics of D-finite Sequences and the Cheetah Algorithm

This talk tackles the following problem:

input : recurrence of a D-finite sequence

output: asymptotics $a_n \sim CA^n n^\alpha$ with closed forms for C, A, α . Several experimental approaches are existing (e.g. packages by Doron Zeilberger, Manuel Kauers, Marc Mez-

zarobba & Bruno Salvy, see also recent work on G-functions by Tanguy Rivoal & Stéphane Fischler), but several problems remain difficult (choosing the right branch, singularity, etc) and to decide if $C = 0$ is conjectured by some people to be undecidable. We use here a mixture of symbolic approach (local behaviour of solution of linear differential equations, as well described by works of Frobenius, Fuchs...) and numerical analysis (the so-called cheetah algorithm) to get C for a lot of problems coming from number theory, physics, probability theory, combinatorics.

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MS16
Lattice Green's Functions of the Higher-Dimensional Face-Centered Cubic Lattices

We study the lattice Green's functions of the face-centered cubic lattice (fcc) in up to six dimensions. We give computer algebra proofs of results that were conjectured by Guttmann and Broadhurst for the four- and five-dimensional fcc lattices. Additionally we derive a differential equation for the six-dimensional fcc lattice, a result that was not believed to be achievable with current computer hardware. We also present some conjectures concerning the nature of the lattice Green's function in arbitrary dimensions.

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MS16
The Method of Brackets: A Heuristic Method for Integration

The method of brackets consists of a small number of rules that provide an effective procedure for the evaluation of many integrals appearing in Feynman diagrams. Examples and heuristic rules will be presented. This is joint work with Ivan Gonzalez, Karen Kohl and Armin Straub.

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MS16
Recent Progress in Automation of Asymptotics

It is well known that coefficient asymptotics depend mainly on the geometry of the algebraic surface $Q=0$. One of the challenges in moving from theorems that handle most cases in practice to automated asymptotics is to combinatorialize the geometric data. This talk concerns some of the infrastructure necessary to carry out the combinatorialization. A completely automated and rigorous algorithm for the smooth bivariate case is described in a forthcoming paper.

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MS16
Hypergeometric Evaluations of the Densities of

Short Random Walks

We consider random walks in the plane which consist of n steps of fixed length each taken into a uniformly random direction. Our interest lies in the probability density function of the distance travelled by such a walk. While Lord Rayleigh's limiting density is an excellent approximation for moderately large n , we seek closed forms for the densities in the case of small n . One of the goals of the talk will be to show that in the cases $n=3$ and $n=4$ hypergeometric evaluations can be given. The basic ingredients are combinatorial properties of the associated even moments, computer algebra as well as a surprising modularity of the densities. This is joint work with Jonathan M. Borwein, James Wan, and Wadim Zudilin.

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MS17
Spectrahedra and Determinantal Representations

Spectrahedra are the feasible sets of semidefinite programming. It is an important task to classify spectrahedra. The algebraic problem behind this question is to write polynomials as determinants of linear matrix polynomials. I will discuss recent results concerning this problem. In particular I will show how such determinantal representations are closely linked to sums of squares decompositions of an associated Hermite matrix. The results are joint work with Daniel Plaumann and Andreas Thom.

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MS17
Jacobian SDP Relaxation for Polynomial Optimization

Consider the optimization problem of minimizing a polynomial function subject to polynomial equalities and/or inequalities. Jacobian SDP Relaxation is new type semidefinite programming method that would solve the problem exactly. Its basic idea is to add some new polynomial equalities from the Jacobian of the polynomials, and then apply the hierarchy of Lasserre type sum of squares relaxations. The main result is that if we apply a sufficiently high relaxation order, then the relaxation will be exact and global optimal solutions for the original polynomial optimization could be obtained.

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MS17
Positivity of Piecewise Polynomials

Real algebraic geometry provides certificates for the positivity of real polynomials under polynomial constraints by expressing them as a suitable combination of sums of squares and the defining inequalities. We show how Putinar's theorem for strictly positive polynomials on compact sets can be applied in the case of strictly positive piecewise polynomials on a simplicial complex. In the 1-dimensional case, we improve this result to cover all non-negative piece-

wise polynomials and give explicit degree bounds.

Daniel Plaumann

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MS17

Semidefinite Programs with Rank Constraints

In 1953 Grothendieck worked on the theory of Banach spaces where he proved the fundamental theorem in the metric theory of tensor product, nowadays called Grothendieck inequality. This inequality is a fundamental and unifying tool in many areas of mathematics and computer science (functional analysis, combinatorics, machine learning, system theory, quantum information theory, numerical linear algebra). With hindsight one can view Grothendieck's inequality and its proof (which is algorithmic) as the first randomized approximation algorithm based on semidefinite programming. In the talk I want to extend Grothendieck's inequality so that it can be used to give approximation algorithms for finding ground states of the n -vector model in statistical mechanics. Grothendieck's inequality itself together with the best known constant (due to Krivine) gives a 0.56 approximation algorithm for the Ising model on the integer lattice. For the three-dimensional Heisenberg model the algorithm achieves a ratio of 0.78. (based on joint work with Jop Briet, Fernando de Oliveira Filho)

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MS17

Reformulating Polynomial Programs as Copositive Programs

We present a canonical convexification procedure which yields an equivalent formulation of polynomial programming problems as linear conic programs over the dual of the cone of copositive forms. This formulation is inspired by Burer's dual copositive formulation of binary quadratic programming, which can be recovered as a special case of our procedure. The convexification procedure is based on new certificates of non-negativity for polynomials over the intersection of an unbounded closed domain and the zero set of a given polynomial.

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MS18

Refined Bounds on Connected Components of Sign Conditions on a Variety

In this talk I will talk about a recent result bounding the

number of connected components of all realizable sign conditions of a family of real polynomials in \mathbf{R}^k , restricted to a real variety of dimension smaller than k . Unlike previous results in this direction, our bound distinguishes the roles of the degrees of the polynomials defining the variety from that of the other polynomials. This distinction appears to be crucial in certain applications – particularly, in certain problems of discrete geometry on bounding the number of incidences. I will also point out connections to previous algorithmic work that led to the new proof.

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MS18

Fast Estimates of Hankel Matrix Condition Numbers and Numeric Sparse Interpolation

We investigate our early termination criterion for sparse polynomial interpolation when substantial noise is present in the values of the polynomial. Our criterion in the exact case uses Monte Carlo randomization which introduces a second source of error. We harness the Gohberg-Semencul formula for the inverse of a Hankel matrix to compute in quadratic arithmetic time estimates for the structured condition numbers of all the arising Hankel matrices, and explain how false ill-conditionedness can arise from our randomizations. Finally, we demonstrate by experiments that our condition number estimates lead to a viable termination criterion for polynomials with about 20 non-zero terms and of degree about 100, even in the presence of noise of relative magnitude 0.00001. Joint work with Wen-shin Lee (Univ. Antwerp) and Zhengfeng Yang (ECNU, Shanghai)

Erich Kaltofen

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MS18

On the Computing Time of the Continued Fractions Method

The maximum computing time of the continued fractions method for polynomial real root isolation is shown to be at least quintic in the degree of the input polynomial. This computing time is realized for an infinite sequence of polynomials of increasing degrees, each having the same coefficients. This is the first non-trivial lower bound for the maximum computing time of the continued fractions method. Until recently such large computing times were not even observed. For each input polynomial under consideration, the continued fractions method recursively calls itself on polynomials that become ever harder to process. In the analysis, the coefficients of those polynomials are traced indirectly, by tracing images of the real and nonreal roots of the input polynomial under certain linear fractional transformations. The recursion tree is completely described; its height is more than half the degree of the input polynomial. The quintic lower bound is proven using a series of about eighty theorems and lemmas. The proof also uses well-known theorems that have not been used in computing time analyses before. Key words: Polynomial real root isolation, computing time lower bounds, symmetric functions, subadditivity, Fibonacci numbers, Mignotte polynomials, loxodromic transformations. This is a report

on recent joint work with G. E. Collins, Madison, WI.

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MS18

Connectivity in Semialgebraic Sets

A semialgebraic set is a subset of real space defined by polynomial equations and inequalities. A semialgebraic set is a union of finitely many maximally connected components. In this talk, we consider the problem of deciding whether two given points in a semialgebraic set are connected, that is, whether the two points lie in a same connected component. In particular, we consider the semialgebraic set defined by $f \neq 0$ where f is a given bivariate polynomial. The motivation comes from the observation that many important/non-trivial problems in science and engineering can be often reduced to that of connectivity. Due to its importance, there has been intense research effort on the problem. We will describe a method based on gradient fields and provide a sketch of the proof of correctness based Morse complex.

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MS18

A Divide-and-conquer Method for Computing Cylindrical Algebraic Decomposition

Cylindrical algebraic formulas (CAF) provide an explicit representation of semialgebraic sets as finite unions of cylindrically arranged disjoint cells bounded by graphs of algebraic functions. For a quantified system of polynomial equations and inequalities, cylindrical algebraic decomposition computes a CAF representation of its solution set. If the system is given as a Boolean combination of formulas, its cylindrical algebraic decomposition can be computed either directly, using the cylindrical algebraic decomposition (CAD) algorithm, or in two steps, by first finding a CAF representation of the solution set of each formula and then computing the required Boolean combination of the CAF representations using the CAFCombine algorithm of [1]. In my talk I will discuss the problem of deciding automatically which of the two methods to use and how to subdivide the input system into a Boolean combination of subformulas for the two-step method. I will describe how graph theory methods can be used for this purpose. [1] Computation with Semialgebraic Sets Represented by Cylindrical Algebraic Formulas, Proceedings of the International Symposium on Symbolic and Algebraic Computation, ISSAC 2010, 61-68, Munich, Germany, July 25-28, 2010. ACM, Stephen M. Watt, ed.

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MS19

Reachability Approach to the Persistence of Reaction Networks

tion Networks

For a reaction network, persistence is the property that no species tend to extinction if all species are initially present. We call vacuous persistence a stronger property: the same asymptotic feature when all species are implicitly present. We will present a necessary and sufficient condition for vacuous persistence in terms of reachability, describe two classes of vacuously persistent networks relevant to biochemistry, and relate our condition to known sufficient conditions for persistence.

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MS19

Identifiability of Species Phylogenies Under the Coalescent Model

A phylogenetic tree is a graph that displays evolutionary relationships among a collection of organisms. The sequence data available for phylogenetic inference often include samples taken from multiple genes within each organism. This necessitates modeling of the evolutionary process at two distinct scales. First, given an overall phylogeny representing the actual evolutionary history of the species, individual genes evolve their own histories, called gene trees. Then, along each gene tree, sequence data evolve, leading to the observed data that is used for inference. The coalescent model provides the link between the evolution of the gene trees given the species tree, and the evolution of the sequence data given the gene trees. Phylogenetic invariants have been proposed as a tool for inferring phylogenies using data from a single gene, and their mathematical properties have been widely studied. In this talk, we consider the development of methods based on phylogenetic invariants developed specifically for species trees, as opposed to gene trees. In particular, we use methods from algebraic statistics to establish identifiability of the species phylogeny.

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MS19

Algebraic Results on the Multispecies Coalescent Model

Phylogenetic models relate a gene tree to sequence data arising on it. To study evolutionary relationships between species, one also needs the coalescent to relate a species tree to the gene trees arising on it. Without strong assumptions, the 'time' units in these models are unrelated, so the distribution of topological gene trees is of interest. In this setting, we describe several recent results on identifiability of species trees which have used a viewpoint of algebraic geometry.

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MS19

New Sufficient Conditions for Ruling out Multiple Steady States in Chemical Reaction Systems

In a chemical reaction system, the concentrations of chemical species evolve in time, governed by the polynomial differential equations of mass-action kinetics. This talk concerns the problem of determining whether a chemical reaction network admits multiple steady states. In general, establishing the existence of (multiple) steady states is challenging, as it requires the solution of a large system of polynomials with unknown coefficients. However, for networks that have special structure, various easy criteria can be applied. This talk will highlight a new criterion for ruling out multiple steady states based on the Jacobian Criterion of Craciun and Feinberg, and will present work on a classification of small multistationary chemical reaction networks. Relevant examples from biochemistry will be given.

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MS20

Certification and Complexity in Solving Nonlinear Systems of Equations.

In this talk I will present some of the recent progress in the problem of solving systems of equations with two requirements: the solutions must be certified, and the complexity of the algorithms must be understood.

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MS20

Numerical Algorithms for Dual Bases of Positive-Dimensional Ideals

An ideal of a local polynomial ring can be described by calculating a standard basis with respect to a local monomial ordering. However if we are only allowed approximate numerical computations, this process is not numerically stable. On the other hand we can describe the ideal numerically by finding the space of dual functionals that annihilate it. There are several known algorithms for finding the truncated dual up to any specified degree, which is useful for zero-dimensional ideals. I present a stopping criterion for positive-dimensional cases based on homogenization that guarantees all generators of the initial monomial ideal are found. This has applications for calculating Hilbert functions.

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MS20

Applications of Numerical Algebraic Geometry to Pure Mathematics

This talk will survey known applications of the homotopy continuation methods and numerical algebraic geometry to the problems arising in several areas of pure mathematics. I will showcase our joint work with Frank Sottile on computing Galois groups of Schubert problems via homotopy continuation. Rigorous certification of results obtained with approximate numerical algorithms will be discussed as well.

Anton Leykin
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MS20

Symbolic-Numeric Algorithms for the Computation of Discrete Invariants

This talk will focus on the use of symbolic-numeric methods to compute discrete invariants of varieties, schemes, and sheaves.

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MS20

Applying Littlewood-Richardson Homotopies to Schubert Problems

Based on Ravi Vakil's geometric proof of the Littlewood-Richardson rule, we developed a numerical homotopy continuation algorithm for finding all solutions to Schubert problems on Grassmannians. For generic Schubert problems the number of paths tracked is optimal. The Littlewood-Richardson homotopy algorithm is implemented using the path trackers of the software package PHCpack.

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Ravi Vakil
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MS21

Search Bounds with Respect to Height in Linear and Quadratic Spaces

In this talk, I will discuss a variety of results on existence of points and subspaces of bounded height, possibly satisfying some additional algebraic conditions, in linear and quadratic spaces over global fields and rings. These results represent some of the recent developments on extensions and generalizations of such classical Diophantine theorems

as Siegel's lemma and Cassels' theorem on small zeros of quadratic forms.

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MS21

A Deterministic Polynomial Time Algorithm for Finding Roots of Polynomials Over Complex Numbers

We present an iterative algorithm that has global convergence, that is, for any univariate polynomial of degree n over complex numbers and any initial point, the algorithm will find a root of the polynomial, and the number of complex operations used is bounded by a polynomial in n and the number of digits for desired accuracy.

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MS21

Higher Mahler Measure and Lehmer's Question

The k -higher Mahler measure of a nonzero polynomial P is the integral of $\log^k |P|$ on the unit circle. I will discuss Lehmer's question for $k > 1$ and show some interesting formulae for 2- and 3-higher Mahler measure of cyclotomic polynomials.

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MS21

Title Not Available at Time of Publication

Abstract not available at time of publication.

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MS21

The Subset Sum Problem

This will be an introduction to some new results and problems on the subset sum problem, particularly those with additional algebraic structure.

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MS22

Hyperdeterminantal Varieties from Tensor Complexes

I will discuss a family of hyperdeterminantal varieties which include varieties cut out by maximal minors of a matrix as well as hypersurfaces given by hyperdeterminants of the boundary format. These varieties arise from an explicit construction of certain free resolutions, called tensor complexes.

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MS22

Markov Bases and Beyond

Describing Markov bases of toric statistical models has been recognized as a rich source of interesting combinatorial problems. We will give an overview over past and ongoing research in this area.

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MS22

Testing Chromosome Proximity Hypothesis Using Log-linear Models

Studying chromosome proximity and its effect on chromosome exchange plays important role in understanding of development of certain types of cancer. The proximity-effect hypothesis states that number of exchanges is larger between chromosomes that are located close to each other. To evaluate previously published data from numerous experiments involving 22 human autosomes of respectively three data sets of lymphocyte cells that have been subject to ionizing radiation we used tools of algebraic statistics together with asymptotic properties of log linear models. Specifically we used Markov basis of a log-linear model which does not have proximity-effect parameters, to sample from a large space of tables that have the same minimal sufficient statistics as the experiment data table. The Markov Chain Monte Carlo approach did not provide sufficient evidence to the reject null hypothesis of no proximity-effect. We considered a modified model, were proximity effect of individual pairs was included as an additional parameter. After using asymptotic tests we could not reject certain modified models.

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MS22**Primary Decomposition of a Class of Conditional Independence Ideals**

We describe (in a combinatorial way) the primary decomposition of a class of ideals arising in the context of conditional independence models. The ideals we consider generalize the ideals considered by Fink (2010) in a way distinct from that of Herzog, Hibi, Hreinsdottir, Kahle, and Rauh (2010). We give a combinatorial description of the minimal components, along with the corresponding prime ideals (they turn out to be the same, although there are embedded primes) of these conditional independence ideals. Along the way we introduce an equivalence relation and recover some other interesting algebra and geometry results as a consequence of the development of the proof of our main result.

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MS22**Pairwise Ranking: Choice of Method Can Produce Arbitrarily Different Order**

We showed that for any two of the three popular methods for ranking by pairwise comparison (HodgeRank, Tropical and Principal Eigenvector), and for any pair of rankings of at least four items, there exists a comparison matrix for the items such that the rankings found by the two methods are the prescribed ones. We discuss the implications, study the geometry and combinatorics, and state some open problems.

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MS23**The Berlekamp/Massey Algorithm and Counting Singular Hankel Matrices over a Finite Field**

We derive a formula for the number of singular $n \times n$ Hankel matrices over a finite field of q elements by observing the Berlekamp/Massey algorithm run on the entries, allowing some entries above or on the anti-diagonal to be fixed. We also derive a formula for the number of $n \times n$ Hankel matrices whose first r leading principal submatrices are non-singular and the rest are singular. This result generalizes to block-Hankel matrices as well.

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MS23**Exact Linear Algebra and Algebraic Topology**

We present exact linear algebra methods for the computation of homology groups of simplicial complexes and in algebraic K-theory for computation of the cohomology of the linear group $GL_7(Z)$. These computations are re-

alised through the computation of the ranks and of the integer Smith normal forms of the boundary matrices of the groups. We present novel methods for the efficient computation of these routines, specialized for the large and sparse matrices arising in these applications.

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MS23**Integral Cohomology of Some Arithmetic Groups**

The cohomology of a discrete group G is the cohomology of the quotient space X/G where X is any contractible space admitting a free action of G . This talk will describe an implemented algorithm for computing the integral cohomology of groups such as $PSL_4(Z)$, $Sp_4(Z)$ and $PSL_2(O)$ for various rings O of quadratic integers. The algorithm can also compute the cohomology of finite index subgroups of these three groups. The talk will discuss bottlenecks that arise when the homology degree or the subgroup index is large.

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MS23**Symbolic-numeric Linear Algebra and Lattice Basis Reduction**

LLL basis reduction is an important algorithm in computer science and mathematics making worthwhile efficiency improvements. Currently fastest reduction algorithms are using both symbolic and numeric linear algebra techniques. Revisiting and improving classical tools from the field of numerical analysis, we propose new reduction algorithms and certificate ([X.-W. Chang, D. Stehlé, G. Villard][A. Novocin, D. Stehlé, G. Villard]). We describe these developments, we show in particular how approximate computations may be introduced at various levels in the overall reduction process.

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MS23**Berlekamp/Massey: Implementations in LinBox**

The Berlekamp/Massey algorithm and its variants comprise a key component to many exact linear algebra methods. Often the computation of the minimal generator of a linearly generated sequence is a vital sub-procedure in methods like linear solving, smith forms and rank computations. A discussion of recent and current work on the implementation of this algorithm within the LinBox library will be given. The algorithm presents interesting choices for the implementation and data structures required during its computations. The different design choices, applications and computational results will be presented.

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MS24**Analysis Aware Representations, Parameterizations, and Models**

Isogeometric Analysis (IA) has been proposed as a methodology for bridging the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). In order to support design and full 3D IA, new ab initio design methods must create suitable representations and approximation techniques must include parameterization methodologies for volumes. This presentation discusses some of the challenges in moving from current representations and datafitting techniques towards this goal and demonstrates initial results and analyses.

Elaine Cohen

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MS24**Splines and Isogeometric Analysis (IGA)**

Finite Element Analysis (FEA) was already established in 1970 when Computer Aided Geometric Design (CAGD) community was established. Until the introduction of IGA by T. J. R. Hughes in 2005 little cooperation took place between the FEA and CAGD communities. IGA has the potential to drastically improve the interoperability of CAGD and FEA and the quality FEA. To achieve this we need improved technology and increased research into spline technology, FEA and visualization.

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MS24**Characterization of Hierarchical Spline Spaces**

The only requirement which characterizes the hierarchical model is a refinable nature of the underlying basis functions defined on nested approximation spaces. The talk will present recent results on the construction of normalized hierarchical spline bases and their properties.

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Bert Jüttler

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MS24**Locally Refined B-splines**

We will address local refinement of tensor product B-splines specified as a sequence of inserted line segments parallel to the knot lines. We obtain a quadrilateral grid with T-junctions in the parameter domain, and a collection of tensor product B-splines on this mesh here named an LR-mesh. The approach applies equally well in dimensions higher than two. Moreover, in the two dimensional case this collection of B-splines spans the full spline space on the LR-mesh.

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MS24**Splines on Rectangular Subdivisions: Dimension and Basis**

Standard parametrisations of surfaces in CAGD are based on tensor product bspline functions, defined from a grid of nodes over a rectangular domain. This representation is not well-adapted to local refinement, which is particularly important in applications such as surface reconstruction or isogeometric analysis. To extend the representation of tensor-product splines while providing local refinement capabilities, some recent works have considered T-subdivisions. These are partitions of an axis-aligned box into smaller axis-aligned boxes. In this talk, we will analyze the space of bivariate functions that are piecewise polynomial of bidegree (m, m') and class $C^{(r, r')}$ over such a planar T-subdivision. We shall show how algebraic techniques yield a good control of its resolution, give a new formula for its dimension and explicit bases for small degree and regularity.

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MS25**Reeb Graph Reconstruction and Zigzag Persistent Homology**

Reeb graph reconstruction is an important problem in topological data analysis. We study the error in reconstructing the Reeb graph of a point cloud sampled from a manifold, using the MAPPER algorithm. Using the language of zigzag persistence, we are able to succinctly capture the difference between the reconstructed and actual Reeb graphs. A worst case bound on the error is derived in terms of the sampling density. We demonstrate practical applications such as dataset classification.

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MS25**PHIsoMap: Estimating Intrinsic Distance Using Persistent Homology**

Many dimension reduction techniques (including IsoMap) start by constructing a graph on the data in order to estimate the geodesic distance from the underlying manifold. We present an algorithm that uses persistent homology (in dimensions one and zero) to construct an ϵ -proximity graph; our initial experiments show that this graph produces strikingly good estimates of the underlying geodesic

distance.

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MS25

Covering and Packing by Diagonal Distortion

We address the problem of covering R^n with congruent balls, while minimizing the number of balls that contain an average point. Considering the 1-parameter family of lattices defined by stretching or compressing the integer grid in diagonal direction, we give a closed formula for the covering density that depends on the distortion parameter. We observe that our family contains the thinnest lattice coverings in dimensions 2 to 5. We also consider the problem of packing congruent balls in R^n , for which we give a closed formula for the packing density as well. Again we observe that our family contains optimal configurations, this time densest packings in dimensions 2 and 3.

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MS26

Implicitization of Surfaces via Geometric Tropicalization

In this talk we discuss recent developments in tropical methods for implicitization of surfaces. This study was pioneered in the generic case by work of Sturmfels, Tevelev and Yu, and is based on the theory of geometric tropicalization, developed by Hacking, Keel and Tevelev, The latter hinges on computing the tropicalization of subvarieties of tori by analyzing the combinatorics of their boundary in a suitable (tropical) compactification. We enhance this theory by providing a formula for computing weights on tropical varieties, a key tool for tropical implicitization. Finally, we address the question of tropical implicitization for non-generic surfaces and illustrate our techniques with several numerical examples in 3-space.

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MS26

Singular Tropical Hypersurfaces

The concept of a singular point of a tropical variety is not well established yet. A natural definition is to declare a point q in a tropical variety V singular if q is the image under the valuation of a singular point of a classical algebraic variety, defined over the field of Puiseux series and with tropicalization V . We present a purely tropical characterization of a singularity of a tropical hypersurface of arbitrary dimension (with fixed support) in terms of tropical Euler derivatives. We also describe in this setting non-transversal intersections of planar tropical curves. We show an algorithm to compute all singularities and we relate our approach to the known tropicalization of the discriminant.

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MS26

Tropical Reparameterisations

It is well known that the tropicalisation of a polynomial map $\phi : A^m \rightarrow A^n$ maps $(R \cup \{\infty\})^m$ into the tropicalisation of the image X of ϕ . Typically this tropical map is not surjective. To remedy this, one may precompose ϕ with other polynomial maps and then tropicalise the result. Ideally, one would like to prove that finitely many such reparameterisations suffice to cover the entire tropicalisation of X . I will give an overview of when this is known to be the case.

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MS26

Computing Tropical Resultants

Tropical resultant varieties arise when fixing a set of Newton polytopes and asking for which set of coefficients their tropical hypersurfaces have a common intersection. In this talk we compare various methods for computing tropical resultants. Some of the algorithms involve traversing subfans of secondary fans while other works by reconstruction tropical hypersurfaces from projections. Using these techniques we also get a new method for computing mixed fiber polytopes.

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MS26

An Effective Computation of Tropical Linear Spaces, with an Application to A-discriminants

We develop and implement a very fast algorithm for computing tropical linear spaces, making some computational applications of tropical geometry now viable. For this purpose, we study a fan structure on the Bergman fan of a matroid which slightly refines its nested set structure, and which is amenable to our computational purposes. Some software implementations will be shown, including an application to the computation of A-discriminants.

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MS27

The Moment Problem for Continuous Linear Func-

tionals

Let V be the countable dimensional real polynomial algebra. Let $\tau\theta$ be a locally convex topology on V . Let K be a closed subset of \mathbf{R}^n , and let $M := M_{\{g_1, \dots, g_s\}}$ be a finitely generated quadratic module in V . We investigate the following question: *When is the cone $\text{Psd}(K)$ (of polynomials nonnegative on K) included in the τ closure of M ?* We give an interpretation of this inclusion with respect to representing continuous linear functionals by measures. We discuss several examples; we compute the closure of the cone of sums of squares with respect to weighted norm- p topologies. We show that this closure coincides with the cone $\text{Psd}(K)$ for certain convex compact K . We use these results to generalize Berg et al. work on exponentially bounded moment sequences. Joint work with M. Ghasemi and E. Samei

Salma Kuhlmann

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MS27**A New Look at Nonnegativity on Closed Sets and Polynomial Optimization**

We provide a new characterization of the convex cone C_K of degree- d polynomials that are nonnegative on a closed set $K \subseteq \mathbf{R}^n$, provided that one knows all moments of a finite Borel measure whose support is exactly K . From this we also provide a hierarchy of *outer* approximations (P_j) of C_K where each convex cone P_j is a spectrahedron described only in terms of the coefficients of the polynomials, i.e., with *no* lifting. As a by-product, with $K = \mathbf{R}_+^n$, we obtain converging outer approximations of the cone of copositive matrices. Finally, and also as a by-product, when K is a simple set like e.g., a box, an ellipsoid, the hypercube $\{-1, 1\}^n$, the positive orthant, or the whole space \mathbf{R}^n , etc., we provide a monotone nonincreasing sequence of upper bounds on the global minimum of any polynomial f on K . These upper bounds complement the sequence of lower bounds obtained from the moment-sos relaxations.

Jean B. Lasserre

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MS27**On the Inverse Moment Problem for Polytopes**

Reconstructing a measure from its moments is known as an inverse moment problem, and has important applications for data processing, e.g. in geophysics, image recognition, etc. The problem is well-studied in dimension 2, where tools of complex analysis are applicable. The basic in the approach there are procedures for expressing coordinates of the vertices $V(P)$ of a polygon P , given moments of a measure supported on P . Higher-dimensional problems are traditionally treated in an approximate way, by working with 2-D slices, and only lead to approximate solutions. We present efficient procedures for reconstructing polynomial measures $d\mu$, supported on convex d -dimensional polytopes P , given finitely many moments $\int_P X^a d\mu$, $a \in \mathbf{Z}_+^d$. Our procedures recover the vertices of P and μ , and can be used in exact (rational inputs, exact rational outputs) as well as in non-exact setting (inputs with errors, approximate outputs). We use tools from toric geometry, and symbolic-numeric computations with representations of $V(P)$ as uni-

variate rational functions.

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MS27**Using Symmetries in Polynomial Optimization**

Solving polynomial optimization problems is known to be a hard task in general. The semidefinite methods which evolved within recent years give possibilities to calculate these hard problems. However, as the sizes of the resulting SDPs grow fast, it is helpful to exploit some structure of the problem. We present approaches for exploiting symmetries within this framework. Our special focus is on problems which are invariant by the symmetric group.

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MS27**Successive Convex Relaxation Methods for Optimization over Semi-Algebraic Sets**

I will discuss various techniques (new and old) used in convergence proofs for successive convex relaxation methods applied to optimization problems expressed as maximization of a linear function over a finite set of polynomial inequality constraints. I will focus on convergence rates and specially structured non-convex optimization problems.

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MS28**Toward a Salmon Conjecture**

Methods from numerical algebraic geometry are applied in combination with techniques from classical representation theory to show that the variety of 3-by-3-by-4 tensors of border rank 4 is cut out by polynomials of degree 6 and 9. Combined with results of Landsberg and Manivel, this furnishes a computational solution of an open problem in algebraic statistics, namely, the set-theoretic version of Allman's salmon conjecture for 4-by-4-by-4 tensors of border rank 4. A proof without numerical computation was given

recently by Friedland and Gross.

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MS28

Geometry of Tensors and Numerical Decomposition

This talk will consider tensor parameter spaces from a geometric point of view. The study of tensors from the vantage point of algebraic geometry leads to symbolic and symbolic-numeric algorithms related to tensor decompositions, tensor approximations, and generic rank. Both a geometric description and basic implementations of several of these algorithms will be presented.

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MS28

Some non-minimal Canonical Representations of Forms as a Sum of Powers of Linear Forms

We will give some illustrations of the title phenomenon, especially Reichstein's beautiful "completion of the cube" from the late 1980s.

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MS28

Stochastic Models, Tensor Rank, and Inequalities

The hyperdeterminant Δ on $2 \times 2 \times 2$ tensors appears in a semialgebraic characterization of the probability distributions arising from binary models of evolution on phylogenetic trees, through its connection to tensor rank. To understand inequality constraints for non-binary models, we construct an explicit representation space of functions which, for $k \times k \times k$ tensors of border rank k , is analogous to Δ . In particular, their non-vanishing on a tensor of border rank k indicates it has rank k .

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MS28

Ranks and Generalized Ranks

The Waring rank of a polynomial of degree d is the least number of terms in an expression for the polynomial as a sum of d th powers. The problem of finding the rank of a given polynomial and studying rank in general is related to secant varieties, and there are applications throughout engineering and the sciences, such as in signal processing and computational complexity; and of course, it has been a central problem of classical algebraic geometry. For example, J.J. Sylvester gave a lower bound for rank in terms of catalecticant matrices in the mid-19th century. While catalecticant matrices and varieties have become objects of study in their own right, there has been relatively little progress in the last 160 years on the problem of bounding or determining the rank of a given (not general) polynomial. I will describe joint work with J.M. Landsberg which gives a new, elementary improvement to the catalecticant lower bound for the rank of a polynomial, in terms of the geometry of the polynomial. I will also describe new work which generalizes this improved bound to multihomogeneous polynomials, corresponding to secant varieties of Segre-Veronese varieties. A further generalization to the recently described Young flattenings (treating secant varieties of arbitrary varieties) is work in progress.

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MS29

A Practical, Principled Approach to Fast Simplification of Tarski Formulas

This talk focuses on the problem of false simplification of Tarski formulas, which means finding meaningful simplifications that occur in practice within a period of time that is small relative to most symbolic computations involving polynomial systems over the reals. An effective algorithm for fast simplification can be used to improve other algorithms, by pre-processing input, post-processing output, and, perhaps most importantly, by reducing the size of intermediate results. The talk will describe extensions of work I have previously published on the subject that, ultimately, provide a fast simplification procedure that is effective at improving the performance and quality of results for several other algorithms. The approach makes some interesting connections between a sub-class of simplifications on Tarski formulas and coding theory; a connection which is exploited in complexity analyses.

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MS29

Certified Global Optimization with Exact Sum-Of-

Squares

Given a multivariate polynomial f with real or rational coefficients, we focus on computing and certifying its lower bounds or its global infimum. The global optimization problem is NP-hard. A well known technique is to use the sum-of-squares(SOS) and semidefinite programming(SDP) to relax it to get lower bounds. We implement the SDP interior-point method in Maple and get high precision SDP results which can not be obtained by the fixed precision SDP solvers in matlab. Then we certify the lower bounds by making projections to get the exact SOS representations. To compute the global infimum, we use the results from the computation of sample points in the smooth real algebraic set, then relax the problem by computing its infima over some finite semialgebraic sets which can be reduced to SDPs. From Artin's affirmative solution of Hilbert's 17th problem, we know that a nonnegative polynomial can be written as a sums of squares of rational functions. Given a fixed degree of the denominators, we give the certification if the polynomial can not be represented as sums of squares of rational functions with denominators of the fixed degree. This is a joint work with Erich Kaltofen, Mohab Safey El Din, Zhengfeng Yang and Lihong Zhi.

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MS29

Multivariate Root Bound

Given a system F of n polynomials in n variables, a bound B is obtained such that if $F(x) = 0$, then $x_1, \dots, x_n \leq B$, for all real x . A root bound gives a useful starting point for various domain decomposition methods used to approximate the solution of systems of polynomials. Such a bound can also be used to estimate the complexity of root finding algorithms. We present a bound which is significantly tighter than those previously studied. The bound is obtained by integrating results from Elimination theory and Eigenvalue inclosure theory.

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MS29

On Hilbert's 16th Problem

At the start of the 20th century David Hilbert proposed 23 problems to be solved over the coming century. The first portion of his 16th problem asks about the arrangement of the connected components of nonsingular algebraic curves in the real projective plane. Despite progress over the last century, the problem has remained unsolved in a general sense. Efforts to resolve this problem have previously focused on constructions of maximal curves by hand through slight permutations of singular curves. Approaching the problem by hand unfortunately tends to produce polynomials with large coefficients and curves which are visually poor when plotted. Instead this talk will approach the problem from a computerized perspective after using grid processing to compute topologies for many generated polynomials. The statistical distribution of curve topologies by degree and coefficient range will be discussed as well as 'nice' witness polynomials for known topological cases.

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MS29

Computing Rational Points in Convex Semi-Algebraic Sets

Let $\mathcal{P} = \{h_1, \dots, h_s\} \subset Z[Y_1, \dots, Y_k]$, $D \geq \deg(h_i)$ for $1 \leq i \leq s$, σ bounding the bit length of the coefficients of the h_i 's, and Φ be a quantifier-free \mathcal{P} -formula defining a convex semi-algebraic set. We design an algorithm returning a rational point in \mathcal{S} if and only if $\mathcal{S} \cap Q^k \neq \emptyset$. It requires $\sigma^{O(1)} D^{O(k^3)}$ bit operations. If a rational point is outputted its coordinates have bit length dominated by $\sigma D^{O(k^3)}$. Using this result, we obtain a procedure deciding if a polynomial $f \in Z[X_1, \dots, X_n]$ is a sum of squares of polynomials in $Q[X_1, \dots, X_n]$. Denote by d the degree of f , τ the maximum bit length of the coefficients in f , $D = \binom{n+d}{n}$ and $k \leq D(D+1) - \binom{n+2d}{n}$. This procedure requires $\tau^{O(1)} D^{O(k^3)}$ bit operations and the coefficients of the outputted polynomials have bit length dominated by $\tau D^{O(k^3)}$. This is joint work with Mohab Safey El Din

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MS30

Using Fiber Products and Numerical Algebraic Geometry to Find Exceptional Mechanisms

An exceptional mechanism is a mechanism having special movement as compared to other mechanisms in the same class. These correspond to points in the parameter space of a given class, which have fiber dimension greater than the generic fiber dimension. Such points are usually obscured by their containment in an irreducible component. We use numerical algebraic geometry, starting with the fiber product method of Sommese and Wampler, to identify these points, thus finding exceptional mechanisms.

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MS30

Beyond Parametrization of Algebraic Varieties by Means of Global Optimization

This research work presents a new global optimization algorithm, Evo-Runge-Kutta, to tackle the open problem of

minimizing a positive function on an algebraic variety. We illustrate the application of Evo-Runge-Kutta, a two-phase evolutionary optimization algorithm using a multicriteria approach, for computing explicit and implicit Runge-Kutta methods. In particular, we present theoretical motivations for the application of the class of evolutionary algorithms: the hardness to find 1) the dimension of the variety and 2) feasible solutions for the parametrization. The mapping between algebraic geometry and global optimization is direct. In order to design methods having specific properties, we expect that many open problems in algebraic geometry will be modeled as constrained multi-objective optimization problems.

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MS30
Solving Principal Agent Problems by Polynomial Programming

We present a new way to solve principal agent problems by polynomial programming techniques. We study the case where the agent's actions are unobservable by the principal but the outcomes are. We assume that the agent's actions lie in an interval and the space of outcomes is a finite set. Furthermore the agent's expected utility is a rational function in his actions. The resulting problem is a bilevel optimization problem with the principal's problem as the upper and the agent's problem as the lower level. The key idea is to find an exact reformulation of the agent's problem as a semidefinite optimization problem. Since this is a convex optimization problem, we then have necessary and sufficient global optimality conditions for the agent's problem. The reformulation can be done by using classical results from real algebraic geometry linking positive polynomials and semidefinite matrices. We obtain a nonlinear program. If all functions are rational functions, we then can solve it to global optimality.

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MS30
Tumor Growth Models and Numerical Algebraic Geometry

Numerical Algebraic Geometry is being used to explore free boundary problems arising from tumor growth models. These methods are used to compute solutions, check their stability, and track the solutions through bifurcations as critical parameters change. The polynomial systems, which involve thousands of variables, pose interesting challenges.

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MS31
Ideals of Graph Homomorphisms and Graph Colorings

We show how ideals of graph homomorphisms in algebraic statistics can be employed to study graph colorings.

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MS31
The Binomial Ideal of the Intersection Axiom for Conditional Probabilities

The binomial ideal associated with the intersection axiom of conditional probability is shown to be radical and is expressed as intersection of toric prime ideals. This resolves a conjecture in algebraic statistics due to Cartwright and Engstrom.

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MS31
Commutative Algebra of Statistical Ranking Models

A model for statistical ranking is a family of probability distributions whose states are orderings of a fixed finite set of items. We represent the orderings as maximal chains in a graded poset. The most widely used ranking models are parameterized by rational function in the model parameters, so they define algebraic varieties. We study these varieties from the perspective of combinatorial commutative algebra. One of our models, the Plackett-Luce model, is non-toric. Five others are toric. For these models we examine the toric algebra, its lattice polytope, and its Markov basis.

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MS31 Polytopes and Imsets

We study polytopes and imsets of generalized tree graphs.

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MS31 Conditional Independence Ideals and Robustness

The talk discusses a class of conditional independence (CI) ideals motivated by conceptual studies of robustness [AY, N., AND KRAKAUER, D. C., “Geometric robustness theory and biological networks,” *Theory in Biosciences*, vol. 125, no. 2, pp. 93 – 121, 2007.]. Consider a collection of $n + 1$ finite random variables X_0, X_1, \dots, X_n with the interpretation that X_0 is the output of the system, computed from the inputs X_1, \dots, X_n . Such a system is robust if the complete knowledge of all input variables is not necessary in order to compute the output. This notion of robustness can be modelled as a collection of CI statements, generalizing the set of CI statements studied in [FINK, A., “The binomial ideal of the intersection axiom for conditional probabilities,” *Journal of Algebraic Combinatorics*, vol. 33, no. 3, pp. 455–463, 2011.]. CI statements give polynomial restrictions on the joint probability distribution of the system. If the output variable is binary, then the resulting CI ideal is a *binomial edge ideal*, as defined in [HERZOG, J., HIBI, T., HREINSDÓTTIR, F., KAHLE, T., AND RAUH, J., “Binomial edge ideals and conditional independence statements,” *Advances in Applied Mathematics*, vol. 45, no. 3, pp. 317 – 333, 2010.]. The results generalize to the case that the output is non-binary. In particular, the CI ideal is radical, and the primary decomposition has a nice interpretation.

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MS32 Linear Algebra over Dense Matrices over $\text{GF}(2)$ and Small Extensions - Albrecht

Linear algebra over F_2 and F_{2^e} where e is a small integer has many applications such as cryptography and coding theory. Since matrices over these base fields can be efficiently bit-packed, dense techniques are often feasible for considerable matrix sizes beyond 100,000 x 100,000. In this talk we will present our work on matrices over these fields and discuss algorithms, implementation techniques and issues.

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MS32 Pivoting Strategies for Sparse Matrices over Finite Fields

Solving a polynomial system of equations over a finite field is at the heart of algebraic cryptanalysis and numerous other applications, including the Quadratic Sieve. Several algorithms for solving such systems of equations over those fields involve reducing a matrix over a finite field. In this paper, we analyze a plethora of pivoting strategies, ranging from the naive to the complex, for performing structured Gaussian Elimination to minimize fill-in and field operation counts. We consider random matrices, as well as matrices of the forms produced by the Quadratic Sieve and also the XL/F4 algorithms. The ability of the various techniques to maintain sparsity is compared for various sizes and input matrices, as well as the running times.

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MS32 Groebner Bases and Linear Algebra

There is a strong interplay between computing efficiently Gröbner bases and linear algebra. In this talk, we focus on several aspects of this convergence:

- Algorithmic point of view: algorithms for computing efficiently Gröbner bases (F_4, F_5, \dots) not only rely heavily on linear algebra but were based on linear algebra techniques. Recent algorithms to change the ordering of a Gröbner basis is also related to multivariate generalization of the Wiedemann algorithm. Mixing Gröbner bases methods and linear algebra technique for solving sparse linear systems leads to an efficient algorithm to solve boolean quadratic equations; this algorithm is faster than exhaustive search by an exponential factor
- Application point of view: for instance, a generalization of the eigenvalue problem to several matrices – the MinRank problem – is at the heart of the security of many multivariate public key cryptosystems.
- Design of library: when implementing Gröbner bases algorithms the crucial task is the linear algebra part; moreover the specific structure of the matrices has to be taken into account to speedup even more the computations.

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MS32 Verified Numerical Linear Algebra: Linear System Solving

Solving numerically a linear system can be performed very efficiently, using optimized routines, but it yields an approximate solution without any indication about its accuracy. Getting an enclosure of the error between the approximate and the exact solutions is called “verification”. We present a verified algorithm which gets an accurate enclosure, with a moderate overhead both in complexity and in practical performance. Its key ingredients are interval arithmetic, iterative refinement and well-chosen computing

precision.

Nathalie Revol

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MS32

Exact Solutions to Mixed-integer Linear Programming Problems

We present a hybrid symbolic-numeric approach for solving mixed-integer-programming problems exactly over the rational numbers. By performing many operations using fast floating-point arithmetic and then verifying and correcting results using symbolic computation, exact solutions can be found without relying entirely on rational arithmetic. Computational results will be presented based on an exact branch-and-bound algorithm implemented within the constraint integer programming framework SCIP.

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MS33

Algebraic Problems in Graphical Modeling

A graphical model is a statistical model associated with a graph whose nodes correspond to random variables. I will give an introduction to these models and provide statistical motivation for algebraic problems that will be addressed by the speakers in the session.

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MS33

Parameter Identifiability via the Half-trek Criterion in Mixed Graphs

In the setting of Gaussian graphical models, mixed graphs allow for both linear effects between the nodes (directed edges) and covariance among the Gaussian errors on each node (bidirected edges). We study parameter identifiability in these models, that is, we ask for conditions that ensure that the edge coefficients and correlations appearing in a linear structural equation model can be uniquely recovered from the covariance matrix of the associated normal distribution. In particular, we are interested in generic parameter identifiability, where recovery is possible for almost every choice of parameters. We give a new graphical criterion that is sufficient for generic identifiability. Unlike prior work on this problem, our criterion does not require the directed part of the graph to be acyclic, and additionally it improves on existing criteria for the acyclic case. We also develop a related necessary condition, and examine the “gap” between sufficient and necessary conditions through

simulations as well as exhaustive algebraic computations for graphs with up to five nodes.

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MS33

Parameter Identification of Structural Equation Models

Structural equation models (SEMs) are used to formalize a variety of causal queries as certain types of probability distributions. A central problem in SEMs is the analysis of identification. A model is identified if it only admits a unique parametrization to be compatible with a given data set. Here, I will introduce a computer algebra software to test identifiability, and discuss recent advances on global identification via the analysis of the Jacobian of the parametrization.

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MS33

Discrete Graphical Models with One Hidden Variable

Kruskal’s theorem applies to a simple latent class model, in which three observed variables are independent when conditioned on a single hidden one. Our aim is to give general conditions for identifiability using Kruskal’s theorem and, when the conditions fail to hold, to characterize the space where identifiability breaks down. We present a technique that, given an arbitrary directed graphical model with a single hidden variables, modifies the model in such a way that we can apply Kruskal’s theorem. The technique is based on the following three operations:

- Clump several variables (all hidden or all observed) into a single one, with larger state space.
- Condition on the state of an observed variable,
- Marginalize over an observed variable (making it hidden).

Each of these operations can be done multiple times, and in combination with one another.

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MS33**Identifiability of Phylogenetic Tree Models for Binary Data**

Our focus in this talk is on undirected discrete graphical tree models when all the variables in the system are binary, where leaves represent the observable variables and where all the inner nodes are unobserved. A novel approach, based on the theory of partially ordered sets, allows us to obtain a convenient parametrization of this model class. A simple product-like form of the resulting parameterization gives insight into identifiability issues associated with this model class. In particular, we provide necessary and sufficient conditions for such a model to be identified up to the switching of labels of the inner nodes. When these conditions hold we give explicit formulas for the parameters of the model. Whenever the model fails to be identified we use the new parameterization to describe the geometry of the unidentified parameter space. We illustrate these results using a simple example.

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MS34**Evasion Paths in Mobile Sensor Networks**

In “Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology,” Vin de Silva and Robert Ghrist use the local connectivity data of a mobile sensor network to determine, in some cases, whether an evasion path exists in the network. We consider examples that show the existence of an evasion path depends not only on the network’s connectivity data but also on its embedding, and we search for invariants of the embedding that provide sharper criteria for the existence of an evasion path.

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MS34**Mapping Spaces in Computational Algebraic Topology**

A basic aspect of the modern perspective on homotopy theory is that the set of maps between spaces can itself be regarded as a topological space, where paths in this mapping space represent homotopies. All of the homotopy theory of spaces can be encoded in the data of the mapping space. This talk will present recent work on combinatorial simplicial mapping spaces in computational topology, emphasizing connections between simplicial subdivision and sampling.

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MS34**Homotopy Theory of Dynamical Systems**

A dynamical system is a space X with a pairing from $X \times S$ to X for some parameter space S , and a map of such dynamical systems is an S -equivariant map. There is an injective

and a projective Quillen model structure for the resulting category of spaces with S -action, and both are easily derived. Simultaneously varying the parameter space S along with the space X up to weak equivalence is more interesting, and requires a new model structure having weak equivalences defined by homotopy colimits, as well as a generalization of Thomason’s model structure for small categories. These techniques give methods of detecting when one dynamical system is close to another, either spatially or combinatorially.

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MS34**Detecting Persistent Knotting**

Given a filtration of spaces $X_0 \subset X_1 \subset \dots \subset X_n \subset \mathbf{R}^3$, we discuss what it means for any knotting of the space X_i to persist in the larger space X_{i+p} . This generalizes techniques from classical knot theory. In particular, this knotting can be characterized by the persistent homotopy groups of its complement. We will discuss techniques to determine if knotting of a complex persists in a larger space or if the knotting is transient.

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MS35**Rank 4 Premodular Categories**

By relaxing the non-degeneracy condition of modular categories one obtains the more general class of premodular categories. These are of interest for several reasons: firstly, as spherical categories they lead to modular categories via the Drinfeld center construction. Secondly, they give rise to link invariants. Finally, recent constructions suggest that to get non-trivial 4-D TQFTs the modularity condition should be removed. We will discuss recent work on the classification of rank 4 premodular categories.

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MS35**Principle of Maximum Entropy and Ground Spaces of Local Hamiltonians**

Correlations are of key importance in understanding many fundamental phenomena of many-body quantum physics. Motivated by the concept of irreducible correlations studied by [Linden et al., PRL 89, 277906] and [Zhou, PRL 101, 180505], which is in turn based on the principle of maximum entropy, we present that, some quantum states

are with certain k -particle correlation if and only if they span a ground space of some k -local Hamiltonian. It establishes a better understanding of k -body correlations and builds an intimate link of correlations of quantum states to ground spaces of local Hamiltonians.

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MS35

Localizing Topological Quantum Computers

The quantum circuit and topological models for quantum computation are computationally equivalent in the sense that suitably chosen models in each setting can efficiently simulate a universal model in the other setting. I will discuss several recent attempts to refine this theoretical result to a more practical, hybrid model. This is based upon joint work with Zhenghan Wang.

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MS35

Hidden Symmetry Subgroup Problems in Quantum Computing

We advocate a new way of addressing hidden structure problems and of finding efficient quantum algorithms. We define and investigate the Hidden Symmetry Subgroup Problem (HSSP), a generalization of the well studied Hidden Subgroup Problem (HSP). Given a group acting on a set and an oracle whose level sets define a partition of the set, the task is to recover the subgroup of symmetries of this partition inside the group. The HSSP provides a unifying framework that, beside the HSP, encompasses a wide range of oracle problems, including hidden polynomial problems. While we show that the HSSP can have instances with exponential quantum query complexity, for various instances we obtain efficient quantum algorithms. To achieve that we design a general framework for reducing the HSSP to the HSP which works efficiently in many interesting cases related to symmetries of polynomials. The HSSP therefore connects in a rather surprising way certain hidden polynomial problems with the HSP. Using this connection, we obtain the first ever efficient quantum algorithm for the hidden polynomial problem, for quadratic polynomials over fields of constant characteristic.

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MS35

SIC-POVMs in Dimension Three via Singular El-

liptic Curves

It is not known whether each complex dimension d contains a set of equiangular lines with the maximum possible cardinality of d^2 (a SIC-POVM). Zauner conjectured that one always exists as an orbit of a finite Heisenberg group. In dimension 3, Hughston observed that the inflection points of any Hesse cubic give a particular Heisenberg SIC-POVM. I give a geometric interpretation for the entire infinite family as the 3-torsion points of singular elliptic curves.

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MS36

Convexity and SOS-Convexity

A multivariate polynomial $p(x) = p(x_1, \dots, x_n)$ is sos-convex if its Hessian $H(x)$ can be factored as $H(x) = M^T(x)M(x)$ with a possibly nonsquare polynomial matrix $M(x)$. The notion of sos-convexity is a sufficient condition for convexity of polynomials that can be checked efficiently with semidefinite programming. We will present three results on the subject: (i) We will show that two other natural sos relaxations for convexity based on the definition of convexity and its first order characterization turn out to be equivalent to sos-convexity. (ii) We will present an explicit example of a convex but not sos-convex polynomial in six variables and of degree four, whose construction comes directly from our recent proof of NP-hardness of deciding convexity of quartic polynomials. (iii) We will show that for polynomials in n variables and of degree d , the notions of convexity and sos-convexity are equivalent if and only if $n = 1$ or $d = 2$ or $(n, d) = (2, 4)$. Remarkably, these are exactly the cases where nonnegative polynomials are guaranteed to be sums of squares, as proven by Hilbert.

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MS36

Positive Semidefinite Gorenstein Ideals

I will explain how positive semidefinite Gorenstein ideals naturally arise as objects that separate nonnegative polynomials from sums of squares. It is possible to deduce all of the cases where nonnegative polynomials are equal to sums of squares as the cases where psd Gorenstein ideals do not exist. I will present applications to decompositions of ternary forms as sums of powers of rational functions and symmetric tensor decompositions

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MS36

Strong Nonnegativity and Sums of Squares on Real Varieties

Motivated by scheme theory, we introduce strong nonnegativity on real varieties, which has the property that a sum of squares is strongly nonnegative. We show that this algebraic property is equivalent to nonnegativity for nonsingular real varieties. Moreover, for singular varieties, we reprove and generalize obstructions of Gouveia and Netzer to the convergence of the theta body hierarchy of convex

bodies approximating the convex hull of a real variety.

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MS36

The Algebraic Boundary of $SO(2)$ -orbitopes

An $SO(2)$ -orbitope is the convex hull of an orbit under some linear action of $SO(2)$ on a finite dimensional real vector space. Such a set is always a compact, convex, semi-algebraic set. We will characterize the Zariski closure of its boundary in the 4-dimensional case. One component is the secant variety to the Zariski closure of the orbit, which is a real algebraic curve, and for the other components, we give explicit equations. Our answer will have implications on the semi-algebraic geometry of these orbitopes and on the question, whether or not they are spectrahedra. This is work in progress.

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MS36

The Central Curve of a Semidefinite Program

The central curve of a semidefinite program is an algebraic curve specified by an affine linear space of symmetric matrices and a cost vector. Here we examine the algebraic properties and beautiful geometry of this curve. We will focus on examples and special cases coming from applications. In the special case of diagonal matrices (linear programming) we will see connections to hyperplane arrangements and matroid theory.

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MS37

Ideal Forms of Coppersmith's Theorem and Guruswami-Sudan List Decoding, Part 2

Abstract not available at time of publication.

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MS37

Ideal Forms of Coppersmith's Theorem and Guruswami-Sudan List Decoding

Coppersmith's algorithm is a celebrated technique for finding small solutions to polynomial equations modulo integers. It has many important applications in cryptography, particularly in the cryptanalysis of RSA. In this talk, we show how the ideas of Coppersmith's theorem can be extended to a more general framework encompassing the original number-theoretic problem, list decoding of Reed-Solomon and algebraic-geometric codes, and the problem of finding solutions to polynomial equations modulo ideals

in rings of algebraic integers.

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MS37

List Decoding for AG Codes Using Gröbner Bases

Finding efficient algorithms for the interpolation step in Guruswami-Sudan list decoding remains a problem of interest. We will discuss how the recent algorithms of [Lee-O'Sullivan, List Decoding of Hermitian Codes, JSC 44 (2009), 1662-1675] and [Trifonov, Efficient Interpolation in the Guruswami-Sudan Algorithm, preprint 2010], which find the desired interpolating polynomial in a certain module Gröbner basis, can be extended to codes from other curves. We will also compare their methods with an FGLM approach.

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MS37

General Bounds for Generalized Toric Codes

Toric codes are algebraic evaluation codes corresponding to linear systems on toric varieties. Their parameters are strongly related to the geometry of the lattice polytope defining a linear system. Recently Diego Ruano introduced, so called, generalized toric codes (GTC) when one takes arbitrary subspaces of linear systems on a toric variety. It is harder to capture their properties as they are defined by arbitrary finite lattice point configurations S which lack the geometry of lattice polytopes. We introduce a general lower bound for the minimum distance of a GTC. The bound involves the minimum distance of GTCs defined by projections of the configuration S and the fibers of the projection.

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MS37

Toric codes and Minkowski Length of 3D Polytopes

The Minkowski sum of two polytopes is the set of all pairwise sums of their points. The central object of my talk is the Minkowski length $L(P)$ of a lattice polytope P which is defined to be the largest number of primitive lattice segments whose Minkowski sum is in P . The Minkowski

length represents the largest possible number of factors in a factorization of polynomials with exponent vectors in P and comes up in lower bounds for the minimum distance of toric codes. I will explain some combinatorial results about $L(P)$ where P is a 3D lattice polytope in connection with 3D toric codes.

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MS38

A Domain Decomposition Algorithm for Computing Multiple Steady States of Differential Equations

Problems in engineering often lead to systems of partial differential equations, for which the only hope of solution is to compute numerical solutions. This talk will describe domain decomposition method for solving polynomial systems derived from the discretization of differential equations based on homotopy continuation. This method divides the domain into subdomains and solves the polynomial system arising from each subdomain. Homotopy continuation then uses these solutions to build solutions for the original domain.

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MS38

Numerical Algebraic Geometry Applied to Various Theoretical Physics Problems

Nonlinear equations arise in theoretical physics naturally and frequently. In general, it is always difficult to solve (i.e., get ALL solutions, either exactly or numerically) them. However, if the non-linearity of the equations is polynomial-like, then one can use the computational and numerical algebraic geometry methods to solve the equations exactly or numerically, respectively. In the talk, this will be explained in more detail by taking a few examples from theoretical physics, namely, in statistical mechanics, string theory and lattice field theories, polynomial equations arise naturally and it will be shown how the algebraic geometry methods can be used to get very interesting physics out.

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MS38

Using Homotopy Continuation to Model Rate-dependent Magnetic Phenomena

Rate-dependent magnetic phenomena occur because thermal fluctuations can drive a system of magnetic moments over energy barriers between stable states. A new method for modeling these transitions uses the Bertini homotopy continuation package. Systems of polynomial equations are solved for the stable states and energy barriers. The solutions are incorporated in a master equation for the time evolution of the probability of each stable state.

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MS38

Point Clouds of Varieties and Persistent Homology

Computing topological invariants of algebraic objects is simultaneously challenging, from an algebraic standpoint, and of large interest to application fields. We approach this challenge with a two-pronged method: first, using techniques from homotopy continuation and numerical algebraic geometry, we generate point clouds – finite sample sets – on the variety, and subsequently, we approach the point cloud with techniques from applied algebraic topology. Numerical algebraic geometry provides techniques to solve systems of polynomial equations and retrieve solution points that are guaranteed to contain the isolated points in the variety defined by the equational system. By intersecting the variety of interest with generic hyperplanes, we can produce equation systems with isolated points located on all components of the original variety. Furthermore, the techniques of persistent homology were originally developed in order to acquire topological information from finite point samples, from point clouds. We apply these techniques to gain computational indications of the topological structure of the point cloud generated from homotopy continuation, and describe some ways to decompose and re-assemble topological information for better computational performance.

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MS39**Approximate Implicitization using Chebyshev Polynomials**

Whereas traditional approaches to implicitization of rational parametric curves have focussed on exact methods, the past two decades have seen increased interest in the application of approximate methods for implicitization. In this talk we will discuss how the properties of the Chebyshev polynomial basis can be used to improve the speed, stability and approximation quality of existing algorithms for approximate implicitization. We will also look at how the algorithm is well suited to parallelization.

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MS39**Blending Natural Quadrics and Associated Volumes**

Natural quadrics are simplest primitive shapes used in CAD: spheres, right circular cones or right circular cylinders. We construct exact rational fixed and variable radius blends of minimal possible parametrization degrees between two natural quadrics using canal surfaces in all cases where it is possible. Bounds on parameters that ensure non-singular blends and associated volumes will be presented.

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MS39**Why Approximate Algebraic Methods?**

The talk will address the potential of the use of approximate algebraic methods in CAGD systems, with a focus on intersection and self-intersection algorithms. An introduction to the original approach to approximate implicitization will be given.

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MS39**Application of Numerical Methods in Implicitization**

Relying on methods for implicit support prediction, we reduce implicitization of rational curves and (hyper)surfaces to interpolating the implicit coefficients. We apply numerical methods, including SVD, in order to compute a matrix kernel. Computational experiments in Maple and Matlab are performed to compare different approaches for constructing a real or complex matrix, either square or rectangular, and for computing its kernel, with respect to robustness and speed. Our methods exploit sparsity and

may compute approximate implicit equations.

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MS39**Approximate Implicitization of Envelope Surfaces**

Given a rational family of parametric surfaces in a certain region of interest, we are interested in computing an implicit representation of the envelope. Although several methods for exact implicitization are known, they tend to be computationally expensive and may even introduce unwanted branches and singularities. We adapt the concept of approximate implicitization to envelopes and formulate an algorithm for computing a piecewise algebraic approximation of low degree.

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MS40**Dynamics of Weakly Reversible Reaction Networks and the Global Attractor Conjecture**

I will discuss some recent work pertaining to weakly reversible chemical reaction networks and, in particular, the Global Attractor Conjecture. I will present some details of the proof of the conjecture in the setting where the underlying reaction diagram consists of a single linkage class, or connected component.

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MS40**QSSA and Algebraic Elimination in Chemical Kinetics**

Chemical mechanisms for reaction network dynamics involve many highly reactive and short-lived species (intermediates), present in small concentration, in addition to the main reactants and products, present in larger concentration. A classic model reduction method known as the quasi-steady-state approximation (QSSA) is often used to eliminate the highly reactive intermediate species and remove the large rate constants that cannot be determined from concentration measurements of the reactants and products. We show that the program usually taught to students for applying the 100 year-old approach of classic QSSA model reduction cannot be carried out in many, perhaps most, relevant kinetics problems. In particular, by using Galois theory, we prove that the required algebraic equations cannot be solved for as few as five bimolecular

reactions between five species. We also describe algorithms that can test any mechanism for solvability, and propose a new strategy for dealing with unsolvable systems, based on rescaling the reactive intermediate species.

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MS40

Catalysis in Reaction Networks

We define *catalytic networks* as chemical reaction networks with an essentially catalytic reaction pathway: one which is “on” in the presence of certain catalysts and “off” in their absence. We show that examples of catalytic networks include synthetic DNA molecular circuits that have been shown to perform signal amplification and molecular logic. Recall that a *critical siphon* is a subset of the species in a chemical reaction network whose absence is forward invariant and stoichiometrically compatible with a positive point. Our main theorem is that all weakly-reversible networks with critical siphons are catalytic. Consequently, we obtain new proofs for the persistence of atomic event-systems of Adleman *et al.*, and normal networks of Gnacadja. We define *autocatalytic networks*, and conjecture that a weakly-reversible reaction network has critical siphons if and only if it is autocatalytic.

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MS40

Linear Conjugacy of Chemical Reaction Networks

Under suitable assumptions, the dynamic behaviour of a chemical reaction network is governed by an autonomous set of polynomial ordinary differential equations over continuous variables representing the concentrations of the reactant species. It is known that two networks may possess the same governing mass-action dynamics despite disparate network structure. To date, however, there has only been limited work exploiting this phenomenon even for the cases where one network possesses known dynamics while the other does not. In this presentation, I will present results which bring these known results together into a broader unified theory which we have called conjugate chemical reaction network theory. In particular, I will present a theorem which gives conditions under which two networks with different governing mass-action dynamics may exhibit the same qualitative dynamics and use it to extend the scope of the well-known theory of weakly reversible systems.

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MS40

Persistence of Chemical Kinetics Systems

It is known that a solution to a chemical kinetics system

can only approach the boundary of the non-negative orthant at points belonging to a semi-locking set. We generalize results of Angeli, De Leenheer and Sontag (2007) which assume that every semi-locking set satisfies a conservation condition or is dynamically non-emptiable by introducing a notion of a weakly dynamically non-emptiable semi-locking set. Systems whose semi-locking sets are weakly dynamically non-emptiable are persistent. The facet result of Anderson and Shiu (2010) also fits into this framework.

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MS41

Pentads and Minors Define the Gaussian Two-factor Model

In 1928, educational psychologist Truman Lee Kelley published *Crossroads in the Mind of Man*, which contained a new *pentad test* for Gaussian factor analysis with two factors. Pentads are degree-five polynomials that vanishes on all symmetric $n \times n$ -matrices that can be expressed as a rank-two matrix plus a diagonal matrix. In their recent algebraic approach to factor analysis, Drton, Sturmfels, and Sullivant raise the problem of determining all such polynomials. I will report on a finite computation that determines a generating set of polynomials for all n . This is joint work with Andries Brouwer.

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MS41

Geometry of Deep Belief Networks

We report on recent progress on the algebraic statistics of deep belief networks. These are graphical models built from layered restricted Boltzmann machines, which in turn correspond to Hadamard products of secant varieties of Segre varieties.

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MS41

Parametrization of Causal Models Associated with Acyclic Directed Mixed Graphs

Acyclic directed mixed graphs may be used to represent causal directed acyclic graph (DAG) models in which some variables are unmeasured (latent). In this talk we present a parametrization of the observed distribution in the multivariate binary case. The resulting model satisfies Markov constraints, and so-called Verma constraints. The parametrization leads to efficient algorithms for computing intervention distributions, as well as allowing scoring-based model search. [Joint work with Robin Evans (University of Washington), James Robins and Ilya Shpitser (Harvard

School of Public Health).]

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MS41

Treks and Determinants of Path Matrices in Gaussian Graphical Models

This talk will explore a connection between combinatorics and algebraic statistics. In particular, we will look at Gaussian graphical models, whose covariance matrices can be given in terms of certain path families called treks. Inspired by classical results in algebraic combinatorics, we develop a graph-theoretic criterion for determining the rank of a submatrix of the covariance matrix.

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MS41

Geometry of Maximum Likelihood Estimation in Gaussian Graphical Models

We study maximum likelihood estimation in Gaussian graphical models from a geometric point of view. In current applications of statistics, we are often faced with problems involving a large number of random variables, but only a small number of observations. An algebraic elimination criterion allows us to find exact lower bounds on the number of observations needed to ensure that the maximum likelihood estimator exists with probability one. This is applied to bipartite graphs and grids, leading to the first instance of a graph for which the maximum likelihood estimator exists with probability one even when the number of observations equals the treewidth of the graph.

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MS42

Basic Notions and Current Trends in Numerical Algebraic Geometry

This talk will overview some of the recent advances within the field of numerical algebraic geometry without discussing applications (covered in an earlier minisymposium). The goal is to describe some of the latest trends and current open problems in advancing this field. Basic notions of numerical algebraic geometry (homotopy continuation, numerical irreducible decomposition, etc.) will be introduced very briefly as this is an introductory talk for this minisymposium. Coming after the 12-15 talks in the

Applications of Numerical Algebraic Geometry minisymposium, though, I will keep the introductory parts short!

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MS42

Numerical Algebraic Geometry via Numerical Polynomial Algebra

Techniques of numerical polynomial algebra are applied to numerical algebraic geometry. These techniques include Sylvester and Macaulay matrices, and local and global duality. Macaulay's H-bases connect the homogeneous and affine cases. The computations are done by use of SVD based numerical linear algebra using Mathematica. The main goal of these methods is to provide explicit polynomials defining the ideals defining algebraic sets which are components of, or unions of, other algebraic sets.

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MS42

Certification of Approximate Multiple Roots

Approximation and certification of multiple roots is a real challenge in numerical computation. In this talk, we will describe a symbolic-numeric algorithm for the certification of singular isolated points. The approach is based on the computation of the inverse systems of the isolated singular point. We derive a one-step deflation technique, from the description of the multiplicity structure in terms of differentials. The deflated system can be used in Newton-based iterative schemes with quadratic convergence. Starting from a polynomial system and a sufficiently small neighborhood, we obtain a criterion for the existence and uniqueness of a singular root of a given multiplicity structure, applying a well-chosen symbolic perturbation. Standard verification methods, based e.g. on interval arithmetic and a fixed point theorem, can be employed to certify that there exists a unique perturbed system with a singular root in the domain.

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MS42

Isosingular Sets and Deflation

This talk will discuss the new concept of isosingular sets, which are pure-dimensional irreducible algebraic subsets of the set of solutions to a system of polynomial equations such that generic points of an isosingular set share a common singularity structure. The definition of these sets depends on deflation, a procedure that uses differentiation to regularize solutions. A weak form of deflation has proven useful in regularizing algebraic sets, making them amenable to treatment by the algorithms of numerical algebraic ge-

ometry. We introduce a strong form of deflation and use it to define deflation sequences, which are similar to the sequences arising in Thom-Boardman singularity theory. Isosingular sets consist of points that have the same deflation sequence. From this, one may define the isosingular set of a given point and also the isosingular local dimension. While isosingular sets are of theoretical interest as constructs for describing singularity structures of algebraic sets, they also expand the kinds of algebraic sets that can be investigated with methods from numerical algebraic geometry.

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MS43

$|Z_{Kup}| = |Z_{Henn}|^2$ for Lens Spaces

M. A. Hennings and G. Kuperberg defined quantum invariants Z_{Henn} and Z_{Kup} of 3-manifolds based on Hopf algebras, respectively. We prove that $Z_{Kup}(L(p, q), H) = Z_{Henn}(L(p, q) \# \overline{L(p, q)}, H)$ for lens spaces when both invariants are based on a factorizable finite dimensional ribbon Hopf algebra H .

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MS43

Quantum Stabilizer Codes from Toric Varieties

The technology to produce classical codes from higher dimensional algebraic varieties has been known for approximately 10 years. The combinatorial structure of toric varieties made them particularly suitable for applying these techniques. Nevertheless, the only algebraic varieties from which one could construct quantum codes seemed to be curves. In this talk, we outline how to construct quantum stabilizer codes from higher dimensional toric varieties. Examples of such codes constructed from toric surfaces will be discussed briefly.

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MS43

Quantum Error-Correcting Codes over Rings

The discretization of errors in quantum error-correcting codes typically relies on a generalization of the Pauli matrices. As a consequence, most quantum codes have been constructed with the help of classical codes over finite fields.

We discuss methods of discretization that use matrices that are parametrized by finite Frobenius rings, and sketch how quantum codes can be constructed in this case.

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MS43

Classifying Small Index Subfactors

I'll describe recent progress on the classification of subfactors with index at most 5, and corollaries for fusion categories.

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MS43

Recent Progress in Exactly Soluble Discrete Models for Topological Phases in Two Dimensions

The study of two-dimensional topological phases in condensed matter systems is a frontier in the field of condensed matter theory as well as topological quantum computation. Discrete or lattice models, which are exactly soluble have been proposed by Kitaev and by Levin and Wen, respectively. Here we present a summary of recent progress, made by us and other groups, in studying these models and their generalizations. The topics to be reviewed include 1) Duality between the Kitaev and Levin-Wen models in certain special cases; 2) General procedure for computing ground state degeneracy when the models are put on a topologically non-trivial surface; 3) More detailed study of the sectors of topological excitations; 4) General framework for topology-preserving renormalization group flow that characterizes these models as fixed points; 5) Generalization of these models to more general graphs or to incorporating more general degrees of freedom. Our approach, though closely related to topological field theory and tensor category theory, could be understood by physicists.

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MS44

Minimum Fuel Multi-impulse Orbit Transfer

Most of the equations in orbital mechanics are expressed in terms of trigonometric functions. However, except for the case of Kepler's equation, all the remaining instances correspond to simple planar or spatial rotations. By representing these rotations by points in the unit circle (for

2D rotations) or unit quaternions (for 3D rotations), all the equations become polynomials, hence tractable with algebraic geometry techniques. In this talk, we will focus mainly in the problem of orbit transfer using a finite sequence of thrusts, and minimum fuel.

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MS44 Reinterpreting Regularization of Collisions through Real Algebraic Geometry

Regularization of binary and/or simultaneous binary collisions in the collinear N -body problem is a common tool used to analyze the geometry of orbits near these kinds of singularities. The Levi-Civita type coordinate and time transformations typically used in regularization are reinterpreted in terms of real algebraic geometry. This reinterpretation applies to Hamiltonians whose potential part is a finite sum of reciprocals of homogeneous polynomials in the position coordinates.

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MS44 Resonances in Celestial Mechanics

A dynamical system is in resonance $p : q$ if there are two frequencies ω_1 and ω_2 such that $\omega_1/\omega_2 = p/q$, with p and q two coprime numbers. Thus, resonances are related with periodic motions, and periodic motions are inherent to Celestial Mechanics problems. Nature presents many examples of resonances, for instance the orbital-rotational motion of the Moon, but there are many other cases of resonances, which appear in the equations of motion and that creates many difficulties in the integration, mainly by analytic methods. For these cases, we present some ways in tackling this problem by means of appropriate sets of variables. Some examples are presented as well as some applications to the stability of the solutions.

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MS44 The Algebra and Geometry of Periodic Satellites Ground Tracks

We survey the algebra and geometry of periodic satellites ground tracks for remote sensing observations from space of

the Earth and other planets. The repetition of the ground tracks allows for multiple scientific observations to be taken of the same column of space all the way to a little below the planet surface at various times of day. The symmetry and geometry of such orbits are governed by finite groups which classify them.

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MS44 Evolution of Flower Constellations Theory

The theory of Flower Constellations consists of a general methodology to design constellations of N_s satellites. The original theory evolved over time and a rich literature was produced with subsequent insights and mathematical reformulations. This talk describes these mathematical progresses making now the theory connected with the 2D and 3D Lattice theory and the Necklaces theory.

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MS45 On the Decodability of Primitive Reed-Solomon Codes

Reed-solomon codes are (list) decodable up to the Johnson-Guruswami-Sudan bound. No polynomial time decoding algorithm is known when number of errors is larger than the JGS bound. It appears hard to establish complexity results for the primitive Reed-Solomon codes. In this talk, I will present several results on this problem. I will also discuss whether the results can be generalized to algebraic geometric codes.

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MS45 Interpolation Decoding of a Class of Quasi-cyclic Codes

For a quasi-cyclic code of index ℓ , we transform the decoding problem into an interpolation problem for a Reed-Solomon code. For each location, ℓ values are assigned (counting multiplicities) and the usual interpolation decoding algorithms may be applied. We compute upper bounds on the decoding capability of the algorithm, showing that it can correct well beyond the minimum distance bound with high probability. Experimental results corroborate our computations.

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MS45 Small Bias Sets from General Algebraic Geometry

Codes

Small bias sets are important in a number of applications, from derandomization of algorithms to hash functions. There is a connection between small bias sets and linear codes. In this talk we consider the construction of small bias sets from general algebraic geometry codes.

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MS45**Goppa-like Codes Beating the Best Known Codes**

The paper presents a construction of subfield subcodes from generalized Reed-Solomon codes that is similar to classical Goppa codes, but simpler. Following an idea of Roseiro et al, we use Delsarte's theorem to create subfield subcodes of larger than expected dimension by ensuring their duals, which are trace codes, have small dimension. This method produced several codes over \mathbf{F}_5 , \mathbf{F}_3 and \mathbf{F}_2 that beat the best known codes, and numerous others that match the best known parameters.

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MS45**Extended Norm-Trace Codes**

Consider the function field $F_{q^r}(x, y)/F_{q^r}$ defined by

$$x^u = L(y),$$

where $u | \frac{q^r-1}{q-1}$ and $L(y) = \sum_{i=0}^d a_i y^{q^i}$ is a linearized polynomial with $a_0, a_d \neq 0$ and q^d distinct roots in q^r . Note that the Hermitian and norm-trace function fields are special cases of this function field. Recent work has yielded explicit bases for certain Riemann-Roch spaces of this function field. In this talk, we apply this progress to construction of codes arising from these spaces.

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MS46**On the Algebraic Representation of Bisectors for Low Degree Surfaces**

Bisectors are geometric constructions appearing very often in CAD. For two given low degree parametric surfaces, it will be shown a new approach to determine an algebraic representation of their bisector by using the so-called generalized Cramer rules. The new introduced approach allows to obtain easily a parametrization of the quadric-plane or quadric-cylinder bisectors, which is rational in most cases (in the remaining cases the parametrization involves one square root which is well-suited for approximation purposes).

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MS46**Toric Degenerations of Toric Varieties and Applications to Modeling**

Toric varieties have interesting combinatorial properties that make them attractive for applications to geometric modeling. They were employed by R. Krasauskas to define toric patches that are a generalization of Bézier patches. Toric degenerations were used by L. D. García-Puente, F. Sottile and C. Zhu to explain some ways that control points govern the shape of toric patches when the weights vary. In this talk we will present our efforts to extend these properties to irrational toric patches. This is a joint project with F. Sottile and N. Villamizar.

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MS46**On the Dimension of Triangular Splines**

We consider the space of spline functions defined on a triangular subdivision of a polygonal domain. Using homological techniques we will present a lower and an upper bound to the dimension of this spline space which are, in many or perhaps most of the cases, more general and give better approximations to the exact value of the dimension than the already existing ones. These results can also be extended to spline spaces on 3-dimensional (simplicial) complexes.

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MS47**Determining Multiple Steady States in Mass Action Networks by Linear Inequality Systems**

Ordinary Differential Equations (ODEs) are an important tool in many areas of Quantitative Biology. For many ODE

systems multistationarity (the existence of ≥ 2 positive steady states) is a desired feature. For mass action networks establishing multistationarity is equivalent to establishing the existence of at least two positive solutions of a polynomial system with unknown coefficients. For networks with certain structural properties, necessary and sufficient conditions for multistationarity that take the form linear inequality systems are presented.

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MS47

Tools From Computational Algebraic Geometry for the Study of Biochemical Reaction Networks

We will illustrate the use of some tools from computational algebraic geometry for the study of (bio)chemical reaction networks, extracted from our joint work with Carsten Conradi, Mercedes Pérez Millán, and Anne Shiu.

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MS47

Complexity of Steady States in Molecular Networks and Numerical Algebraic Geometry

One of the most pressing questions in modern biology is how molecular interactions at the cellular level lead to physiology, particularly disease. The most accessible mathematical approach uses the law of mass-action to describe the underlying molecular network in terms of a polynomial dynamical system. Using tools from projective algebraic geometry coupled with high performance numerical analysis, we investigate the complexity of steady states in networks of established biological relevance, such as the Wnt pathway, and translate the parameter problem of biology into a question about complex structure moduli of the underlying algebraic variety.

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MS47

Persistence and the Global Attractor Conjecture: Recent Approaches

We describe recent approaches to proving the Persistence Conjecture (which describes a class of mass-action systems for which variables do not approach zero) and the Global Attractor Conjecture (which describes a class of mass-action systems for which trajectories converge to a single positive equilibrium). We show that an extended version of the Persistence Conjecture holds for systems with two-dimensional stoichiometric subspace and bounded trajectories and we prove the Global Attractor Conjecture for systems with three-dimensional stoichiometric subspace.

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MS47

Sequential and Distributive Multisite Phosphorylations Have Toric Steady States

We show that the steady states of the sequential and distributive multisite phosphorylations system, are described by binomial equations, and can thus be explicitly parametrized by monomials. This result is implicit in [Wang and Sontag, 2008] and it is a particular case of [Thomson and Gunawardena, 2009]. We moreover give sufficient conditions for any chemical reaction system to have *toric steady states*. This is joint work with Alicia Dickenstein, Anne Shiu and Carsten Conradi.

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MS48

Finite Groebner Bases in Infinite Dimensional Polynomial Rings and Applications

We discuss the theory of monoidal Groebner bases, a concept which generalizes the familiar notion in a polynomial ring and allows for a description of Groebner bases of ideals that are stable under the action of a monoid. The main motivation for developing this theory is to prove finiteness theorems in commutative algebra and its applications. A major result of this type is that ideals in infinitely many indeterminates stable under the action of the symmetric group are finitely generated up to symmetry. We discuss how this machinery gives new proofs of some classical finiteness theorems in algebraic statistics as well as a proof of the independent set conjecture of Hosten and the second author.

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MS48

Finiteness Theorems for Chains of Lattice Ideals

We study chains of lattice ideals that are invariant under a symmetric group action. In our setting, the ambient rings for these ideals are polynomial rings which are increasing in (Krull) dimension. Thus, these chains will fail to stabilize

in the traditional commutative algebra sense. However, we prove a general theorem which says that "up to the action of the group", these chains stabilize up to monomial localization. This gives a partial resolution to a conjecture of Aschenbrenner and Hillar.

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MS48

Defining Equations of Secant Varieties to Segre-Veronese Varieties

We describe the defining ideal of the r th secant variety of $\mathbf{P}^2 \times \mathbf{P}^n$ embedded by $\mathcal{O}(\infty, \epsilon)$, for arbitrary n and $r \leq 5$. We also present the Schur module decomposition of the space of generators of each such ideal. Our main results are based on a more general construction for producing explicit matrix equations that vanish on secant varieties of products of projective spaces. This extends previous work of Strassen and Ottaviani.

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MS48

Algebraic Identification of Binary-Valued Hidden Markov Processes

The complete identification problem is to decide whether a stochastic process (X_t) is a hidden Markov process and if yes to infer a corresponding parametrization. So far only partial answers to either the decision or the inference part have been given all of which depend on further assumptions on the processes. Here we present a full, general solution for binary-valued hidden Markov processes. Our approach is rooted in algebraic statistics hence geometric in nature. We demonstrate that the algebraic varieties which describe the probability distributions associated with binary-valued hidden Markov processes are zero sets of determinantal equations which draws a connection to well-studied objects from algebra. As a consequence, our solution provides immediate algorithmic access where tests come in form of elementary (linear) algebraic routines.

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MS48

Finiteness on Homogeneous Markov Chain Models

In 2010, Takemura and Hara showed a complete description of a Markov basis for conditional tests of the toric homogeneous Markov chain model, the envelope exponential family for the homogeneous discrete time Markov chain model, with the following cases; (1) two-state, arbitrary length; (2) arbitrary finite state space and length of three; and (3) two-state, arbitrary length without initial parameters. Motivated by these results, we study finiteness of Markov bases for the toric homogeneous Markov chain model. In this talk we consider the toric homogeneous Markov chain model without initial parameters and without any loops for the state space S with $|S| = 3, 4$. A key tool is a *state graph*, that is, the directed multigraph $G(x)$ such that vertices are given by the states in S and the directed edges are given by the transitions from state i to j in x , a summary of moves in Markov chains.

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MS49

Mahonian Partition Identities Via Polyhedral Geometry

In a series of papers, George Andrews and various coauthors successfully revitalized seemingly forgotten, powerful machinery based on MacMahon's Omega operator to systematically compute generating functions of integer partitions. Our goal is to geometrically prove and extend many of the Andrews et al theorems, by realizing a given family of partitions as the set of integer lattice points in a certain polyhedron. This is joint work with Matthias Beck and Nguyen Le.

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MS49

Lattice Points in Polyhedra and the Summation Method for Mixed-Integer Optimization

We present a new fully polynomial-time approximation scheme for the problem of optimizing non-convex polynomial functions over the mixed-integer points of a polytope of fixed dimension [J.A. De Loera, R. Hemmecke, M. Kppe, R. Weismantel, FPTAS for optimizing polynomials over the mixed-integer points of polytopes in fixed dimension, Math. Prog. Ser. A 118 (2008), 273–290]. The algorithm also extends to a class of problems in varying dimension. This is the culmination of an effort to extend the efficient theory of discrete generating functions of lattice points in polyhedra to the mixed-integer case in a practical way, without using discretization, and to bring it to use in optimization.

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MS49

Vector Partition Functions and Applications to Lie Representation Theory

Given a finite set I of non-zero integral vectors with non-negative coordinates, and a vector γ with coordinates $(\gamma_1, \dots, \gamma_n)$, the vector partition function $P_I(\gamma)$ is the number of ways we can split γ as an integral sum with non-negative coefficients of the vectors in I . We formulate an algorithm for computing the vector partition function as quasipolynomials in $(\gamma_1, \dots, \gamma_n)$ over a finite set of combinatorial chambers and demonstrate an on-line implementation. We explain some problems from Lie representation theory motivating this study.

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MS49

Ehrhart Theory and Lecture Hall Polytopes

For a sequence $s = (s_1, \dots, s_n)$ of positive integers, define the s -lecture hall polytope to be the set $\{x \in R^n \mid 0 \leq \frac{x_1}{s_1} \leq \frac{x_2}{s_2} \leq \dots \leq \frac{x_n}{s_n}\}$. We prove a theorem relating the Ehrhart series of the s -lecture hall polytope to ascent statistics on s -inversion sequences. We show how the theorem can be refined and specialized to yield known results and derive new ones. This includes joint work with Michael Schuster, Gopal Viswanathan, and Thomas Pensyl.

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MS49

An Efficient Calculation of the Top Ehrhart Coefficients for the Knapsack.

Let \mathbf{A} be a sequence of $N + 1$ positive integers. For t an integer, let $E(\mathbf{A})(t)$ be the number of solutions in non-negative integers x_i of the equation $\sum_{i=1}^{N+1} A_i x_i = t$. Then $E(\mathbf{A})(t) = \sum_{i=0}^N E_i(t) t^i$ where $E_i(t)$ is a periodic function of t . Fixing k , we show that the residue theorem in one variable, and the signed unimodular decomposition of a cone, leads to an algorithm of polynomial complexity to compute the top k -coefficients of $E(\mathbf{A})(t)$.

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MS50

Expected Complexity of Bisection Methods for Real Solving

We examine degree- d polynomials with iid coefficients under two zero-mean normal distributions, such that the i -th coefficient has variance $\binom{d}{i}$, or $1/i!$. The expected bit complexity of the Sturm solver is $O^*(rd^2\tau)$, where r bounds the number of real roots and τ the coefficient bitsize. For Bernstein polynomials with iid coefficients of moderate standard deviation, we show $E[r] = \sqrt{2d} \pm O(1)$.

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MS50

Hybrid Method for Solving Bivariate Polynomial System

Many problems in science and engineering can be reduced to that of solving bivariate polynomial systems of equations. Intensive research efforts have yielded reliable symbolic methods and efficient numeric methods. We combine the advantages of symbolic methods (in particular resultant) and numeric methods (in particular Hansen-Sengupta operator with slope form). The resulting method can reliably approximate all the solutions more efficiently than the other (symbolic, numeric) methods in most cases.

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MS50

Accurate Path Tracking

Tracking solution paths defined by a family of polynomial systems is a basic operation for many algorithms in numerical algebraic geometry. Version 2.3.55 of PHCpack incorporated the QD-2.3.9, a software library of Y. Hida, X.S.

Li, and D.H. Bailey for quad double arithmetic. In this talk we will explain the adaptive use of double double and quad double arithmetic in the path trackers of PHCpack.

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MS50

An Efficient Exact Numerical Algorithm for Isotopic Approximation of NonSingular Surfaces

We describe a subdivision algorithm for computing an isotopic ϵ -approximation of a nonsingular surface $f^{-1}(0)$ where $f : R^3 \rightarrow R$ is a C^1 function. Our algorithm (called Cxyz) is based on numerical predicates which are easy to implement and does not suffer from implementation gaps. Cxyz combines the technique of non-local isotopy from Plantinga-Vegter (2004) and parametrizability from Snyder (1996). Our implementation shows that this is more efficient than either approach separately. The correctness of Cxyz is non-trivial.

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MS51

Polyhedral Methods For Positive Dimensional Solution Sets

We present a polyhedral algorithm to manipulate positive dimensional solution sets. Using facet normals to Newton polytopes as pretropisms, we zero in at the first two terms of a Puiseux series expansion. We illustrate how this polyhedral algorithm gives insight into the structure of a tropical prevariety. This polyhedral algorithm is well suited for exploitation of symmetry, when it arises in systems of polynomials. Initial form systems with pretropisms in the same group orbit are solved only once, allowing for a systematic filtration of redundant data. The computational results will be illustrated on cyclic n-roots polynomial systems.

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MS51

Hom4ps in Parallel

When solving systems of polynomial equations by the polyhedral homotopy methods, while the path tracing part is naturally parallel, the mixed cell computation which provides the starting points for the homotopy paths is highly serial. Recently, we have successfully reformulated the mixed cell computation and achieved very high speed-ups in parallel. In this talk, we will report our most updated parallel implementation of both path tracing and mixed cell computation algorithms. The parallel implementations are designed for a variety of parallel architectures such as multi-core processors, symmetric multiprocessors, nonuniform memory access architectures, clusters, distributed computers, and the exciting new technology of general purpose GPUs.

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MS51

Real Solutions and Numerical Algebraic Geometry

In many applications, the real solutions to polynomial systems are the ones of interest. This talk will consider using classical approaches with methods in numerical algebraic geometry to compute real solutions of a given polynomial system. Examples arising from applications will be presented to demonstrate the practicality of the methods.

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MS51

Multithreaded Newton's Method and Path Tracking

We have developed a multithreaded implementation of Newton's method on a multicore workstation in a static memory allocation multiprecision environment using the quad double arithmetic in the QD-2.3.9. library. Furthermore, based on this implementation, we obtained a complete parallel version of a path tracker. In my talk I will report on the details of work load distribution among the threads and of synchronization in our parallel implementation. I will also discuss to what extent our use of multithreading compensates the overhead caused by employing higher precision arithmetic, when tracking a solution path.

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MS52**AG Codes for Secure Multiparty Computation**

Chen and Cramer (Crypto 2006) use algebraic curves to construct linear secret sharing schemes that are suitable for secure multiparty computation. We describe the construction and we analyse its main properties in terms of the geometry of the curve.

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MS52**Groebner Bases and Linear Codes**

Many classes of good codes have been constructed from algebraic curves. These constructions, however, have restrictions on the alphabet size (say q has to be a square) and on the block length (which depends on the number of rational points). We are interested in a more general approach that avoids these restrictions. More precisely, given any prime power q and any block length n , Farr and Gao (2003) shows how Groebner basis theory can be used to construct linear codes of length n over $\text{GF}(q)$ from any set of n points from an affine space. In this talk, we shall discuss various computational problems related to this class of codes, including decoding via Groebner bases.

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MS52**Bounding the Number of Points on a Curve using a Generalization of Weierstrass Semigroups and an Application to Toric Codes**

In this talk we use techniques from coding theory to derive upper bounds for the number of rational places of an algebraic curve defined over a finite field. The used techniques yield upper bounds if the (generalized) Weierstrass semigroup for an n -tuple of places is known. This sometimes enables one to get an upper bound for the number of rational places for families of function fields. We consider an application to toric codes.

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MS52**Bent Functions on a Galois Ring and Systematic Authentication Codes**

Authentication codes with secrecy and without secrecy, produced by different techniques, have received consider-

able attention lately. A subclass of the latter is the Systematic Authentication Codes (SACs). Several types of SACs have appeared constructed by using highly nonlinear functions over finite fields or non-degenerated and rational functions on a Galois ring. In this talk a class of bent functions over a Galois ring of characteristic p^2 is introduced and based on these functions a class of SACs is given. These SAC's generalize some appearing in the literature for finite fields.

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MS52**Complexity in Coding Theory**

We will give an introduction to the complexity issues in coding theory, both for general codes and for specific codes such as algebraic geometric codes of low genus.

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MS53**Some Examples of Non-finitely Generated $\text{Sym}(N)$ -invariant Ideals**

We are concerned with $\text{Sym}(N)$ -invariant (or similar) ideals of subalgebras of polynomial algebras in countably infinitely many variables. These ideals arose recently from applications to statistics and chemistry. Aschenbrenner, Hillar, Sullivant and Draisma have proved that in certain subalgebras all $\text{Sym}(N)$ -invariant ideals are finitely generated (as such). In this talk, we will present some examples of non-finitely generated $\text{Sym}(N)$ -invariant ideals in some subalgebras closed to ones mentioned above.

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MS53**Tensors of Bounded Rank are Defined in Bounded Degree**

An element of a p -fold tensor product of vector spaces has border rank k if it may be approximated by a sum of k (pure) tensors. The variety of tensors of a given border rank is a classical object of algebraic geometry (a higher secant variety) with important modern applications in various diverse areas (algebraic geometry, statistics, complexity theory). Except for small k no system of defining equations for this variety is known. We show that for each k there exists a universal degree bound d such that for each p there are defining equations of degree at most d for the variety of p -dimensional tensors of border rank k . The cru-

cial ingredient is to remove the dependence on p by passing to the limit $p \rightarrow \infty$ and using the action of some natural group of symmetries. In fact, we show that up to these symmetries, there are finitely many equations defining the variety for all p -dimensional tensors as long as p is large enough.

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MS53

Associative Algebras and Letterplace Embedding

We introduce an algebra embedding $\iota : K\langle X \rangle \rightarrow S$ from the free associative algebra $K\langle X \rangle$ generated by a finite or countable set X into the skew monoid ring $S = P * \Sigma$, where $P = K[X \times N]$ and $\Sigma = \langle \sigma \rangle$ is the monoid, generated by a suitable endomorphism $\sigma : P \rightarrow P$. We present a general Gröbner bases theory for graded two-sided ideals of the graded algebra $S = \bigoplus_i P\sigma^i$, where $P = K[Y]$ is any commutative polynomial ring and $\sigma : P \rightarrow P$ is an abstract endomorphism satisfying two explicit conditions. We obtain a bijective correspondence, preserving Gröbner bases, between graded Σ -invariant ideals of P and a class of graded two-sided ideals of S . As an application we show, that up to a given degree, Gröbner bases of finitely generated graded ordinary difference ideals can be computed by the proposed methods in S in a finite number of steps.

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MS53

Secant Varieties of Segre-Veronese Varieties

Secant varieties of Segre and Veronese varieties are classical objects that go back to the Italian school in the nineteenth century. Surprisingly, very little is known about their equations. Inspired by experiments related to algebraic statistics, Garcia, Stillman and Sturmfels gave a conjectural description of the generators of the ideal of the secant line variety $Sec(X)$ of a Segre variety X . This generalizes the familiar result which states that matrices of rank two are defined by the vanishing of their 3×3 minors. For a Veronese variety X , it was known by work of Kanev that the ideal of $Sec(X)$ is generated in degree three by minors of catalecticant matrices. I will introduce a new technique for studying the equations of the secant varieties of Segre-Veronese varieties, based on the usual representation theoretic approach to this problem. I will explain how this technique applies to show that for X a Segre-Veronese variety, the ideal of $Sec(X)$ is generated in degree three by minors of matrices of flattenings, and to give a description of the decomposition into irreducible representations of the homogeneous coordinate ring of $Sec(X)$. This will recover as special cases the conjecture of Garcia, Stillman

and Sturmfels, and the result of Kanev.

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MS53

Twisted Commutative Algebras and Delta-modules

Twisted commutative algebras and Δ -modules are two kinds of algebraic structures that can be interpreted as having infinitely many variables. I will explain what they are and state two finiteness theorems about them: the first is a noetherianity result, the second a rationality result for Hilbert series. If time permits, I will explain how these results can be applied to study the syzygies of certain families of varieties.

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MS54

The Convex Geometry of Linear Inverse Problems

Building on the success of generalizing compressed sensing to low-rank matrix recovery, we extend the catalog of structures that can be recovered from partial information. We describe algorithms to decompose signals into sums of atomic signals from a simple, not necessarily discrete, set. These algorithms are derived in a convex optimization framework that generalizes previous methods based on ℓ_1 -norm and nuclear norm minimization. We discuss general recovery guarantees for our approach and several example applications.

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MS54

Extended BIC and Bayesian Marginal Likelihood in Sparse Graphical Models

We consider the problem of recovering sparse graphical structure in the setting of Gaussian or discrete graphical modeling. While the total number of possible sparse models is often too large for exhaustive search, information criteria may be used to select a sparse model from a reduced set of candidate models (for instance, the “path” of models selected by a parameterized procedure such as the graphical lasso). Modern applications of graphical models often have a number of nodes that is comparable to or greater than the number of data points observed. In our treatment of an extended Bayesian information criterion (BIC) we thus consider asymptotic scenarios in which the number of nodes p to grows as the sample size n increases. Our work examines the consistency properties of the extended BIC as well as its relationship to the marginal likelihood

arising in fully Bayesian approaches.

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MS54

Diagonal/Low-Rank Decomposition and Correlation Matrices

Suppose an $n \times n$ matrix X is the sum of a diagonal and a psd low-rank matrix. Decomposing X into these unknown constituents has applications in statistics, signal processing, and elsewhere. We give a simple condition on the column space of the low rank matrix that ensures a tractable convex program can correctly decompose X . Our analysis highlights connections between this problem and the structure of the set of correlation matrices.

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MS54

Chromosome Packing in Cell Nucleus

During most of the cell cycle each chromosome occupies a roughly spherical domain called a chromosome territory. Chromosome territories can overlap and their radial and relative positions are non-random and similar among similar cell types. A chromosome arrangement can be viewed as a packing of overlapping spheres of various sizes inside an ellipsoid, the cell nucleus. We present a non-convex model for chromosome arrangements together with an alternating minimization algorithm. Using this model, we simulate chromosome arrangements and study the resulting number and volume of internal 'holes', which make chromosomes deep inside accessible to regulatory factors.

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MS54

Finding Approximately Rank-One Submatrices with the Nuclear Norm and l_1 -Norm

We propose a tractable method to find large approximately rank-one submatrices which can be used in nonnegative matrix factorization. We apply a proximal point algorithm to solve the resulting optimization problem which is formulated with the nuclear norm and l_1 -norm. We report numerical results on realistic datasets of decomposable bitmap images and nearly separable corpus.

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MS55

Continuous Amortization for the Complexity of Adaptive Subdivision Methods

Adaptive subdivision algorithms subdivide more times near challenging features and fewer times elsewhere. Computations of the size of the subdivision tree should consider adaptivity: e.g., bounds on the maximum depth do not reflect situations where the tree has a few deep paths, but remains modest in overall size. I use the new analysis technique of continuous amortization to show that the tree size of a simple, evaluation-based root isolation algorithm is nearly optimal.

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MS55

Patterns in Roots of the Derivatives of Random Univariate Polynomials

I will first recall old and recent results on distributions of roots of random polynomials and eigenvalues of random matrices. Then I will present original observations of patterns of roots of the derivatives of random polynomials. I will set some conjectures enforced by experiments and outline proofs of some claims. This can be applied to design root isolation algorithms.

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MS55

Subadditivity in Polynomial Real Root Isolation

Isac Schoenberg's theory of variation-diminishing linear transformations, published in 1930, is used to prove the following subadditivity property for all univariate real polynomials A ,

$$\text{var}(T(A)) + \text{var}((T \circ R)(A)) \leq \text{var}(A).$$

Here, T , R and var are, respectively, the translation by 1, the reciprocal transformation, and the number of sign variations. The subadditivity property is useful in the implementation and analysis of the continued fractions method for polynomial real root isolation. This research is joint work with G. E. Collins.

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MS55**Subdivision Methods for the Topology and Arrangement of Semi-algebraic Curves and Surfaces**

Computing the topology and arrangement of algebraic curves and surfaces appears in many geometric modelling problems, such as surface-surface intersection, self-intersection, ... In the presentation, we will describe subdivision methods which exploit information on the boundary of regions and which only require the isolation of characteristic points. We will show how topological degree computation can help analysing the number of branches at singular points. Combining regularity criterion with subdivision strategies, this yields a complete algorithm for computing the topology of (singular) algebraic curves. Extension of this approach to arrangement computation of curves or surfaces will be described. Experimentation with the algebraic-geometric modeller AXEL will shortly be demonstrated.

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PP1**The Geometry of Tree Reconstruction**

Tree agglomeration methods such as the Neighbor-Joining and UPGMA algorithms continue to play a significant role in phylogenetic research. We use polyhedral geometry to analyze the natural subdivision of Euclidean space induced by classifying the input vectors according to tree topologies returned by an algorithm. We also use lattice theory to analyze disparities between observed and predicted shape statistics on reconstructed trees.

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PP1**The Maximum Likelihood Degree for the Random Effects Model**

Given a statistical model, the maximum likelihood degree (ML degree) is the degree of the variety defined by the likelihood equations. It measures the algebraic complexity of the computation to find model parameters that best explain a given data point. In this poster, we give an explicit formula for the ML degree of the analysis of variance model with random effects. This is joint work with Mathias Drton and Sonja Petrovic.

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PP1**Unexpected Reality: the Monotone-Secant Conjecture****ture**

The Monotone Secant Conjecture poses a rich class of polynomial systems, all of whose solutions are real. These systems come from the Schubert calculus on flag manifolds, and the Monotone Secant Conjecture is a compelling generalization of the Shapiro Conjecture for Grassmannians (Theorem of Mukhin, Tarasov, and Varchenko). We present the Monotone Secant Conjecture as a generalization of the Shapiro Conjecture and explain the massive computational evidence in its favor.

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PP1**Dual Space Computations for Homotopy Continuation Using H-Bases**

Using homotopy continuation methods, certain systems end up with a number of paths tending toward infinity. Using dual space computations through H-bases, our goal is to pre-condition these systems. By moving to the dual space and using H-bases, we can cut out many of these extraneous paths. When we come back from the dual space, we have a system with fewer paths tracking to infinity thereby saving time and computation. We examine how many times to efficiently implement this for optimal time and results.

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PP1**Markov Bases for the Analysis of Partially Ranked Data**

Suppose survey respondents have been asked to give a complete ranking of a collection of items. Diaconis and Sturmfels, and Diaconis and Eriksson, have shown how Markov bases can be used to better understand relationships between natural summary statistics associated with the resulting *fully ranked data*. In this talk, we show how their ideas can be fruitfully extended to situations in which respondents are asked to provide rankings of only a subset of the items.

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PP1**Algebraic Ecological Inference**

Ecological inference is the process of inferring individual level behavior from aggregate data. The problem of ecological inference occurs frequently in many applied sciences. In the present work, we present an algebraic solution to the problem of ecological inference using tools from computational algebra. This method can handle multi-dimensional contingency tables, deterministically respects bounds and can incorporate information from many different sources. We illustrate the method by a few examples and provide

R code.

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PP1

The Convex Hull of Circles

We describe the boundaries of the convex hulls of perhaps the simplest compact curves in three-dimensional space, namely pairs of circles. We show that the edge surface of such a convex hull is in general an irrational ruled surface whose ruling forms a (2,2)-curve, and conjecture that any real (2,2)-curve can arise from the edge surface of the convex hull of two circles.

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PP1

Developing Water Loss Prevention Robot System Geometrical Design

This research describes the geometrical design of an in-pipe robot system able to redevelop fresh water pipes of different diameters. The robots geometry is calculated in conjunction of its working environment, the size of the pipes, and its physical properties, such as weight and required force to move and redevelop the pipe.

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PP1

Perturbation Analysis to Design a Robust Decoupling Geometric Technique in Linear Multi-Input Multi-Output Systems

The linear state-feedback control is one of the basic and fundamental part of the linear control theory. Eigenvectors are used through the singular value decomposition (svd) in robust analysis and synthesis of robust controllers in multi-input multi-output systems. In this paper the problem of robustness of a decoupling controller is analyzed. In particular, it is shown that the robustness of a decoupling controller is achieved using subspaces closer to the eigenvectors associated to the system. The analysis of the robustness is based on a perturbation method and a synthesis of a controller is shown. An application concerning an electrical motor is shown.

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PP1

A Stochastic Framework for Discrete Models in

Systems Biology

We present a new modeling framework for gene regulatory networks that incorporates state dependent delays and that is able to capture the cell-to-cell variability. We present this framework in the context of polynomial dynamical systems where we can use computational algebra to analyze our model. The state dependent delays represent the time delays of activation and degradation. One of the new features of this framework is that it allows a finer analysis of discrete models and the possibility to simulate cell populations. We applied our methods to one of the best known stochastic regulatory networks, that is involved in controlling the outcome of lambda phage infection of bacteria.

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PP1

An Implementation of Parameter Homotopies and their Applications

There are several implementations of homotopy continuation-based polynomial system solvers, some of which allow for so-called parameter homotopies between polynomial systems that differ only in coefficients. Some applications from science and engineering require the repeated solution of polynomial systems with slightly different coefficients. The new software package described in this poster is a parallelized implementation of parameter homotopies, making use of Bertini (D. Bates, J. Hauenstein, A. Sommese, C. Wampler). An example considering bistability in biochemical kinetic reactions is provided to show the usefulness of these methods. This is joint work with Dan Bates and Dan Brake.

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PP1

Computing the Variety for K-Algebra Homomorphisms

Let k be an algebraically closed field. Let A and B be arbitrary commutative (unitary) k -algebras. Assume $V \subset A$ and $W \subset B$ are finite dimensional k -linear subspaces. Denote the subalgebras of A and B generated by V and W as $A(V)$ and $B(W)$. Then the set $\mathbf{R}Hom(A, B, V, W)$ of k -algebra homomorphisms $f : A(V) \rightarrow B(W)$ such that $f(V) \subset W$ is an affine k -variety in a natural way. The structure of the proof of this claim suggests an algorithm could be developed to allow software to calculate the affine variety $Hom(A, B, V, W)$. The goal of my research is to develop software capable of doing this calculation and use this software to compute some classical algebras (e.g. group algebras, monomial algebras etc).

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