

**IP0****Dénes König Prize Lecture: Ramsey Numbers of Graphs and Hypergraphs**

Determining or estimating Ramsey numbers is one of the central problems in combinatorics. In this talk we discuss recent progress on some longstanding conjectures in this area which have played an important role in the development of Ramsey theory.

Jacob Fox

Princeton University  
jacobfox@math.princeton.edu

**IP1****Binary Matroid Minors**

The graph minors project is a sequence of extraordinary theorems proved by Neil Robertson and Paul Seymour in the 1980s. The banner theorem of that project is that every minor-closed family of graphs is characterized by a finite set of excluded minors. The project also has a number of remarkable algorithmic consequences, for example, the membership testing problem can be efficiently solved for any minor-closed class of graphs. Much of the work in the graph minors project goes into proving a technical theorem that constructively characterizes all minor-closed classes of graphs. The graph minors project can be interpreted in the context of matroid theory by considering the class of graphic matroids. Over the past decade, in joint work with Bert Gerards and Geoff Whittle, we have extended much of the graph minors project from the class of graphic matroids to the class of binary matroids. This talk will be introductory, no prior knowledge of matroid theory is assumed. We give a broad overview of the results and focus on potential applications to graph theory, coding theory, and quantum computing.

Jim Geelen

University of Waterloo  
jggeelen@math.uwaterloo.ca

**IP2****The Combinatorics of Discrete Random Matrices**

In random matrix theory, both continuous random matrix ensembles (e.g. the gaussian unitary ensemble, GUE) and discrete random matrix ensembles (e.g. the Bernoulli ensemble of random sign matrices, or the adjacency matrices of random graphs) are of interest. However, the discrete case contains additional difficulties that are not present in the continuous case. For instance, it is obvious that continuous random square matrices are almost surely invertible, but this is not true in the discrete case. Nevertheless, in recent years several tools of an additive combinatorics nature have been developed to close the gap between our understanding of discrete random matrices and continuous random matrices, and in particular inverse Littlewood-Offord theory, which roughly speaking asserts that discrete random walks behave much like their continuous counterparts, except in highly arithmetically structured cases, such as when the step sizes of the random walk all lie in an arithmetic progression. We survey these developments, and their applications, in this talk.

Terence Tao

Department of Mathematics, UCLA  
tao@math.ucla.edu

**IP3****Extremal Problems for Convex Lattice Polytopes**

In this survey I will present several extremal problems, and some solutions, concerning convex lattice polytopes. A typical example is to determine the minimal volume that a convex lattice polytope can have if it has exactly  $n$  vertices. Other examples are the minimal surface area, or the minimal lattice width in the same class of polytopes. These problems are related to a question of V I Arnold from 1980 asking for the number of (equivalence classes of) lattice polytopes of volume  $V$  in  $d$ -dimensional space, where two convex lattice polytopes are equivalent if one can be carried to the other by a lattice preserving affine transformation.

Imre Bárány

Rényi Institute  
University College London  
barany@renyi.hu

**IP4****Optimizing in a Strategic World: A Survey of Recent Research in Algorithmic Game Theory**

The goal of discrete optimization is to design systems with optimal or near-optimal performance. In the age of the Internet, however, we must take into account the fact that many of the users of our systems are driven by an economic goal, and interact with varying degrees of collaboration and competition. Moreover, the strategic nature of interactions in online dynamic marketplaces means that the roll-out of a new algorithm designed with the expectation of improved performance can end up degrading performance due to unanticipated responses by strategic users. The field of algorithmic game theory addresses this issue, as well as a wide variety of other problems at the intersection of game theory, economics and computer science. In this talk, we survey recent research and open problems in this field.

Anna R. Karlin

University of Washington  
karlin@cs.washington.edu

**IP5****The Method of Multiplicities**

In 2008, Zeev Dvir achieved a breakthrough in combinatorial geometry by giving a stunningly simple, and sharp, bound on the size of "Kakeya Sets" in  $F_q^n$ , the  $n$ -dimensional vector space over the finite field on  $q$  elements. (A Kakeya set in any vector space is a set that contains a line in every direction.) Dvir proved this bound by setting up an  $n$ -variate low-degree non-zero polynomial that vanished on every point in the set, and then used the algebraic nature of a Kakeya set to argue that this polynomial was zero too often if the set was too small. In addition to resolving a long-studied problem in combinatorial geometry, this method also led to new constructions of "randomness extractors". In this talk I will describe algebraic methods to improve the analysis of Dvir, by using polynomials that vanish with "high multiplicity" on every point on the given set. This method, based on prior work with Guruswami (1998), ends up yielding extremely tight (to within a factor of 2) bounds on the size of Kakeya sets; and, in combination with a host of other techniques, state-of-the-art "extractors" (algorithms that purify randomness). In this talk I will describe the (simple) idea behind the method of multiplicities and some of the applications. Based on joint

works with Shubhangi Saraf (Analysis & PDE, 2009); and with Zeev Dvir, Swastik Kopparty, and Shubhangi Saraf (FOCS 2009).

Madhu Sudan  
Microsoft Research  
madhu@microsoft.com

## IP6

### Configurations in Large $t$ -connected Graphs

I will discuss a technique that allows us to establish the existence of certain configurations in large  $t$ -connected graphs, even though such configurations need not exist in small graphs. Two configurations of interest are complete minors and disjoint paths connecting prescribed pairs of vertices. We prove that for every integer  $t$  there exists an integer  $N$  such that every  $t$ -connected graph on at least  $N$  vertices with no minor isomorphic to the complete graph on  $t$  vertices has a set of at most  $t-5$  vertices whose deletion makes the graph planar. This is best possible, except for the value of  $N$ . We also prove that the  $k$  Disjoint Paths Problem is feasible in every sufficiently big  $(2k+3)$ -connected graph. This is joint work with Sergey Norin.

Robin Thomas  
Georgia Tech  
thomas@math.gatech.edu

## IP7

### Hypergraphs with Low Dimension

Any hypergraph can be viewed as an object with geometric properties, by considering a geometric realisation of its associated abstract simplicial complex. It is well-known that any  $k$ -uniform hypergraph has such a realisation in  $(2k-1)$ -dimensional real space. We focus in particular on  $k$ -uniform hypergraphs that have a geometric realisation in  $k$ -dimensional space (so when  $k=2$  this is the class of planar graphs). We consider some properties of planar graphs that naturally extend to this class of ‘low-dimensional’ hypergraphs.

Penny Haxell  
University of Waterloo  
pehaxell@math.uwaterloo.ca

## IP8

### A Survey of Alternating Permutations

A permutation  $a_1 a_2 \cdots a_n$  of  $1, 2, \dots, n$  is *alternating* if  $a_1 > a_2 < a_3 > a_4 < a_5 > \cdots$ . If  $E_n$  is the number of alternating permutations of  $1, 2, \dots, n$ , then

$$\sum_{n \geq 0} E_n \frac{x^n}{n!} = \sec x + \tan x.$$

We will discuss several aspects of the theory of alternating permutations. Some occurrences of the numbers  $E_n$ , such as counting orbits of group actions and volumes of polytopes, will be surveyed. The behavior of the length of the longest alternating subsequence of a random permutation will be analyzed, in analogy to the length of the longest increasing subsequence. We will also explain how various classes of alternating permutations, such as those that are also fixed-point free involutions, can be counted using umbral techniques arising from a certain representation of the

symmetric group  $S_n$  whose dimension is  $E_n$ .

Richard Stanley  
Massachusetts Institute of Technology  
rstan@math.mit.edu

## CP1

### A Solution to Alspach’s Problem for Complete Graphs of Large Odd Order

In 1981 Alspach posed the problem of proving that a complete graph of odd order  $n$  can be decomposed into edge-disjoint cycles of specified lengths  $m_1, m_2, \dots, m_t$  whenever the obvious necessary conditions that  $3 \leq m_1, m_2, \dots, m_t \leq n$  and  $m_1 + m_2 + \cdots + m_t = \binom{n}{2}$  are satisfied. In this talk I will give a brief outline of a solution to Alspach’s problem for sufficiently large odd values of  $n$ .

Daniel Horsley  
Memorial University of Newfoundland  
danhorsley@gmail.com

Darryn Bryant  
University of Queensland  
db@maths.uq.edu.au

## CP1

### When Every $k$ -Cycle Has at Least $f(k)$ Chords

Chordal graphs can be characterized by every  $k$ -cycle having at least  $k-3$  chords. Similarly, requiring  $\geq 2\lfloor \frac{k-3}{2} \rfloor$  chords characterizes the house-hole-domino-free graphs, and requiring  $\geq 2k-7$  chords characterizes graphs whose blocks are trivially perfect. Moreover, these three functions  $f(k)$  are optimum for their graph classes—there are always graphs in each class that have  $k$ -cycles with exactly  $f(k)$  chords. The functions  $3\lfloor \frac{k-3}{3} \rfloor$  and  $3k-11$  characterize similar graph classes without being optimum.

Terry McKee  
Wright State University  
Department of Mathematics & Statistics  
terry.mckee@wright.edu

## CP1

### 4-Cycle Systems of $K_n$ with An Almost 2-Regular Leave

An almost 2-regular leave of  $K_n$  is a subgraph of  $K_n$  in which each vertex except one has degree two; the exceptional vertex has arbitrary degree. In this talk, we will provide the necessary and sufficient conditions for the existence of 4-cycle systems of  $K_n$  with an almost 2-regular leave. The solution to this problem has applications in neighbor designs used by serology researchers.

Nidhi Sehgal, C. A. Rodger  
Auburn University  
sehgani@auburn.edu, rodgec1@auburn.edu

## CP1

### Approaching Kelly’s Conjecture

A Hamilton decomposition of a digraph  $G$  is a set of edge-disjoint Hamilton cycles which together cover all the edges of  $G$ . A conjecture of Kelly from 1968 states that every

regular tournament has a Hamilton decomposition. We recently proved the following approximate version of Kelly's conjecture: Every regular tournament on  $n$  vertices contains  $(1/2 - o(1))n$  edge-disjoint Hamilton cycles. I will discuss some of our techniques as well as some related open problems.

Andrew Treglown, Daniela Kuehn, Deryk Osthus  
University of Birmingham  
treglowa@maths.bham.ac.uk, kuehn@maths.bham.ac.uk,  
osthus@maths.bham.ac.uk

## CP2

### Packing of Degree Sequences

We consider the question of when graphic sequences can be simultaneously realized by edge-disjoint graphs on the same vertex set. Results will be presented in both the bipartite and non-bipartite cases.

Stephen Hartke  
Department of Mathematics  
University of Nebraska - Lincoln  
hartke@math.unl.edu

## CP2

### The Embedded Graphs of a Knot and the Partial Duals of a Plane Graph

Recently O. Dasbach et. al. introduced a way to construct an embedded graph from a knot. Two fundamental questions arise from this construction: "which embedded graphs represent knots?" and "how are knots presented by the same embedded graph related to each other?". In this talk I will answer both of these questions through the use of a characterization of the partial duals of a plane graph in terms of separability.

Jain Moffatt  
University of South Alabama  
imoffatt@jaguar1.usouthal.edu

## CP2

### Progress on Hendry's Tables of Ramsey Numbers

In 1989, George R. T. Hendry presented a table of two-color graph Ramsey numbers  $R(G, H)$  for all pairs of graphs  $G$  and  $H$  having five vertices, except seven cases:  $R(K_5 - K_{1,3}, K_5) = R(K_4, K_5)$ ,  $R(W_5, K_5 - e)$ ,  $R(B_3, K_5)$ ,  $R(W_5, K_5)$ ,  $R(K_5 - P_3, K_5)$ ,  $R(K_5 - e, K_5)$  and  $R(K_5, K_5)$ . The values  $R(K_4, K_5) = 25$  and  $R(W_5, K_5 - e) = 17$  are known since 1995. In this talk we overview the other two solved cases  $R(B_3, K_5) = 20$  and  $R(W_5, K_5) = 27$ , where  $B_3 = K_2 + \overline{K}_3$  and  $W_5 = K_1 + C_4$ , and present the progress (or lack of it) on the remaining three open cases.

Stanislaw P. Radziszowski  
Department of Computer Science  
Rochester Institute of Technology  
spr@cs.rit.edu

## CP2

### How to Draw a Tait-Colored Graph

Suppose  $G$  is a cubic graph with a Tait-coloring. We wish to draw  $G$  in the plane in such a way that (a) every edge is represented by a line segment, (b) all of the lines supporting the segments sharing a common color are concurrent, and

(c) every supporting line has exactly two vertices. Using a seemingly unrelated result about 3-connected Tait-colored graphs, we shall see a sufficient (and possibly necessary) condition for when  $G$  has such a drawing.

David Richter  
Western Michigan University  
david.richter@wmich.edu

## CP3

### Mobius Inversion for Functions in Several Variables and Combinatorial Identities Involving Mertens and Mobius Functions

Dirichlet's convolution and Möbius inversion can be extended to arithmetical multivariable functions in many ways. We mention two examples. If  $\alpha$  is an arithmetical function in one variable and  $F$  and  $G$  be arithmetical function of  $k$  variables, then we define two convolutions  $\alpha \bullet F$  and  $\alpha \diamond F$ :

$$(\alpha \bullet F)(m, n_1, n_2, \dots, n_{k-1}) = \sum_{d|m} \alpha(d) F\left(\frac{m}{d}, \left\lfloor \frac{n_1}{d} \right\rfloor, \left\lfloor \frac{n_2}{d} \right\rfloor, \dots, \left\lfloor \frac{n_{k-1}}{d} \right\rfloor\right),$$

$$(\alpha \diamond F)(n_1, n_2, \dots, n_k) = \sum_{d|(n_1, n_2, \dots, n_k)} \alpha(d) F\left(\frac{n_1}{d}, \frac{n_2}{d}, \dots, \frac{n_k}{d}\right).$$

For simplicity we let

$$\overline{n}_k = (n_1, n_2, \dots, n_k) \text{ and } \left\lfloor \frac{n}{d} \right\rfloor_k = \left(\left\lfloor \frac{n_1}{d} \right\rfloor, \dots, \left\lfloor \frac{n_k}{d} \right\rfloor\right).$$

**Theorem.** Let  $\alpha$  be arithmetical having an inverse  $\alpha^{-1}$  and let  $F$  and  $G$  are arithmetical of  $k$  variables. Then (a)

$$G(m, \overline{n}_{k-1}) = \sum_{d|m} \alpha(d) F\left(\frac{m}{d}, \left\lfloor \frac{n}{d} \right\rfloor_{k-1}\right) \Leftrightarrow F(m, \overline{n}_{k-1}) = \sum_{d|m} \alpha^{-1}(d) G\left(\frac{m}{d}, \overline{n}_{k-1}\right)$$

(b)

$$G(\overline{n}_k) = \sum_{d|(n_1, n_2, \dots, n_k)} \alpha(d) F\left(\frac{n}{d}\right)_k \Leftrightarrow F(\overline{n}_k) = \sum_{d|(n_1, n_2, \dots, n_k)} \alpha^{-1}(d) G\left(\frac{n}{d}\right)_k$$

As applications we will use these extensions to count for any nonempty finite set of integers the number of its subsets which are relatively prime to a fixed positive integer and we count the number of such subsets having some fixed cardinality. Other applications are combinatorial identities involving Mertens and Mobius functions

Mohamed El Bachraoui  
United Arab Emirates University  
Department of Mathematical Sciences  
melbachraoui@uaeu.ac.ae

## CP3

### Effect of Seasonal Forces and Cross-Immunity on the Transmission Dynamics of Two Strains of Dengue

A deterministic model for the transmission dynamics of two strains of dengue disease is presented. The model, consisting of mutually-exclusive epidemiological compartments representing the human and vector dynamics, has a locally-asymptotically stable, disease-free equilibrium whenever the maximum of the associated reproduction numbers of the two strains is less than unity. The model can have infinitely many co-existence equilibria if infection with one strain confers complete cross-immunity against

the other strain and the associated reproduction number of each strain exceeds unity. On the other hand, if infection with one strain confers partial immunity against the other strain, disease elimination, competitive exclusion or co-existence of the two strains can occur. The effect of seasonality on dengue transmission dynamics is explored using numerical simulations, where it is shown that the oscillation pattern differs between the strains, depending on the degree of the cross-immunity between the strains.

Salisu M. Garbu  
University of Manitoba  
Department of Mathematics  
garba@cc.umanitoba.ca

### CP3

#### Limit Theorems on Weakly Logarithmic Random Assemblies

We are concerned with the asymptotic value distribution problems for additive mappings defined on the random decomposable structures called assemblies. Extending the known approximations of the component structure vector by independent random variables in the total variation metrics to weakly logarithmic classes, we seek sufficient and at the same time necessary convergence conditions when the sizes increase. The one-dimensional, functional, and strong convergence will be discussed.

Eugenijus Manstavicius  
Vilnius University  
Dept. of Mathematics and Informatics  
eugenijus.manstavicius@mif.vu.lt

### CP3

#### The Distribution of the $\delta$ -Transformation for All Derangements of Order $n$ with a Single Cycle

We find a general formula for this distribution. The algorithm was obtained by studying patterns in the unique outputs, obtained from the Burrows-Wheeler Transform for all possible permutations of order  $n$ . We start with an initial distribution and then subtract appropriate elements by making connections with indices in appropriately constructed matrices. We also find some interesting rules and patterns related to these derangements.

Amy Mihnea  
Florida Atlantic University  
amihnea@fau.edu

### CP4

#### Perfect Matchings in Grid Graphs after Vertex Deletions

We consider the  $d$ -dimensional grid graph  $G = G_m^d$  on vertices  $\{1, 2, \dots, m\}^d$  (a subset of  $\mathbf{Z}^d$ ) where two vertices are joined if and only if their coordinates differ in one place and have a difference of just 1. The graph is bipartite and the  $m^d$  vertices have bipartition  $W$  and  $B$  (sets  $W, B$  can be determined by the parity of their sum of coordinates). We show that there are constants  $a_d, b_d$  so that for every even  $m$ , if we choose subsets  $B' \subseteq B$  and  $W' \subseteq W$  in the  $d$ -dimensional grid graph  $G$  which satisfy three conditions (i)  $|B'| = |W'|$ , (ii) for any  $x, y \in B'$   $d_G(x, y) \geq a_d m^{1/d} + b_d$  and (iii) for any  $x, y \in W'$   $d_G(x, y) \geq a_d m^{1/d} + b_d$  then  $G$  with the vertices  $B' \cup W'$  deleted has a perfect matching.

The factor  $m^{1/d}$  is best possible.

Richard P. Anstee  
Mathematics Department  
University of British Columbia  
anstee@math.ubc.ca

Jonathan Blackman, Hangjun Yang  
University of British Columbia  
blackman@interchange.ubc.ca, gavin.yang@sauder.ubc.ca

### CP4

#### Minimum Degree Threshold for Bipartite Graph Tiling

Graph tiling problems involve finding many vertex disjoint copies of  $H$  in a larger graph  $G$ . We answer a question of Zhao [*SIAM J. Disc. Math.* **23** vol.2, (2009), 888-900] that determines the minimum degree threshold for a bipartite graph  $G$  to contain an  $H$ -factor (a perfect tiling of  $G$  with  $H$ ) for any bipartite graph  $H$ . We also show that this threshold is best possible up to a constant depending only on  $H$ . This result can be viewed as an analog to Kuhn and Osthus' result [*Combinatorica* **29** (2009), 65-107] for bipartite graphs.

Albert Bush  
Georgia Institute of Technology  
albertbush@gmail.com

Yi Zhao  
Georgia State University  
matyxx@langate.gsu.edu

### CP4

#### Rainbow Matchings in Edge-Colored Graphs

The *color degree* of a vertex  $v$  in an edge-colored graph is the number of distinct colors appearing on edges incident to  $v$ . Wang and Li conjectured that for  $k \geq 4$ , edge-colored graphs with minimum color degree at least  $k$  contain rainbow matchings of size at least  $\lfloor k/2 \rfloor$ . We guarantee rainbow matchings of size  $\lfloor k/2 \rfloor$  and prove the conjecture for triangle-free graphs and properly edge-colored graphs whose complements have non-leaf vertices.

Timothy D. LeSaulnier, Christopher Stocker, Paul Wenger  
University of Illinois, Urbana  
tlesaul2@uiuc.edu, stocker2@uiuc.edu,  
pwenger2@uiuc.edu

Douglas B. West  
University of Illinois, Urbana  
Department of Mathematics  
west@math.uiuc.edu

### CP4

#### $K_6$ -Minors in Triangulations on Surfaces

If  $n \leq 5$ , graphs with no  $K_n$ -minor have already been characterized. (In particular, the case of  $n = 5$  was solved by Wagner.) However, it seems to be difficult to characterize graphs with no  $K_6$ -minor, in general. For the projective plane and the torus, we characterize triangulations with no  $K_6$ -minor by a unique forbidden subgraph. In our talk, we shall characterize triangulations with no  $K_6$ -minor for

surfaces with low genus.

Raiji Mukae  
Yokohama National University  
mkerij@gmail.com

#### CP4

##### Edge-Connectivity, Eigenvalues, and Matchings in Regular Graphs

The matching number of a graph is the maximum size of a matching in it. We previously characterized the graphs having the smallest maximum number among connected  $(2k+1)$ -regular graphs with  $n$  vertices; the extremal graphs have cut-edges. In this talk, we prove a lower bound for the maximum matching in a  $t$ -edge-connected  $r$ -regular graph with  $n$  vertices, for  $t \geq 2$  and  $r \geq 4$ ; various special cases were obtained earlier. We also characterize the graphs achieving equality. We also study the relationship between eigenvalues and matchings in  $t$ -edge-connected  $r$ -regular graphs. We give a condition on an appropriate eigenvalue that guarantees a lower bound for the matching number in a  $t$ -edge-connected  $r$ -regular graph; this generalizes a recent result of Cioaba, Gregory, and Haemers.

Suil O  
University of Illinois, Urbana  
suilo2@math.uiuc.edu

Douglas B. West  
University of Illinois, Urbana  
Department of Mathematics  
west@math.uiuc.edu

Sebastian Cioaba  
University of Delaware at Newark  
cioaba@math.udel.edu

#### CP5

##### Dynamic Forests and Load Balancing in Wireless Networks

We define a dynamic forest in a DAG to be a random forest generated by independently choosing a parent from the out-neighborhood of each non-sink node with respect to a given probability distribution. We present results and algorithms for solving the bi-criteria optimization problem of balancing the load on the nodes for longer lifetime and the load on the edges incident with each node for greater reliability. Our results have applications to wireless networks.

Kenneth A. Berman, Aravind Ranganathan  
Dept. of Computer Science  
University of Cincinnati  
bermanka@ucmail.uc.edu, rangana@mail.uc.edu

#### CP5

##### Markovian Network Interdiction and the Four Color Theorem

Applications such as supply chains management, traffic monitoring and disease control give rise a discrete optimization problem termed The Unreactive Markovian Evader Interdiction Problem (UME). UME asks to optimally place sensors on a network to detect Markovian motion by one or more "evaders". We prove that the problem is NP-hard with just 2 evaders using a connection to coloring of pla-

nar graphs. The result suggests that approximation algorithms are needed even in applications where the number of evaders is small.

Alexander Gutfraind  
Theoretical Division  
Los Alamos National Laboratory  
gfriend@lanl.gov

#### CP5

##### A Multi-Hub Theory for Spectral-Based System Design

There are a variety of systems whose properties are governed by the leading eigenvectors of the underlying system matrix. If the set of dominant eigenvalues are clearly separated from the next largest eigenvalues, we can characterize the system properties. We will describe approach which can assure clear separation in leading eigenvalues by imposing a proper structure of the underlying matrix. Specifically, we provide bounds on eigenvalues for hierarchical system connection structure. Based on these results, we can design hierarchical systems with assured clustering behavior or absence of it.

Bruce W. Suter  
Air Force Research Laboratory  
bruce.suter@rl.af.mil

H. T. Kung  
Harvard University  
kung@harvard.edu

#### CP5

##### The Coloring Methods and the Rearrangeability of Banyan-Type Networks

The Banyan-type networks are built by basic 2?2 directional couplers with the network structure topology derived from a horizontal expansion and vertical stacking of 2?2 Banyan networks. Some necessary and sufficient conditions of Banyan-type networks which are rearrangeably nonblocking that have been proposed for optical interconnection network fabrics, where these rearrangeably nonblocking conditions are well-supported assuming that all the connections are not allowed to have any crosstalk. In this talk, we use the coloring methods to study the necessary and sufficient conditions for Banyan-type networks, which are rearrangeable nonblocking by considering all possible degrees of crosstalk constraints.

Li-Da Tong  
National Sun Yat-sen University, Taiwan  
ldtong@math.nsysu.edu.tw

#### CP6

##### Aperiodic Sequences and Self-Stabilizing Algorithms

Self-stabilizing algorithms represent an extension of distributed algorithms in which nodes of the network have neither coordination, synchronization, nor initialization. Recently we provided a novel algorithm for determining the size of a unidirectional ring where all nodes except one use constant space. This algorithm exploits a token-circulation idea due to Afek and Brown and uses the concept of aperiodic sequences. In this talk we review the algorithm and discuss generating aperiodic sequences using

self-stabilization.

Wayne Goddard  
School of Computing  
Clemson University  
goddard@clemson.edu

Pradip Srimani  
Clemson University  
srimani@clemson.edu

### CP6

#### Some Contributions in Biform Games

Brandenburger and Stuart (2006) have introduced the biform game (BG). The mixed extension of BG, needs cores comparison. The convex combination of non empty cores is analyzed. We mentioned results obtained by Zhao (2000) and Cesco (2010). Cesco studies when the core is a point. The unidimensional core of a (0,1)-reduction game is located in the boundary. New dominations in the sense of Marchi and Auriol (2010) are presented.

Ezio Marchi  
IMASL-UNSL  
emarchi@speedy.com.ar

### CP6

#### A Measure of the Connection Strengths Between Graph Vertices with Applications

We present a simple iterative strategy for measuring the connection strength between a pair of vertices in a graph. The method is attractive in that it has a linear complexity and can be easily parallelized. Based on an analysis of the convergence property, we propose a mutually reinforcing model to explain the intuition behind the strategy. The practical effectiveness of this measure is demonstrated through several discrete optimization problems on graphs and hypergraphs.

Ilya Safro  
Argonne National Laboratory  
safro@mcs.anl.gov

Jie Chen  
Department of Computer Science and Engineering  
University of Minnesota  
jchen@cs.umn.edu

### CP6

#### A Realistic Approach to Space-Time Performance of Computational Algorithms.

Computational complexity is widely researched in Theoretical Computer Science. However, only a few of them have treated complexity as applied to the real-life computation. Time and space complexities jointly depict the overall computational scenario. Without the treatment of either one of these, the analysis is incomplete, and the model may not depict the actual computation. Using a realistic approach, this paper has treated both of these complexities due to a wide variety of algorithmic data structures arising frequently in practice.

Ahmed Tarek  
Asso Prof of Math and Computer Science, Comp Sci Prog Chair

California University of Pennsylvania, 250 Uni Ave PA  
15419  
atarek64@gmail.com

### CP7

#### A Note on Antimagic Labelings of Graphs

In this talk, we will discuss the conjecture proposed by M. Miller and M. Bača in 2000. It said that the generalized Petersen graphs  $P(n, k)$  are  $(\frac{5n+5}{2}, 2)$ -antimagic for odd  $n$ ,  $n \geq 5$  and  $2 \leq k \leq \frac{n-1}{2}$ . We will show that it is wrong when  $n = 5$ .

Nam-Po Chiang  
Department of Applied Mathematics, Tatung University  
npchian@ttu.edu.tw

Tsung-Han Wu  
Department of Applied Mathematics  
Tatung University  
limit621@msn.com

### CP7

#### The Graph Induced by All Vertices in Some Critical Independent Set

An independent set  $I_c$  of a graph  $G$  is a *critical independent set* if  $|I_c| - |N(I_c)| \geq |J| - |N(J)|$ , for any independent set  $J$ . Critical independent sets are of interest for both computational and theoretical reasons. It will be shown that the graph induced on the set  $H$  of all vertices in some critical independent set of  $G$  is a König-Egerváry graph whose components are either isolated vertices or which have perfect matchings. Furthermore, if  $I_0$  is the set of isolated vertices in  $G[H]$ , then  $\alpha(G[H]) = |I_0| + \frac{1}{2}|H \setminus I_0|$ .

Craig E. Larson  
Virginia Commonwealth University  
clarson@vcu.edu

Ermelinda DeLaVina  
University of Houston  
DelavinaE@uhd.edu

### CP7

#### Bounds on Steiner Trees in the Hypercube

Let  $S$  be a set of vertices in the  $n$ -dimensional hypercube  $Q_n$ , and let  $L(S)$  be the minimum number of edges in any connected subgraph  $H$  of  $Q_n$  containing  $S$ . We obtain the following results by probabilistic methods. (1) If  $S$  is of size  $k$ , then  $L(S) \leq \frac{1}{3}(k+1+ln(k-1))n$ . (2) We show the above bound is nearly best possible for a certain range in  $k$  as follows. Let  $\epsilon > 0$  be a fixed small real number, and let  $n$  be sufficiently large as a function of  $\epsilon$ . Further let  $k$  lie in the range  $K_1 \leq k \leq K_2 c^n$ , where  $K_1$ ,  $K_2$ , and  $c$  constants which depend only on  $\epsilon$ , with  $1 < c < 2$ . Then there exist sets  $S$  in  $Q_n$  of size  $k$  such that  $L(S) \geq (\frac{1}{3} - \epsilon)kn$ .

Zevi Miller  
Dept. of Mathematics & Statistics  
Miami University  
millerz@muohio.edu

Tao Jiang  
Department of Mathematics and Statistic  
Miami University  
jiangt@muohio.edu

Dan - Pritikin  
Miami University  
pritikd@muohio.edu

### CP7

#### An Approximate Version of Sumner's Universal Tournament Conjecture

Sumner's Universal Tournament Conjecture, first posed in 1971, states that any tournament on  $2n-2$  vertices contains any directed tree on  $n$  vertices. In this talk, I will explain how a randomized embedding algorithm can be used to prove an approximate version of this conjecture, and also a stronger result for trees of bounded maximum degree. I will also outline progress towards a proof of the conjecture for large  $n$ .

Richard Mycroft  
Queen Mary  
University of London  
r.mycroft@qmul.ac.uk

### CP7

#### Every Longest Hamiltonian Path in Odd N-Gons

This presentation concerns the resolution of the  $\lfloor \frac{n}{2} \rfloor$  different Longest Euclidean Hamiltonian path problems on the vertices of an odd regular polygon. Firstly, we show the lengths of every longest Euclidean Hamiltonian path problem. Secondly, we determine the composition of the directed sides that should accomplish those longest traveled lengths. Finally, we single out the Euclidean Hamiltonian paths that solve every problem. This paper is developed with our methodology, which has been proposed in "*Finding Every Longest Hamiltonian Path on a Regular Polygon*" and already submitted to *Discrete Mathematics*.

Blanca I. Niel  
Departamento de Matemática  
Universidad Nacional del Sur - Baha Blanca - Argentina  
biniel@criba.edu.ar

Walter Reartes  
Depto de Matematica - Universidad Nacional del Sur  
(UNS)  
Bahia Blanca - Argentina  
reartes@uns.edu.ar

Nelida Brignole  
PLAPIQUI (UNS-CONICET) Bahia Blanca - Argentina  
Lab de Invest y Desarrollo en Computacion Cientif  
(LIDeCC)  
dybrigno@criba.edu.ar

### CP8

#### Degree Sequences, Vertex Substitutions, and Matrogenic Graphs

Given graphs  $G$  and  $H$  and a vertex  $v$  of  $H$ , *substituting  $G$  for  $v$*  is done by deleting  $v$  and making each vertex of  $G$  adjacent to all neighbors of  $v$  in  $H$ . The *substitution closure* of a graph class  $\mathcal{C}$  is the smallest class that contains  $\mathcal{C}$  and is closed under substitutions. Motivated by questions on degree sequences, we characterize graphs in the substitution closures of the split graphs and the matrogenic graphs.

Michael Barrus

Black Hills State University  
Department of Mathematics  
michael.barrus@bhsu.edu

### CP8

#### Identification of Universal Vertex in Threshold Component for Strict 2-Threshold Graphs

An efficient algorithm to identify universal vertex for each of the two threshold components for strict 2-threshold (S2T) graph is proposed. S2T graph is a prominent sub-class of a class of perfect graph named 2-threshold graph, which has wide applications including synchronization of processes. Previous method assumes vertex of maximum degree is one of the said universal vertices and applies recognition algorithm for threshold to conduct further checking. In this paper, an efficient method is proposed to achieve the same purpose simply by degrees of two vertices. The time consumed compares favorably to previous method.

Wei-Da Hao  
Department of Electrical Engineering and Computer Science  
Texas A/&M University-Kingsville  
kfw000@tamuk.edu

Chung Leung  
Dept of Electrical Engineering and Computer Science  
Texas A/&M University-Kingsville  
c-leung@tamuk.edu

Lin-Yu Tseng  
Department of Computer Science and Engineering  
National Chung-Hsing University, ROC  
lytseng@cs.nchu.edu.tw

### CP8

#### Finding a Sun in Building-Free Graphs

A *sun* is a graph with a Hamiltonian cycle  $(x_1, y_1, \dots, x_n, y_n)$ ,  $n \geq 3$ , where each  $x_i$  has degree two and the  $y_i$  vertices form a clique. Deciding whether an arbitrary graph contains a sun is NP-complete, while for some graph classes (e.g. chordal, hhd-free) the problem is polynomial time. We give a polynomial-time algorithm to test for suns in *building-free* graphs. Building-free graphs generalize Meyniel graphs (and hence, hhd-free, i-triangulated, and parity graphs).

Elaine M. Eschen  
Lane Department of CSEE  
West Virginia University  
Elaine.Eschen@mail.wvu.edu

Chinh Hoang  
Wilfrid Laurier University  
Dept of Physics and Computing Science  
choang@wlu.ca

Jeremy Spinrad  
Vanderbilt University  
spin@vuse.vanderbilt.edu

R. Sritharan  
The University of Dayton  
srithara@notes.udayton.edu

**CP8****Elimination Orderings of Graphs of Bounded Asteroidal Number**

The asteroidal number of a graph is the size of a largest subset of vertices such that for no vertex in the set, its closed neighbourhood disconnects the remaining vertices of the set. We characterize graphs of bounded asteroidal number by means of a vertex elimination ordering. Similar characterizations are known for chordal, interval, co-comparability, and path-orderable graphs, last three being, in fact, proper subclasses of AT-free graphs (asteroidal number two).

Juraj Stacho  
Wilfrid Laurier University  
jstacho@cs.sfu.ca

Derek Corneil  
University of Toronto  
dgc@cs.utoronto.ca

**CP9****On Enumeration of Simple Polytopes**

Two simple polytopes are said to be distinct if their face posets are non-isomorphic. We consider the problem of enumerating distinct  $n$ -facet simple polytopes in  $R^d$ , denoted  $(n, d)$ -polytopes. Let the number of distinct  $(n, d)$ -polytopes be  $\#(n, d)$ . In this talk, we present a new algorithm to generate all  $(n, d)$  simple polytopes in time polynomial in  $\#(n, d)$ . The algorithm operates by computing the effect of a hyperplane sweep on the face lattice of a sequence of polytopes. Further, we present new bounds on the value of  $\#(n, d)$ .

Anand P. Kulkarni  
Department of Industrial Engineering and Operations  
Research  
University of California, Berkeley  
anandk@berkeley.edu

Sandeep Koranne  
Mentor Graphics Corporation  
sandeep\_koranne@mentor.com

**CP9****Polytope Numbers**

Polytope numbers for a polytope are a sequence of nonnegative integers which are defined by the facial information of a polytope. This is a higher dimensional generalization of polygonal number. It is well known that every polygon can be decomposed into triangles. A higher dimensional analogue of this fact states that every polytope has a triangulation, namely, it can be decomposed into simplices. Thus it may be possible to represent polytope numbers as sums of simplex numbers, which gives another way of calculating polytope numbers. In this talk, we define polytope

numbers and calculate polytope numbers for several polytopes, and we introduce decomposition theorem, which is a way of representing polytope numbers as sums of simplex numbers. Joint work with Prof. Hyun Kwang Kim,

POSTECH, Korea.

Joon Yop Lee  
ASARC, KAIST

flutelee@postech.ac.kr

**CP9****The Cunningham-Geelen Algorithm in Practice: Branch-Decompositions and Integer Programs**

Consider the integer program  $\max(c^T x : Ax = b, x \geq 0)$ , where  $A$  is non-negative and the column-matroid of  $A$  (denoted by  $M(A)$ ) has constant branch width. Cunningham and Geelen introduce a pseudo-polynomial time algorithm for solving this integer program that takes a branch decomposition  $T$  of  $M(A)$  as input. We describe a heuristic for finding  $T$  and report on computation results of a C++ implementation of this algorithm, where the input branch decomposition  $T$  is produced by this heuristic.

Susan Margulies  
Computational and Applied Mathematics  
Rice University  
susan.margulies@rice.edu

**CP9****Dominating Circuits in Matroids**

In 1971, Nash-Williams proved that if  $G$  is a simple 2-connected graph on  $n$  vertices having minimum degree  $\delta_G \geq \frac{1}{3}(n+2)$ , then any longest cycle  $C$  in  $G$  is also edge-dominating; that is, each edge of  $G$  has at least one end-vertex belonging to  $C$ . We say that a circuit  $C$  in a matroid  $M$  is dominating if each component of  $M/C$  has rank at most one. In this talk, we shall describe an analogous theorem for regular matroids: we show that if each cocircuit in a simple connected regular matroid  $M$  has at size at least  $\frac{1}{3}r(M) + 1$ , then any longest circuit is dominating.

Sean Mcguinness  
Thompson Rivers University  
smcguinness@tru.ca

**CP9****On the Convexity of the Tutte Polynomial of a Paving Matroid Along Line Segments**

We prove that the Tutte polynomial of a coloopless paving matroid is convex along the line segments  $x + y = p$  for  $p \geq 0$  and  $0 \leq y \leq p$ . Every coloopless paving matroids is in the class of matroids which contain two disjoint bases or whose ground set is the union of two bases of  $M^*$ . For this latter class we give a proof that  $T_M(a, a) \leq \max\{T_M(2a, 0), T_M(0, 2a)\}$  for  $a \geq 2$ . We conjecture that  $T_M(1, 1) \leq \max\{T_M(2, 0), T_M(0, 2)\}$  for the same class of matroids. We also prove this conjecture for some families of graphs and matroids.

Criel Merino  
UNIVERSIDAD NACIONAL AUTONOMA DE  
MEXICO  
merino@matem.unam.mx

Laura Chavez  
UNIVERSIDAD AUTONOMA  
METROPOLITANA  
laurachav@gmail.com

noble steve  
BRUNEL UNIVERSITY  
mastsdn@brunel.ac.uk

marcelino ramirez  
UNIVERSIDAD NACIONAL  
AUTONOMA DE MEXICO  
marchelino@gmail.com

#### CP10

##### On Enumerating Diverse Feasible Solutions to a Combinatorial Optimization Problem

This talk summarizes efforts at the National Renewable Energy Laboratory to enumerate diverse building designs that satisfy an energy goal, and are ranked by cost. The mathematical setting is optimization over discrete variables, each possible combination of which represents a valid design that may or may not be feasible according to a non-linear function. The origin acts as a baseline value, and diversity is obtained by partitioning the decision space into functional categories.

Elaine T. Hale

National Renewable Energy Laboratory  
Advanced Commercial Buildings Research Group  
elaine.hale@nrel.gov

#### CP10

##### The Minimum Cost Flow Problem in Dynamic Multi Generative Network Flows

This paper consists in constructing and modeling Dynamic Multi Generative Network Flows in which the flow commodities is dynamically generated at source nodes and dynamically consumed at sink nodes. It is assumed that the source nodes produce the flow commodities according to  $k$  time generative functions and the sink nodes absorb the flow commodities according to  $k$  time consumption functions. The minimum cost dynamic flow problem in such networks that extend the classical optimal flow problems on static networks, for a pre-specified time horizon  $T$  is defined and mathematically formulated. Moreover, it is showed that the dynamic problem on these networks can be formulated as a linear program whose special structure permits efficient computations of its solution and can be solved by one minimum cost static flow computation on an auxiliary time-commodity expanded network. By using flow decomposition theorem, we elaborate a different model of the problem to reduce its complexity. We consider the problem in the general case when the cost and capacity functions depend on time and commodity.

Seyed Ahmad Hosseini

Department of Mathematics and Computer Science  
AhmadHosseini1984@gmail.com

#### CP10

##### Using Markov Chains Properties to Analyze the Effectiveness of Local Search Algorithms for Hard Discrete Optimization Problems

The performance of local search algorithms (guided by the best-to-date solution at each iteration) in visiting suboptimal solutions for hard discrete optimization problems is explored. The  $\beta$ -acceptable solution concept is used to capture how effectively an algorithm has performed to date and how effectively an algorithm can be expected to perform in the future in visiting suboptimal solutions. Markov chain state pooling is introduced and used to obtain an estimator for the expected number of iterations to visit a  $\beta$ -acceptable solution. Convergence results for this es-

imator are provided. Computational experiments with the Lin-Kernighan-Helsgaun algorithm applied to medium and large traveling salesman problem instances taken from TSPLIB (all with known optimal solutions) are reported to illustrate the application of this estimator.

Sheldon H. Jacobson

University of Illinois  
Dept of Computer Science  
shj@uiuc.edu

Alexander Nikolaev

Northwestern University  
a-nikolaev@northwestern.edu

#### CP10

##### A Lower Bounding Scheme for a Maximum Dispersion Territory Design Problem

We present a lower bounding scheme for a territory design problem that arises in the recollection of waste electric and electronic equipment. The problem consists of assigning recollection points to companies so as to minimize company monopoly in specific region. Our proposed scheme is used for measuring the quality of some heuristics for this NP-hard problem.

Jabneel R. Maldonado-Flores

Graduate Program in Systems Engineering  
Universidad Autonoma de Nuevo Leon  
jabneel@yalma.fime.uanl.mx

Roger Z. Rios-Mercado

Universidad Autonoma de Nuevo Leon  
roger@yalma.fime.uanl.mx

#### CP10

##### Territory Design with a Routing Constraint by Iterated Greedy Local Search

An iterated greedy local search heuristic for a commercial territory design optimization problem is presented. The problem consists on finding a partition of city blocks so as to minimize territory compactness subject to node activity balancing, territory connectivity, and a routing budget constraint. The empirical results show the effectiveness of the proposed approach.

Roger Z. Rios-Mercado

Universidad Autonoma de Nuevo Leon  
roger@yalma.fime.uanl.mx

Juan C. Salazar-Acosta

Graduate Program in Systems Engineering  
Universidad Autonoma de Nuevo Leon  
jcsa@yalma.fime.uanl.mx

Ruben Ruiz

Department of Statistics and Operations Research  
Universitat Politecnica de Valencia  
rruiz@eio.upv.es

#### CP11

##### $\mathbf{Z}_4$ Linear Kerdock Codes Are Relative Difference Sets

In this talk, we will demonstrate that  $\mathbf{Z}_4$  linear Kerdock codes are equivalent to abelian  $(2^m, 2^m, 2^m, 1)$ - relative dif-

ference sets in Galois rings.

Yuqing Chen

Department of Mathematics and Statistics  
Wright State University  
yuqing.chen@wright.edu

### CP11

#### On The Existence Of Perfect Quaternary Codes\*

In this work, we investigate parameters of perfect quaternary codes with respect to the Lee metric. In order to gain some insight, we establish a binomial formula that gives the number of codewords of a particular Lee weight. Further, by making use of the fact that the balls centered at codewords of a  $t$  error correcting perfect code must intersect at only zero codeword, we are able to exclude many parameters. Here we only present some initial results of an ongoing research problem. \* This research is supported by TUBITAK-Grant No:109T328.

Irfan Siap

Yildiz Technical University  
isiap@yildiz.edu.tr

Mehmet Ozen, Vedat Siap

Sakarya University  
ozen@sakarya.edu.tr, vsiap@sakarya.edu.tr

### CP11

#### New Bounds for the Minimum Density of An $r$ -Identifying Code in Some Infinite Grids

An  $r$ -identifying code on a graph  $G$  is a set  $C \subset V(G)$  such that for every vertex in  $V(G)$ , the intersection of the radius- $r$  closed neighborhood with  $C$  is nonempty and unique. On a finite graph, the density of a code is  $|C|/|V(G)|$ , which naturally extends to a definition of density in certain infinite graphs which are locally finite. We present improved bounds for the minimum density of a code on the infinite hexagonal and square grids.

Brendon Stanton

Iowa State University  
bstanton@iastate.edu

### CP11

#### The Covering Problem for Galois Rings with Respect to the RT-Metric

One of the important problems in Coding Theory is the covering problem which is related to the optimal error correcting of the code. Also this problem is related to the packing problem in finite (or infinite) structures. Lately codes over Galois rings and newly defined RT metric independently has been studied. In this work, the cardinality of the minimal  $R$ -covers of Galois rings with respect to the RT-metric is established. By generalizing the result obtained by Nakaoko and Santos (2009), the minimal cardinalities of zero-short coverings of Galois rings are calculated. The connection between  $R$ -short coverings of Galois rings with respect to the RT metric and the zero-short coverings of these rings are demonstrated and with the help of this connection, the problem of finding the minimal cardinalities of  $R$ -short coverings of Galois finite rings is solved.

Gursel Yesilot

Yildiz Technical University

gyesilot@yildiz.edu.tr

Bahattin Yildiz

Fatih University  
byildiz@fatih.edu.tr

Irfan Siap

Yildiz Technical University  
isiap@yildiz.edu.tr

Tevfik Bilgin

Fatih University  
tbilgin@fatih.edu.tr

### CP11

#### The MacWilliams Theorem for Four-Dimensional Modulo Metric

In this paper, the MacWilliams theorem is stated for codes over finite field with four-dimensional modulo metric

Mehmet zen

Sakarya University  
ozen@sakarya.edu.tr

Murat Güzeltepe

Sakarya University  
mguzeltepe@sakarya.edu.tr

### CP12

#### Counting Bordered Partial Words by Critical Positions

The concepts of primitivity and borderedness are highly connected in areas including combinatorics on words, coding theory, formal languages and text algorithms. A primitive word is a sequence that cannot be written as a power of another sequence, while a bordered word is a sequence such that at least one of its proper prefixes is one of its suffixes. The numbers of primitive and bordered words of a fixed length over an alphabet of a fixed size are well known, the number of primitive words being related to the Möbius function. Here we give formulas for the number of bordered *partial words* (partial words are sequences over a finite alphabet that may contain some undefined positions called "holes"). When dealing with bordered partial words, two types of borders are identified: *simple* and *overlapping*. A partial word is called unbordered if it does not have any border. In the case of words without holes, all borders are simple. Our problem is made extremely more difficult by the failure of that combinatorial property. For the finite alphabet  $\{a, b, c\}$ , the partial word  $a\diamond b$  has both a simple border  $ab$  and a nonsimple border  $aab$ , while the partial word  $a\circ bc$  is unbordered (the  $\diamond$  symbol represents an undefined position or a hole, and matches every character of the alphabet). Our approach is based on the notion of critical positions that once changed into holes create borders.

Francine Blanchet-Sadri

University of North Carolina  
blanchet@uncg.edu

Emily Allen

Carnegie Mellon University  
eaallen@andrew.cmu.edu

John Lensemire

University of Minnesota-Twin Cities

lensm003@umn.edu

### CP12

#### Cyclically $t$ -complementary Uniform Hypergraphs

Consider a partition of the set of all  $k$ -element subsets of an  $n$ -element set  $V$  into  $t$  parts, such that there is a permutation of  $V$  which permutes the  $t$  parts cyclically. Such a permutation is called a  $(t, k)$ -complementing permutation, and each part in the partition is called a *cyclically  $t$ -complementary  $k$ -hypergraph*. When  $t = k = 2$  these are self-complementary graphs, which are well studied due to their connection to the graph isomorphism problem. We give necessary and sufficient conditions on the order of cyclically  $t$ -complementary  $k$ -hypergraphs by characterizing their  $(t, k)$ -complementing permutations.

Shonda Gosselin

University of Winnipeg

Department of Mathematics and Statistics

s.gosselin@uwinnipeg.ca

### CP12

#### Computing Periods in Partial Words

Computing periods in *words*, or finite sequences of symbols from a finite alphabet, has important applications in several areas including data compression, coding, computational biology, string searching and pattern matching algorithms. We give an extension of Fine and Wilf's well known periodicity result in the context of *partial words*, or sequences that may have undefined positions or holes. Here any word with  $h$  holes and having periods  $p_1, \dots, p_m$  and length at least the so-denoted  $L_h(p_1, \dots, p_m)$  has also  $\gcd(p_1, \dots, p_m)$  as a period. We investigate optimal words for the bound  $L_h(p_1, \dots, p_m)$ , that is, partial words  $u$  with  $h$  holes of length  $L_h(p_1, \dots, p_m) - 1$  such that  $p_1, \dots, p_m$  are periods of  $u$  but  $\gcd(p_1, \dots, p_m)$  is not a period of  $u$ . We give closed formulas for  $L_h(p_1, \dots, p_m)$  in a number of cases. Our approach is based on connectivity in graphs associated with sets of periods. World Wide Web server interfaces have been established at [www.uncg.edu/cmp/research/finewilf4](http://www.uncg.edu/cmp/research/finewilf4) and [/finewilf5](http://www.uncg.edu/cmp/research/finewilf5) for automated use of programs which given a number of holes and a period set, compute the optimal bound and an optimal word for that bound.

Francine Blanchet-Sadri

University of North Carolina

blanchet@uncg.edu

James Carraher

University of Nebraska-Lincoln

carraher\_g04it@yahoo.com

Travis Mandel

The University of Texas at Austin

tmandel@math.utexas.edu

Brian Shirey

NC State University

Department of Computer Science

bjshirey@ncsu.edu

Gautam Sisodia

University of Washington-Seattle

gautas@math.washington.edu

### CP12

#### A Generalization of Larman-Rogers-Seidel's Theorem

Let  $X$  be an  $s$ -distance set in the Euclidean space  $\mathbf{R}^d$ , and  $A(X) = \{\alpha_1, \alpha_2, \dots, \alpha_s\}$  be the set of Euclidean distances between two distinct elements of  $X$ . For  $s = 2$ , Larman-Rogers-Seidel proved that if  $|X| \geq 2d + 4$ , then there exists an integer  $k$  such that  $\alpha_1^2/\alpha_2^2 = (k - 1)/k$ . In this talk, for any  $s$ , we give a generalization of the theorem due to Larman-Rogers-Seidel.

Hiroshi Nozaki

University of Texas at Brownsville

Hiroshi.Nozaki@utb.edu

### CP12

#### The Total Weak Discrepancy of a Partially Ordered Set

We consider the total (fractional) weak discrepancy of a partially-ordered set and compare it to our earlier results for weak and fractional weak discrepancy. We prove that, unlike the earlier case, the total fractional weak discrepancy is always an integer. In the proof, we express the problem as a linear program and solve it using the dual, which is a circulation problem with side constraints on a directed graph.

Alan Shuchat

Mathematics Department

Wellesley College

ashuchat@wellesley.edu

Randy Shull

Department of Computer Science

Wellesley College

rshull@wellesley.edu

Ann N. Trenk

Wellesley College

atrenk@wellesley.edu

### CP13

#### Distinguishing and Distinguishing Chromatic Numbers

The distinguishing number of a graph was first defined by Mike Albertson and the presenter in 1996, and the distinguishing chromatic number was first defined by the presenter and Ann Trenk in 2006. This talk will compare and contrast known results about each parameter from the point of view of how the automorphism group of a graph affects its distinguishing and distinguishing chromatic numbers, and discuss these numbers for particular families of graphs.

Karen Collins

Wesleyan University

kcollins@wesleyan.edu

### CP13

#### Harmonious Colorings of Digraphs

Let  $D$  be a directed graph with  $n$  vertices and  $m$  edges. A function  $f : V(D) \rightarrow \{1, 2, 3, \dots, k\}$  where  $k \leq n$  is said to be *harmonious coloring* of  $D$  if for any two edges  $xy$  and  $uv$  of  $D$ , the ordered pair  $(f(x), f(y)) \neq (f(u), f(v))$ . If the pair  $(i, i)$  is not assigned, then  $f$  is said to be a

*proper harmonious coloring* of  $D$ . The minimum  $k$  is called the *proper harmonious coloring number* of  $D$ . We investigate the proper harmonious coloring number of graphs such as unidirectional paths, unicycles, inspoken (outspoken) wheels,  $n$ -ary trees of different levels, union of unidirectional paths, alternating paths and alternating cycles etc.

Suresh M. Hegde  
National Institute of Technology Karnataka  
Surathkal  
smhegde59@yahoo.com

Lolita Castelino  
National Institute of Technology Karnataka  
Surathkal, India  
plolita@yahoo.com

### CP13 Chromaticity of a Family of $k$ -Bridge Graphs

Let  $P(G, \lambda)$  denote the chromatic polynomial of a graph  $G$ . Two graphs  $G$  and  $H$  are chromatically equivalent, written  $G \sim H$ , if  $P(G, \lambda) = P(H, \lambda)$ . A graph  $G$  is chromatically unique written  $\chi$ -unique, if for any graph  $H$ ,  $G \sim H$  implies that  $G$  is isomorphic with  $H$ . In this paper we prove the chromatic uniqueness of a new family of  $k$ -bridge graphs.

Abdul Jalil M. Khalaf  
University of Kufa  
am\_maths@yahoo.com

### CP13 5-Coloring Graphs with 4 Crossings

We disprove a conjecture of Oporowski and Zhao stating that every graph with crossing number at most 5 and clique number at most 5 is 5-colorable. However, we show that every graph with crossing number at most 4 and clique number at most 5 is 5-colorable.

Bernard Lidicky  
Charles University, Prague  
bernard@kam.mff.cuni.cz

Rok Erman  
University of Ljubljana  
rok.erman@gmail.com

Frederic Havet  
Projet Mascotte, I3S(CNRS/UNSA) and INRIA  
fhavet@sophia.inria.fr

Ondrej Pangrac  
Department of Applied Mathematics  
Charles University in Prague  
pangrac@kam.mff.cuni.cz

### CP13 Acyclic List Edge Coloring of Graphs

A proper edge coloring of a graph is said to be acyclic if any cycle is colored with at least three colors. An edge-list  $L$  of a graph  $G$  is a mapping that assigns a finite set of positive integers to each edge of  $G$ . An acyclic edge coloring  $\phi$  of  $G$  such that  $\phi(e) \in L(e)$  for any edge  $e$  is called an acyclic  $L$ -edge coloring of  $G$ . A graph  $G$  is said

to be acyclically  $k$ -edge choosable if it has an acyclic  $L$ -edge coloring for any edge-list  $L$  that satisfies  $|L(e)| \geq k$  for each edge  $e$ . The acyclic list chromatic index is the least integer  $k$  such that  $G$  is acyclically  $k$ -edge choosable. We develop techniques to extend acyclic list edge colorability and apply them to obtain bounds for the acyclic list chromatic indexes of outerplanar graphs, subcubic graphs, and subdivisions of Halin graphs.

Hsin-Hao Lai  
National Kaohsiung Normal University, Taiwan  
hsinhaolai@nknucc.nknu.edu.tw

Ko-Wei Lih  
Institute of Mathematics, Academia Sinica, Taipei  
makwlih@sinica.edu.tw

### CP14 Magic Sum Spectra of Group Magic Graphs

For a positive integer  $k \geq 2$ , let  $Z_k = (Z_k, +, 0)$  be the additive abelian group of integer congruences modulo  $k$  with identity 0. We call a finite simple graph  $G = (V(G), E(G))$  to be  $Z_k$ -magic if it admits an edge labeling ranging in  $Z_k \setminus \{0\}$  such that the induced vertex sum (sum of all incident edge labels at a vertex) is constant, and such constant is called a magic sum index. For each integer  $k$ , we define  $I_k(G)$  as the set of all magic sum indices  $r$  such that  $G$  is  $Z_k$ -magic with an index  $r$ . We call  $I_k(G)$  the **magic sum spectrum** of  $G$  with respect to  $Z_k$ . In this talk, we present the properties of the magic sum spectra of a graph  $G$ . In particular we show that, for a regular graph  $G$  admitting a 1-factor, the magic sum spectrum  $I_k(G)$  is full  $Z_k$ , for each  $k \geq 3$ . We also give examples of regular graphs without 1-factor whose magic sum spectrum is not full  $Z_k$  for some  $k \geq 3$ . We prove that the magic sum spectra with respect to  $Z_k$  of complete bipartite graphs are isomorphic to the additive cyclic subgroups of  $Z_k$ , for each  $k \geq 3$ . Among others, the magic sum spectra of Cartesian product and lexicographic product of graphs are presented, and the magic sum spectra of fans, wheels, and circulant graphs are completely determined. Some open problems will be mentioned in the concluding remarks.

Tao-Ming Wang  
DIMACS, Rutgers University  
wang@thu.edu.tw

Chia-Ming Lin  
Department of Mathematics, Tunghai University, Taiwan  
lcm@thu.edu.tw

### CP14 The Spectrum of 2-Idempotent 3-Quasigroups with Conjugate Invariant Subgroups

A ternary quasigroup (or 3-quasigroup) is a pair  $(N, q)$  where  $N$  is an  $n$ -set and  $q(x, y, z)$  is a ternary operation on  $N$  with unique solvability. A 3-quasigroup is called 2-idempotent if it satisfies the generalized idempotent law:  $q(x, x, y) = q(x, y, x) = q(y, x, x) = y$ . A conjugation of a 3-quasigroup, considered as an  $OA(3, 4, n)$ ,  $(N, B)$ , is a permutation of the coordinate positions applied to the 4-tuples of  $B$ . The subgroup of conjugations under which  $(N, B)$  is invariant is called the conjugate invariant subgroup of  $(N, B)$ . Recently, we determined the existence of 2-idempotent 3-quasigroups of order  $n$ ,  $n \equiv 7$  or  $11 \pmod{12}$  and  $n \geq 11$ , with conjugate invariant subgroup consisting of a single cycle of length three. This result completely

determined the spectrum of 2-idempotent 3-quasigroups with conjugate invariant subgroups.

Ruizhong Wei  
Department of Computer Science  
Lakehead University  
rwei@lakeheadu.ca

Lijun Ji  
Department of Mathematics  
Suzhou University, CHINA  
jilijun@suda.edu.cn

### CP15

#### On the Upper Bound of the Modulus of Subdominant Eigenvalue of Stochastic Matrix.

The subdominant eigenvalues of the transition probability matrix of a Markov chain are determining factor in the speed of transition of the chain to a stationary state. In this talk, we introduce the notion of the scrambling index of a directed graph, and discuss some of its properties. We then present an upper bound on the scrambling index of primitive stochastic matrix  $T$  in terms of Boolean rank  $b(T)$  and characterize all the primitive matrices that can achieve this upper bound. Furthermore by using the scrambling index of the directed graph of a stochastic matrix, we give an attainable upper bound on the modulus of subdominant eigenvalue of  $T$ .

Mahmud Akelbek, Sandra Fital  
Weber State University  
amahemuti@weber.edu, sfitalakelbek@weber.edu

### CP15

#### Weak Sense of Direction Labelings and Graph Embeddings

In distributive computing, when the edge-labeling of a graph has a *sense of direction*, the communication complexity of many distributed problems significantly improves. We consider a weaker version of this property called *weak sense of direction* (WSD). Cayley graphs play a central role in WSD-labelings. They are exactly the regular graphs that have minimal symmetric WSD-labelings. We present new results on WSD-labelings that extend the connections of Cayley graphs and WSD-labelings to non-regular directed graphs.

Christine T. Cheng, Ichiro Suzuki  
University of Wisconsin-Milwaukee  
ccheng@cs.uwm.edu, suzuki@cs.uwm.edu

### CP15

#### Riemann-Roch for Sub-Lattices of the Root Lattice $A_n$

Recently, Baker and Norine (*Advances in Mathematics*, 215(2): 766-788, 2007) found new analogies between graphs and Riemann surfaces by developing a Riemann-Roch machinery on a finite graph  $G$ . We develop a general Riemann-Roch Theory for sub-lattices of the root lattice  $A_n$  by following the work of Baker and Norine, and establish connections between the Riemann-Roch theory and the Voronoi diagrams of lattices under certain simplicial distance functions. In this way, we rediscover the work of Baker and Norine from a geometric point of view and generalise their results to other sub-lattices of  $A_n$ . In par-

ticular, we provide a geometric approach for the study of the Laplacian of graphs. Applications to the classification of lattices are discussed.

Madhusudan Manjunath  
Max Planck Institute for Informatics  
manjun@mpi-inf.mpg.de

Omid Amini  
CNRS / DMA-École Normale Supérieure Paris  
omid.amini@ens.fr

### CP15

#### The Tammes and Related Problems

The Tammes problem asks to find maximal radius of  $N$  equal size nonoverlapping spherical caps in the unit 2-sphere. In this talk we are going to discuss a computer based solution of the Tammes problem for  $N=13$  as well as for several greater  $N$ . This is a joint work with Alexey Tarasov.

Oleg Musin  
University of Texas at Brownsville  
oleg\_musin@hotmail.com

### CP15

#### Planar Reachability in Log-Space Complexity Classes

Log-space computation is a central tool in complexity theory. The requirement of using  $O(\log n)$  space gives rise to local algorithms. Deciding reachability in different classes of graphs gives complete problems for different log-space classes, such as the extremes of reachability in undirected graphs is L-complete while reachability in directed graphs is NL-complete. Separating these classes is the space-bounded equivalent of the P=NP question. The complexity of directed planar graphs is currently unknown. This talk presents recent developments in the reachability problem for several classes of planar graphs.

Derrick P. Stolee  
University of Nebraska-Lincoln  
s-dstolee1@math.unl.edu

### CP16

#### Reflexive Injective Oriented Colourings

We study oriented colourings of directed graphs which satisfy the local injectivity property that no two in-neighbours of any vertex are assigned the same colour, but in which adjacent vertices need not be assigned different colours. We shall discuss complexity, obstructions, bounds, and related homomorphism problems.

Gary Macgillivray  
Mathematics and Statistics  
University of Victoria  
gmacgill@math.uvic.ca

Andre Raspaud  
Universite Bordeaux I  
raspaud@labri.fr

Jacobus Swarts  
Mathematics  
Vancouver Island University

cobus@math.uvic.ca

### CP16

#### On the Edit Distance of $\text{Forb}(K_{2,t})$

The edit distance from a graph  $G$  to a hereditary property  $\mathcal{H}$  is the minimum number of edge-additions/deletions required to transform  $G$  into a member of  $\mathcal{H}$ . The maximum of this quantity, over all density- $p$ ,  $n$ -vertex graphs, normalized by dividing by  $\binom{n}{2}$  and letting  $n \rightarrow \infty$ , is the *edit distance function*. We will give bounds on its value for hereditary properties of the form  $\mathcal{H} = \text{Forb}(\mathcal{K}_{\epsilon, \sqcup})$ , computing it exactly in the case  $t = 3$ .

Tracy J. McKay, Ryan Martin  
Iowa State University  
tmckay16@iastate.edu, rymartin@iastate.edu

### CP16

#### L(2,1)-Labeling of Unigraphs

An L(2,1)-labeling for a graph  $G$  consists of assigning non-negative integers, 0s, to the nodes of  $G$  so that adjacent nodes get values at least two apart and nodes at distance two get different values. Minimize  $s$  is an NP-complete problem. Here is conjectured that this problem remains NP-complete also for unigraphs, i.e. graphs uniquely determined by their own degree sequence up to isomorphism, and a linear time algorithm for L(2,1)-labeling unigraphs is designed.

Rossella Petreschi, Tiziana Calamoneri  
Dipartimento di Informatica, Sapienza Università di Roma  
petreschi@di.uniroma1.it, calamo@di.uniroma1.it

### CP16

#### Weighted Well-Covered Graphs Without Cycles of Lengths 4, 6 and 7

Let  $w$  be a linear set function defined on the vertices of a graph  $G$ . The graph  $G$  is  $w$ -well-covered if all its maximal independent sets have the same weight. Recognizing  $w$ -well-covered graphs is known to be **co-NP**-complete. We prove that the vector space of weight functions under which an input graph is  $w$ -well-covered can be found in polynomial time, if the input graph does not contain cycles of lengths 4, 6 and 7.

David Tankus  
Ariel University Center of Samaria, Israel  
davidta@ariel.ac.il

Vadim E. Levit  
Ariel University Center of Samaria and Holon Inst. of Tech.  
Dept. of Computer Science & Mathematics  
levitv@ariel.ac.il

### CP17

#### On the Mixing Time of Geometric Threshold Graphs

We study the mixing time of random graphs generated by the geographical threshold graph (GTG) model, a generalization of random geometric graphs (RGG). In a GTG, nodes are distributed in a Euclidean space, and

edges are assigned according to a threshold function involving the distance between nodes as well as randomly chosen node weights. If the weight distribution function decays with  $\Pr[W \geq x] = O(x^{-d-\nu})$  for an arbitrarily small constant  $\nu > 0$  then the mixing time of GTG is  $O(n^{2/d}(\log n)^{(d-2)/d})$ . This matches the known mixing bounds for RGG.

Andrew J. Beveridge  
Department of Mathematics, Statistics and Computer Science  
Macalester College  
abeverid@macalester.edu

Milan Brandonjic  
Theoretical Division, and Center for Nonlinear Studies  
Los Alamos National Laboratory  
brandonjic@gmail.com

### CP17

#### The Typical Structure of $H$ -Colourings of Regular Bipartite Graphs

For a graph  $H$  (perhaps with loops), an  $H$ -colouring of a simple graph  $G$  is a function from the vertices of  $G$  to the vertices of  $H$  which maps adjacent vertices to adjacent vertices. With suitable choices of  $H$ ,  $H$ -colourings can encode, for example, weighted independent sets and proper colourings of  $G$ .  $H$ -colourings are also referred to as *graph homomorphisms*. We address the following question: in a typical (uniformly chosen)  $H$ -colouring of a regular bipartite graph  $G$ , what proportion of the vertices of  $G$  get mapped to each vertex of  $H$ ? For a very large class of graphs  $H$ , we can give a quite precise answer to this question. For example, we can say that in almost all proper  $2k$ -colourings of a regular bipartite graph on  $N$  vertices, each colour will appear very close to  $N/2k$  times. The approach is through entropy, and extends work of J. Kahn from 2001 (who considered the size of randomly chosen independent sets of a regular bipartite graph).

David Galvin, John Engbers  
University of Notre Dame  
dgalvin1@nd.edu, jengbers@nd.edu

### CP17

#### A Counterexample to the Alon-Saks-Seymour Conjecture and Related Problems

Consider a graph obtained by taking an edge disjoint union of  $k$  complete bipartite graphs, Alon, Saks, and Seymour conjectured that such graphs have chromatic number at most  $k+1$ . This well known conjecture remained open for almost twenty years. In this talk, we will show a counterexample to this conjecture. This construction will also lead to some related results in combinatorial geometry and communication complexity. In particular, it implies a nontrivial lower bound of the non-deterministic communication complexity of the ‘clique versus independent set’ problem. Joint work with Benny Sudakov.

Hao Huang  
University of California-Los Angeles  
Department of Mathematics  
huanghao@math.ucla.edu

Choongbum Lee, Benjamin Sudakov  
Dept. of Mathematics  
UCLA

abdesire@math.ucla.edu, bsudakov@math.ucla.edu

### CP17

#### Resilient Pancyclicity of Random Graphs

A graph is called pancyclic if it contains cycles of all lengths. A classical theorem of Bondy states that every  $n$ -vertex graph  $G$  with  $\delta(G) > n/2$  is pancyclic. In this talk, we discuss extensions of this and another pancyclicity result to random graphs. In particular, our results show that random graphs remain pancyclic even after deletion of constant proportion of edges incident to every vertex.

Choongbum Lee

University of California, Los Angeles  
choongbum.lee@gmail.com

### CP17

#### Analysis of Scale-Free Network Models Based on $k$ -Trees

We introduce various growth-models for the combinatorial family of  $k$ -trees (introduced by Beineke & Pippert, 1969), which all lead to an asymptotic power-law degree distribution. Based on two descriptions of  $k$ -trees, firstly via a bottom-up approach due to the growth process, and secondly via a top-down approach due to a combinatorial decomposition, we are able to give a quite precise analysis of these network models leading to exact and asymptotic results of various parameters, as node degrees, distance parameters, number of descendants, and the clustering coefficient.

Alois Panholzer, Georg Seitz

Vienna University of Technology  
Institute of Discrete Mathematics and Geometry  
Alois.Panholzer@tuwien.ac.at, georg.seitz@tuwien.ac.at

### CP18

#### On Dobrynin-Gutman Conjecture

Let  $G$  be a connected graph and  $\eta(G) = Sz(G) - W(G)$ , where  $W(G)$  denotes the Wiener index and  $Sz(G)$  denotes the Szeged index of  $G$ . A well-known result of Klavžar, Rajapakse and Gutman states that  $\eta(G) \geq 0$  and by a result of Dobrynin and Gutman  $\eta(G) = 0$  if and only if each block of  $G$  is complete. In this paper an edge-path matrix for the graph  $G$  is presented by which it is possible to present a new characterization for the graphs in which the Wiener and Szeged are the same. It is also shown that  $\eta(G) \neq 1, 3$  and a classification of all graphs with  $\eta(G) = 2, 4, 5$  are presented. Finally, it is proved that for a given positive integer  $k, k \neq 1, 3$ , there exists a graph  $G$  with  $\eta(G) = k$ .

A. R. Ashrafi

University of Kashan  
Department of Mathematics  
alir.ashrafi@gmail.com

### CP18

#### Cospectral Graphs for the Normalized Laplacian

Two graphs are cospectral with respect to some matrix associated with the graph if the two matrices have the same eigenvalues. We will focus on the problem of cospectral graphs for the normalized Laplacian, and in particular give the first construction of an infinite family of non-bipartite

non-regular graphs which are cospectral with respect to the normalized Laplacian.

Steve Butler

UC Los Angeles  
USA  
butler@math.ucla.edu

Jason Grout

Drake University  
jason.grout@drake.edu

### CP18

#### Construction of Maximal-Determinant Binary Matrices

Hadamard matrices have maximal determinant among matrices with elements  $-1$  and  $1$ , but cannot exist when the size  $n$  is not divisible by 4. I describe matrices with large or maximal determinant when  $n \not\equiv 0 \pmod{4}$ . When  $n \equiv 1 \pmod{4}$  an  $(n-1) \times (n-1)$  Hadamard matrix with suitable properties is used. When  $n \equiv 2$  or  $3 \pmod{4}$ , a pair of binary sequences whose autocorrelation functions have sharply-peaked sum is needed. Brent, Osborn, Zimmerman, and I have proved maximality of some of these determinants.

William P. Orrick

Department of Mathematics  
Indiana University  
worrick@indiana.edu

### CP18

#### Matrix Interdiction Problem

In the matrix interdiction problem, a real-valued matrix is given, and a row value is defined at each row as the maximum value of the row elements. The objective is to remove  $k$  columns such that the sum of row values over all rows is minimized. This combinatorial problem is closely related to bipartite network interdiction problem which can be applied to prioritize the border checkpoints in order to minimize the probability that an adversary can successfully cross the border. After introducing the matrix interdiction problem, we will prove the problem is NP-hard, and even NP-hard to approximate with any sub-linear (in  $n$ ) factor. We also present an algorithm for this problem that achieves a factor of  $(n-k)$  approximation.

Feng Pan, Shiva Kasiviswa

Los Alamos National Laboratory  
fpan@lanl.gov, kasivisw@lanl.gov

### MS1

#### Large Cliques and Stable Sets Excluding Paths and Antipaths

For every fixed graph  $H$ , if a graph  $G$  does not contain  $H$  as a minor, then one can say a lot about the structure and properties of  $H$ . Unfortunately, results of that kind do not seem to be true if we replace the minor containment by induced subgraph containment. One of the few conjectures about general behavior of graphs with certain induced subgraphs forbidden is the Erdos Hajnal Conjecture. It states that for every fixed graph  $H$  there exists a constant  $\delta(H)$ , such that if a graph  $G$  has no induced subgraph isomorphic to  $H$ , then  $G$  contains a big clique or a big stable set of size  $|V(G)|^{\delta(H)}$ . The Erdos Hajnal Conjecture is known to

be true for graphs  $H$  on at most four vertices, but there are some five-vertex graphs for which the conjecture is still open. One of such graphs is a path of length four (edges). We prove that if a graph  $G$  does not contain as induced subgraphs a path of length four or the complement of a path of length five, then  $G$  contains a clique or a stable set of size  $|V(G)|^{\frac{1}{6}}$ .

Maria Chudnovsky  
Columbia  
mchudnov@columbia.edu

Yori Zwols  
Columbia University  
yz2198@columbia.edu

### MS1

#### Edge List Colouring Class II Plane Graphs with Few Long Lists

Given an integer-valued edge weighting  $w : E(G) \rightarrow N$ , we say that  $G$  is  $w$ -edge-choosable if, for every list assignment  $L : E(G) \rightarrow 2^N$  satisfying  $|L(e)| \geq w(e)$ , there exists a proper edge coloring  $c$  for which  $c(e) \in L(e)$  for every edge  $e$ . We show how to use the Combinatorial Nullstellensatz to prove choosability results for regular graphs which are not 1-factorable. For example, every planar cubic graph  $G$  is  $w$ -edge-choosable, for some  $w : E(G) \rightarrow \{1, 2, 3, 4\}$  where the number of edges  $e$  with  $w(e) = 4$  is linear in the number of cut-edges in  $G$ .

Luis A. Goddyn  
Department of Mathematics  
Simon Fraser University  
goddyn@math.sfu.ca

### MS1

#### Fractional Total Colourings of Graphs of High Girth

In 1993, Kilakos and Reed proved that the fractional total chromatic number of a  $d$ -regular graph is between  $d+1$  and  $d+2$ . Reed recently conjectured that for fixed  $d$  and  $\epsilon$ , a graph of sufficiently high girth and maximum degree  $d$  has fractional total chromatic number at most  $d+1+\epsilon$ . We prove a stronger result of this type for  $d \in \{3, 4, 6, 8, \dots\}$ , i.e. that a graph of sufficiently high girth (depending on  $d$ ) has fractional total chromatic number exactly  $d+1$ . We prove this conjecture using the approach of 1-path decompositions and a randomized method for choosing total stable sets of  $G$ .

Tomas Kaiser  
University of West Bohemia  
kaiser@kma.zcu.cz

Andrew D. King  
Columbia University  
andrew.d.king@gmail.com

Daniel Kral  
Charles University  
kral@kam.mff.cuni.cz

### MS1

#### The Last Fraction of a Fractional Conjecture and

### Open Problems

A total coloring of a graph is composed of a (proper) vertex coloring and a (proper) edge coloring with the additional constraint that vertices and edges that are incident are assigned different colors. Fractional total colorings are the linear relaxation of total colorings. Reed conjectured that

for fixed  $\Delta$  and  $\epsilon$ , a graph of maximum degree  $\Delta$  and sufficiently high girth has fractional total chromatic number at most  $\Delta + 1 + \epsilon$ . Kaiser, King and Král' proved that for  $\Delta = 3$  and  $\Delta \in \{4, 6, 8, \dots\}$ , every graph of maximum degree  $\Delta$  and sufficiently high girth has fractional total chromatic number *exactly*  $\Delta + 1$ . With F. Kardoš and D. Král', we proved the conjecture for the remaining cases. Our proof relies on first establishing the result for cyclically  $\Delta$ -edge-connected graphs (using a suitable partition of the edges) and then dealing with the general case by induction on the number of edges (using a tree-recoloring lemma). I will give a sketch of the proof and a part of the

talk will be devoted to open problems.

František Kardoš  
University of Pavol Jozef {  
v S}af  
'arik  
Ko{  
v s}ice, Slovakia  
frantisek.kardos@upjs.sk

Daniel Král'  
Institute for Theoretical Computer Science (ITI)  
Charles University, Prague, Czech Republic  
kral@kam.mff.cuni.cz

J. S. Sereni  
CNRS, Paris, France  
(LIAFA - Univ. Diderot)  
sereni@liafa.jussieu.fr

### MS2

#### Filtered Hopf Algebras

We present a theory of non-homogeneous quasi-symmetric invariants of combinatorial Hopf algebras. It extends previous results by the author with Bergeron and Sottile and is motivated by Stanley's theory of P-kernels (which becomes a special case) and work of Billera and Brenti on Kazhdan-Lusztig polynomials.

Marcelo Aguiar  
Texas A&M  
maguiar@math.tamu.edu

Louis J. Billera  
Cornell University  
billera@math.cornell.edu

### MS2

#### Hopf Algebras, $g$ -polynomials and Kazhdan-Lusztig Polynomials

We apply the newly developed theory of non-homogeneous quasi-symmetric invariants of combinatorial Hopf algebras to the special cases of  $g$ -polynomials of regular CW-spheres and Kazhdan-Lusztig polynomials of Bruhat intervals as

well as some extensions of these.

Louis J. Billera  
Cornell University  
billera@math.cornell.edu

Marcelo Aguiar  
Texas A&M  
maguiar@math.tamu.edu

## MS2

### New Hopf Algebra Structures on Compositions.

Compositions, or bijectively boolean subsets, form the graded basis for the very important combinatorial Hopf algebra of quasisymmetric functions. Now, via a description of the simplices as cellular projections of associahedra, we introduce a new graded algebra with the  $n^{\text{th}}$  component of its basis the vertices of the standard  $(n - 1)$ -simplex. We extend this new algebra to a new graded Hopf algebra based upon the full face posets of simplices: the boolean posets. Next, from a description of the cubes as projections of the multiplihedra, comes another new Hopf algebra based upon the compositions illustrated as painted trees. Mysteriously, it appears that our two new algebras are in fact dual to each other.

Stefan Forcey  
Tennessee State University  
sforcey@tnstate.edu

Aaron Lauve, Frank Sottile  
Texas A&M University  
lauve@math.tamu.edu, sottile@math.tamu.edu

## MS2

### Lagrange's Theorem: from Groups to Hopf Monoids in Species

Recall that if  $K \subseteq H$  are finite groups, then  $|K|$  divides  $|H|$ . The literature contains several Hopf variations on this theme. We recount a few, then introduce our own: if  $K \subseteq H$  are Hopf monoids in species, then  $H$  is free over  $K$ . We apply our theorem to enumeration questions in combinatorial Hopf algebras.

Aaron Lauve  
Texas A&M University  
lauve@math.tamu.edu

Marcelo Aguiar  
Texas A&M  
maguiar@math.tamu.edu

## MS2

### The Hopf Algebra of Strict Multiset Compositions and Structure Transfer

We explore the structure of the cocommutative Hopf algebra freely generated (as an algebra) by the coalgebra of finite subsets of natural numbers. We determine explicit formulas for its primitive elements and its antipode. We also develop machinery to transfer this structure to several combinatorial Hopf algebras appearing in the literature, including the Bergeron-Reutenauer-Rosas-Zabrocki Hopf algebra of symmetric functions in noncommuting variables.

Mitja Mastnak  
St. Mary's University  
Halifax  
mmastnak@cs.smu.ca

Aaron Lauve  
Texas A&M University  
lauve@math.tamu.edu

## MS3

### Polynomial Littlewood-Offord Type Theorems

In their studies of the roots of random polynomials Littlewood and Offord came across a question which can be worded probabilistically as follows: If  $a_1, a_2, \dots, a_n$  are fixed nonzero constants, and  $x_1, x_2, \dots, x_n$  are independently set equal to 1 or  $-1$  with equal probability, what is the maximum concentration of the "random walk" function  $a_1x_1 + a_2x_2 + \dots + a_nx_n$  on any given value? Motivated by a question regarding the singularity probability of random symmetric matrices (whose determinant is a quadratic form in the entries of any particular row or column of the matrix), we consider an analogue of this question where the random walk is replaced by a polynomial function of the inputs.

Kevin Costello  
Department of Mathematics  
Georgia Tech.  
kcostell@math.gatech.edu

## MS3

### Sum-product Phenomenon and Expanders - Part I of II

We present a new approach to establishing expansion and spectral gap bounds based on using tools from arithmetic combinatorics, in particular sum-product estimates (joint work with Jean Bourgain)

Alex Gamburd  
Department of Mathematics  
UCSC  
agamburd@ucsc.edu

## MS3

### Sum-product Phenomena and Expanders - Part II of II

We present a new approach to establishing expansion and spectral gap bounds based on using tools from arithmetic combinatorics, in particular sum-product estimates (joint work with Jean Bourgain)

Alex Gamburd  
Department of Mathematics  
UCSC  
agamburd@ucsc.edu

## MS3

### Automated Bounds for Some Problems in Extremal Additive Combinatorics

I will show that how an automated approach can be used to improve some previously known bounds for problems in

extremal additive Combinatorics.

Hamed Hatami  
Department of Mathematics  
Princeton Univ.  
hatami@gmail.com

### MS3

#### Finite Point Configurations in Discrete, Continuous and Arithmetic Settings

We shall discuss the general problem of finding congruent copies of finite point configurations inside sufficiently large subsets of vector spaces and connections between discrete, continuous and arithmetic phenomena that arise in the process.

Alex Iosevich  
University of Missouri  
iosevich@math.missouri.edu

### MS4

#### A Construction of Infinite Sets of Intertwiners for Pairs of Matroids

Intertwiners arise naturally when considering the excluded minors of a union of minor-closed classes. An intertwiner of a pair of matroids is a matroid such that it, but none of its proper minors, has minors that are isomorphic to each matroid in the pair. Intertwiners can be defined likewise for graphs. From the graph minors theorem of Robertson and Seymour (specifically, that graphs are well-quasi-ordered under minors), it follows that any pair of graphs has only finitely many intertwiners. In contrast, addressing a problem first posed by Tom Brylawski, Dirk Vertigan showed that certain pairs of matroids have infinitely many intertwiners. We will approach the construction of intertwiners from a new perspective, namely, that of cyclic flats and their ranks; this leads to an expanded range of results as well as new open problems.

Joseph Bonin  
The George Washington University  
jbonin@gwu.edu

### MS4

#### Fragility in Matroids

For a matroid  $M$  with a minor  $N$ , we say that  $M$  is  $N$ -fragile if, for every element  $e$  in the ground set of  $M$ , either  $M \setminus e$  or  $M/e$  does not contain  $N$  as a minor. Understanding the structure of  $N$ -fragile matroids is necessary for thinking about Rota's conjecture. In this talk, we present a characterization of the binary, Fano-fragile matroids.

Carolyn Chun, Dillon Mayhew  
Victoria University of Wellington  
New Zealand  
carolyn.chun@msor.vuw.ac.nz,  
dillon.mayhew@msor.vuw.ac.nz

Stefan van Zwam  
University of Waterloo  
stefan.van.zwam@cwil.nl

Geoff Whittle  
Victoria University of Wellington  
New Zealand

geoff.whittle@msor.vuw.ac.nz

### MS4

#### Linear Rank Inequalities on Five or More Variables

Ranks of subspaces of vector spaces satisfy all linear inequalities satisfied by entropies (including the standard Shannon or polymatroid inequalities) and an additional inequality due to Ingleton; such additional inequalities are a useful way to prove the nonrepresentability of matroids. It is known that the Shannon and Ingleton inequalities generate all such linear rank inequalities on up to four variables, but it had been an open question whether additional inequalities hold for the case of five or more variables. Here we present a list of 24 inequalities which, together with the Shannon and Ingleton inequalities, generate all linear rank inequalities on five variables. (This is proved by showing that all of the extreme rays of the polytopal cone bounded by these inequalities are given by representable polymatroids.) We also present a partial list of linear rank inequalities on six variables (numbering several hundred at this point) and general results which produce such inequalities on an arbitrary number of variables; we prove that there are essentially new inequalities at each number of variables beyond four (a result also proved recently by Kinser).

Randall Dougherty  
Center for Communications Research  
San Diego  
rdough@ccrwest.org

Christopher Freiling  
Department of Mathematics  
California State University, San Bernardino  
cfreilin@csusb.edu

Kenneth Zeger  
Department of Electrical and Computer Engineering  
University of California, San Diego  
zeger@ucsd.edu

### MS4

#### Constructing all Triangle-free 3-connected Matroids

Tutte's Wheel Theorem guarantees that any simple 3-connected graph is obtained from an appropriated wheel by adding edges or splitting vertices. This result is equivalent to the following: if  $G$  is a simple 3-connected graph non-isomorphic to a wheel, then  $G$  has an edge  $e$  such that  $G \setminus e$  or  $G/e$  is a simple 3-connected graph. Kriesell obtained a similar result for the class of simple triangle-free 3-connected graph. Instead of two basic reductions, that is, an edge contraction or deletion, one need to apply six small operations to reduce a simple triangle-free 3-connected graph, without leaving this class in any step, to a graph isomorphic to  $K_{3,3}$  or to a double-wheel. Tutte extended his result to 3-connected matroids in a fundamental paper in Matroid Theory. We are able to extend Kriesell's theorem to triangle-free 3-connected matroids. We present this result in this talk.

Manoel Lemos  
Universidade Federal de Pernambuco  
Brazil  
manoel@dmat.ufpe.br

## MS4

**Is the Missing Axiom of Matroid Theory Lost Forever?**

In Whitney's seminal paper on matroids, he says that "the fundamental question of completely characterizing systems which represent matrices is left unsolved". Given that his example of a non-matrix matroid is the Fano plane, it seems clear that Whitney was effectively referring to a characterization of real-representable matroids. In 1978, Vámos addressed this question in a paper entitled "The missing axiom of matroid theory is lost forever" giving the clear impression that Whitney's problem is hopeless. But it turns out that the situation is far from being as clear as the title of Vámos' paper suggests. I propose to discuss this issue as well as discussing some recent results on matroid representation over infinite fields.

Geoff Whittle

Victoria University of Wellington  
New Zealand  
geoff.whittle@vuw.ac.nz

## MS5

**Circumferences of  $k$ -Connected Graphs Involving Independence Numbers**

Let  $G$  be a  $k$ -connected graph of order  $n$ ,  $\alpha := \alpha(G)$  the independence number of  $G$ , and  $c(G)$  the circumference of  $G$ . Chvátal and Erdős proved that if  $\alpha \leq k$  then  $G$  is hamiltonian. For  $\alpha \geq k \geq 2$ , Fouquet and Jolivet in 1978 made the conjecture that  $c(G) \geq \frac{k(n+\alpha-k)}{\alpha}$ . Fournier proved that the conjecture is true for  $\alpha \leq k+2$  or  $k=2$  in two different papers. Manoussakis recently proved that the conjecture is true for  $k=3$ . We will show that if  $G$  is a  $k$ -connected graph,  $k \geq 4$ , of order  $n$  and independence number  $\alpha \geq k$ , then  $c(G) \geq \frac{k(n+\alpha-k)}{\alpha} - \frac{(k-3)(k-4)}{2}$ . Consequently, the Fouquet-Jolivet Conjecture holds for  $k=4$ . In addition, we confirm the conjecture for  $\alpha = k+3$ . Inspired by a result of Kouider, we conjecture that, for every graph  $G$  and any two distinct vertices  $u$  and  $v$ , there is a  $u-v$  path  $P$  such that  $\alpha(G - V(P)) \leq \alpha(G) - 1$  unless  $V(G)$  have a partition  $V_1 \cup V_2$  satisfying  $G = (G[V_1]) + (G[V_2])$ . In this paper, we obtain a partial result regarding this conjecture. Let  $G$  be a  $k$ -connected graph and  $k \geq 2$ . In study the intersection of longest cycles, J. Chen et al conjectured that, for any two cycles  $C_1$  and  $C_2$  of  $G$ , there are two cycles  $D_1$  and  $D_2$  such that  $V(D_1) \cup V(D_2) \supseteq V(C_1) \cup V(C_2)$  and  $|D_1 \cap V(D_2)| \geq k$ . We show that the combination of the above two conjectures implies the Fouquet-Jolivet Conjecture.

Guantao Chen

Georgia State University  
Dept of Mathematics and Statistics  
gchen@gsu.edu

Zhiquan Hu

Central China Normal University  
Wuhan, China  
hu\_zhiq@yahoo.com.cn

Yaping Wu

Central China Normal University  
Wuhan, China  
wypdp@sina.com

## MS5

**Disjoint Cycles and Dicycles in Digraphs**

We study the following problem: Given a digraph  $D$ , decide if there is a cycle  $B$  in  $D$  and a cycle  $C$  in its underlying undirected graph such that  $V(B) \cap V(C) = \emptyset$ . Whereas the problem is  $\mathcal{NP}$ -complete if, as an additional part of the input, a vertex  $x$  is prescribed to be contained in  $C$ , one can decide the existence of  $B, C$  in polynomial time under the (mild) additional assumption that  $D$  is strongly connected.

Matthias Kriesell

Department of Mathematics  
University of Hamburg  
kriesell@math.uni-hamburg.de

Joergen Bang-Jensen

Department of Mathematics and Computer Science  
University of Southern Denmark  
jbj@imada.sdu.dk

## MS5

**2-connected Subgraphs After Deleting Many Disjoint Paths**

Motivated by the well-known conjecture by Lovász on the connectivity after the path removal, we study the following problem: There exists a function  $f = f(k, l)$  such that the following holds. For every  $f(k, l)$ -connected graph  $G$  and two distinct vertices  $s$  and  $t$  in  $G$ , there are  $k$  internally disjoint paths  $P_1, \dots, P_k$  with endpoints  $s$  and  $t$  such that  $G - \bigcup_{i=1}^k V(P_i)$  is  $l$ -connected. When  $k=1$ , this problem corresponds to Lovász conjecture, and it is open for all the cases  $l \geq 3$ . We show that  $f(k, 1) = 2k+1$  and  $f(k, 2) \leq 3k+2$ . The connectivity " $2k+1$ " for  $f(k, 1)$  is best possible. Thus our result generalizes the result by Tutte for the case  $k=1$  and  $l=1$  (the first settled case of Lovász conjecture), and the result by Chen, Gould and Yu, Kriesell, Kawarabayashi, Lee, and Yu, independently, for the case  $k=1$  and  $l=2$  (the second settled case of Lovász conjecture). When  $l=1$ , our result also improves the connectivity bound " $2k+2$ " given by Chen, Gould and Yu. This is the joint work with K. Kawarabayashi.

Kenta Ozeki, Ken-ichi Kawarabayashi

National Institute of Informatics, Japan  
ozeki@comb.math.keio.ac.jp, k\_keniti@nii.ac.jp

## MS5

**Color-Critical Graphs Have Logarithmic Circumference**

A graph  $G$  is  $k$ -critical if every proper subgraph of  $G$  is  $(k-1)$ -colorable, but the graph  $G$  itself is not. We prove that every  $k$ -critical graph on  $n$  vertices has a cycle of length at least  $\log n / (100 \log k)$ , improving a bound of Alon, Krivelevich and Seymour from 2000. Examples of Gallai from 1963 show that the bound cannot be improved to exceed  $2(k-1) \log n / \log(k-2)$ . We thus settle the problem of bounding the minimal circumference of  $k$ -critical graphs, raised by Dirac in 1952 and Kelly and Kelly in 1954.

Asaf Shapira, Robin Thomas

Georgia Tech  
asafico@math.gatech.edu, thomas@math.gatech.edu

**MS5****A Shorter Proof of the Unique Linkage Theorem**

Let  $G$  be a graph and  $P_1, \dots, P_k$  disjoint paths in  $G$  such that  $\bigcup_1^k V(P_i) = V(G)$ . The unique linkage theorem of Robertson and Seymour states that either there exist disjoint paths  $P'_1, \dots, P'_k$  such that  $P_i$  and  $P'_i$  have the same endpoints for  $1 \leq i \leq k$  and  $\bigcup_1^k V(P'_i)$  is a proper subset of  $V(G)$ , or, alternatively, the tree width of  $G$  is bounded by a function of  $k$ . The unique linkage theorem is a major tool in the proof of the polynomial running time of Robertson and Seymour's algorithm for the  $k$  disjoint paths problem. The original proof of Robertson and Seymour of the unique linkage theorem is quite difficult and uses the structure theorem for graphs excluding a clique minor. I will describe a new and simpler proof that avoids many of these difficulties; specifically, the proof does not rely on the excluded minor structure theorem of Robertson and Seymour.

Paul Wollan

University of Rome "La Sapienza"  
paul.wollan@gmail.com

Ken-ichi Kawarabayashi

National Institute of Informatics, Japan  
k\_keniti@nii.ac.jp

**MS6****The cd-index of Bruhat and Balanced Graphs**

The **cd**-index is a noncommutative polynomial which compactly encodes the flag vector data of an Eulerian poset. There are two major classes of Eulerian poset: face lattices of convex polytopes (and more generally face posets of regular spherical CW-complexes) and intervals of the strong Bruhat order of Coxeter groups. Billera and Brenti introduced the notion of the complete **cd**-index of a Bruhat interval which encodes more information than the classical **cd**-index of the interval. Motivated by their work, we extend the notion of Bruhat graphs to balanced labeled graphs and prove the existence of the **cd**-index. We end with a discussion about open problems including non-negativity questions. This is joint work-in-progress with Margaret Readdy.

Richard Ehrenborg

University of Kentucky  
jrge@ms.uky.edu

**MS6****Topological Methods for Cellular Resolutions**

A cellular resolution of an ideal is a resolution where all the maps are boundary maps of a supporting cellular complex. Whenever new types of resolutions of monomial ideals are found, it is also common to look for cellular complexes supporting them. In a joint work with Anton Dochtermann, we have tried to understand from a topological perspective how to construct explicit supporting complexes and describe the maps between them. For example, we generalize and explain resolutions studied by Corso, Froberg, Nagel, and Reiner.

Alex Engstrom

UC Berkeley  
alex@math.berkeley.edu

**MS6****Combinatorics and Topology of Face Posets**

This talk will give an overview on results about face posets of simplicial complexes and closure posets of regular CW complexes, including recent work using a mixture of combinatorics of face posets together with codimension one topology to study which stratified spaces are regular CW complexes.

Patricia L. Hersh

North Carolina State University  
plhersh@ncsu.edu

**MS6****A Survey of f-vectors since Bjorner's 2005 NSF-CBMS San Francisco Lecture Series, "Algebraic and Topological Combinatorics of Ordered Sets" - Part II of II**

In 2005, Bjorner gave a lecture series at an NSF-CBMS regional conference in San Francisco titled, "Algebraic and Topological Combinatorics of Ordered Sets." This is part II of a two-part survey covering advances in face enumeration since then.

Isabella Novik

University of Washington  
novik@math.washington.edu

**MS6****A Survey of f-vectors since Bjorner's 2005 NSF-CBMS San Francisco Lecture Series, "Algebraic and Topological Combinatorics of Ordered Sets" - Part I of II**

In 2005, Bjorner gave a lecture series at an NSF-CBMS regional conference in San Francisco titled, "Algebraic and Topological Combinatorics of Ordered Sets." This is part I of a two-part survey covering advances in face enumeration since then.

Ed Swartz

Cornell University  
ebs22@cornell.edu

**MS7****Inverse Littlewood-Offord Theory**

Let  $V = \{v_1, \dots, v_n\}$  be a multiset of  $n$  real numbers. Let  $\eta_i$  be i.i.d. Bernoulli random variables. The concentration probability  $P(V)$  of  $V$  is defined as  $P(V) := \sup_v P(\eta_1 v_1 + \dots + \eta_n v_n = v)$ . A classical result of Littlewood-Offord and Erdos from the 1940s asserts that if the  $v_i$  are non-zero, then the concentration probability of  $V$  is  $O(n^{-1/2})$ . In the reverse direction, Tao and Vu proved that any set of large concentration probability must have structure. In this talk, we will provide a general approach that gives an almost best possible characterization for all such  $V$ . This allows us to recover several previous forward Littlewood-Offord results, including a significant result of Stanley from the 1980s on the optimal value of  $P(V)$  when  $v_i$  are distinct. (Joint with Van Vu, Rutgers University)

Hoi Nguyen

Department of Mathematics  
Rutgers Univ.  
hoi@math.rutgers.edu

Van Vu  
Rutgers University  
vanhavu@yahoo.com

MS7

### A Sharp Estimate for the Square Dependence Problem

Motivated by applications to fast integer factoring algorithms, Pomerance had raised the so-called *square dependence problem* in 1994: *Select integers  $a_1, a_2, \dots$ , at random from the interval  $[1, x]$ , until some (non-empty) subsequence has product equal to a square. Find a tight estimate for the expected stopping time of this process.* Using an idea of Schroepel (1985), Pomerance showed that, with probability  $1 - o(1)$ , the square dependence occurs after at least  $J_0^{1-o(1)}$  integers have been selected, but no more than  $J_0$ , for an appropriate explicitly determined  $J_0 = J_0(x)$ . In joint work with Croot, Granville and Pemanle, we determine this expected stopping time up to a small *constant factor*, tightening Pomerance's interval to

$$[(\pi/4)(e^{-\gamma} - o(1))J_0, (e^{-\gamma} + o(1))J_0],$$

where  $\gamma = 0.577\dots$  is the Euler-Mascheroni constant.

Prasad Tetali

School of Mathematics, Georgia Institute of Technology  
Atlanta, Ga 30332-0160  
tetali@math.gatech.edu

MS7

### Structural Approach to Problems to Combinatorial Number Theory - Part I of II

We give a brief survey on an emerging approach for solving problems in combinatorial number theory. The core of this approach are structural (or inverse, or rigidity) theorems, which characterize sets with certain additive properties. Examples will include: (1) Olson-type subset sums problem in  $Z_p$ . (2) Erdos-Ginzburg-Ziv, Caychy-Davenport, and Erdos-Heilbronn. (3) Erdos-Folkman conjecture on complete sequences (4) Erdos square-free conjecture.

Van Vu

Rutgers University  
vanhavu@yahoo.com

MS7

### Structural Approach to Problems to Combinatorial Number Theory - Part II of II

We give a brief survey on an emerging approach for solving problems in combinatorial number theory. The core of this approach are structural (or inverse, or rigidity) theorems, which characterize sets with certain additive properties. Examples will include: (1) Olson-type subset sums problem in  $Z_p$ . (2) Erdos-Ginzburg-Ziv, Caychy-Davenport, and Erdos-Heilbronn. (3) Erdos-Folkman conjecture on complete sequences (4) Erdos square-free conjecture.

Van Vu

Rutgers University  
vanhavu@yahoo.com

MS7

### The Inverse Erdős-Heilbronn Problem

The Erdős-Heilbronn conjecture (proved by Dias da Silva

and Hamidoune in 1994) asserts that if  $A$  is a subset of  $\mathbf{Z}/p\mathbf{Z}$ , the cyclic group of the integers modulo a prime  $p$ , then  $A\hat{+}A$ , the set of all sums of distinct elements, has cardinality at least  $\min\{2|A| - 3, p\}$ . The bound is sharp when  $A$  is an arithmetic progression. A natural question to ask is whether, conversely,  $A\hat{+}A$  being small implies that  $A$  is close to an arithmetic progression. In this talk, we will discuss some recent progress towards answering this question. Joint work with Van Vu.

Philip Matchett Wood

Stanford University  
pmwood@math.stanford.edu

Van Vu

Rutgers University  
vanvu@math.rutgers.edu

MS8

### Developments in Nash Equilibrium Computation

According to Aumann, zero-sum games are "one of the few areas in game theory, and indeed in the social sciences, where a fairly sharp, unique prediction is made". We provide a generalization of the minmax theorem to \*multi-player games\*. The games we consider are zero-sum polymatrix—that is, every pair of players plays a (potentially different) two-player game, and every outcome of the global interaction has zero sum of all players' payoffs. Our generalization of the minmax theorem to this setting implies convexity of equilibria, polynomial-time tractability, and convergence of no-regret learning algorithms to equilibria. We show that our class of games is essentially the broadest class of multiplayer zero-sum games to which we can hope to push tractability results. And what about extending our results beyond zero-sum games? Previous work has established that computing exact Nash equilibria is computationally intractable, and research on approximation algorithms has made no progress beyond finite values of approximation. Nevertheless, inapproximability results have been evading current techniques. We provide the first inapproximability result for Nash equilibria in two-player games, for constant values of relative approximation.

Constantinos Daskalakis

MIT  
costis@csail.mit.edu

MS8

### Computational Mechanism Analysis: Towards a "CPLEX for Mechanisms"

Many mechanisms become important "in the wild" despite a lack of theoretical arguments in their favor, chiefly because their complexity precludes analysis by existing techniques. This talk advocates the creation of computational tools for describing and empirically analyzing such mechanisms. This agenda raises a host of new theoretical problems: identifying representations that compactly represent interesting classes of games; determining encodings of mechanisms of interest into these representations; and deriving efficient procedures for computing solution concepts of interest given these encodings. Recently, we have begun to take steps to make this agenda concrete. Specifically, I will describe "Action Graph Games", a compact encoding for perfect-information games in which agents' payoffs exhibit context-free independencies. I will show how this representation can be used to encode sponsored search mechanisms, and will characterize resulting equilib-

rium behavior both in terms of social welfare and revenue. I will also outline some recent extensions of our representation to encompass Bayesian games.

Kevin Leyton-Brown  
UBC  
kevinlb@cs.ubc.ca

## MS8

### Game Theory with Costly Computation

We develop a general game-theoretic framework for reasoning about strategic agents performing possibly costly computation. In this framework, many traditional game-theoretic results (such as the existence of a Nash equilibrium) no longer hold. Nevertheless, we can use the framework to provide psychologically appealing explanations to observed behavior in well-studied games (such as finitely repeated prisoner's dilemma and rock-paper-scissors). Furthermore, we provide natural conditions on games sufficient to guarantee that equilibria exist. As an application of this framework, we develop a definition of protocol security relying on game-theoretic notions of implementation. We show that a natural special case of this this definition is equivalent to a variant of the traditional cryptographic definition of protocol security; this result shows that, when taking computation into account, the two approaches used for dealing with “deviating” players in two different communities—Nash equilibrium in game theory and zero-knowledge “simulation” in cryptography—are intimately related. Joint work with Joseph Halpern

Rafael Pass  
Cornell University  
rafael@cs.cornell.edu

## MS8

### Intrinsic Robustness of the Price of Anarchy

The price of anarchy is a measure of the inefficiency of decentralized behavior that has been successfully analyzed in many systems. It is defined as the worst-case ratio between the welfare of an equilibrium and that of an optimal solution. Seemingly, a bound on the price of anarchy is meaningful only if players successfully reach an equilibrium. Our main result is that for most of the classes of games in which the price of anarchy has been studied, results are “intrinsically robust” in the following sense: a bound on the worst-case price of anarchy for equilibria \*necessarily\* implies the exact same worst-case bound for a much larger set of outcomes, such as the possible sequences generated by no-regret learners. We also describe recent applications to the analysis of Bayes-Nash equilibria in (non-truthful) mechanisms.

Tim Roughgarden  
Stanford University  
Computer Science Department  
tim@cs.stanford.edu

## MS8

### Two Dimensional Matching Markets

We study heterogeneous matching or marriage markets where the payoff of the individuals involved in a matching is determined by the type of their mutual partner. Previously, models with one-dimensional type are considered. These models lead to complete homophily in the emerging equilibrium: individuals tend to match with the people of

their own type (or attractiveness). Moreover, in the presence of search frictions in such markets, we will observe block segregation. We show that the block segregation (and homophily in general) is an artifact of the assumption that the types are one-dimensional. We investigate such markets in a more natural setting where the individual types can have multiple dimensions (in our model, two) and each individual cares about a certain dimension of his or her partners type. We characterize the equilibrium in such markets (in both frictional and frictionless instances) and show that a combination of homophily and mixing appears in the equilibrium. Our result explains the mixing observed in both experimental results and real-world studies. Joint work with Daron Acemoglu, Arash Asadpour, Christian Borgs, and Jennifer Chayes

Amin Saberi  
Management Science and Engineering  
Stanford University  
saberi@stanford.edu

## MS9

### Linear Extension Diameter and Random Posets

The linear extension diameter of a poset is the maximum distance between two linear extensions of the poset, where distance is counted as the number of pairs of elements in opposite order. We shall discuss some recent results on this topic. We shall focus on the use of random partial orders to obtain upper bounds on the linear extension diameter for posets in which most pairs are incomparable.

Graham Brightwell  
Department of Mathematics  
London School of Economics  
g.r.brightwell@lse.ac.uk

## MS9

### On the Size of Maximal Chains and the Number of Pairwise Disjoint Maximal Antichains

Fix integers  $n$  and  $k$  with  $n \geq k \geq 3$ . We prove that if  $P$  is a finite poset and  $n \leq |C| \leq n + (n - k)/(k - 2)$  for every maximal chain in  $P$ , then  $P$  must contain  $k$  pairwise disjoint maximal antichains. We construct examples showing that these inequalities are tight. The result has a dual version, the subject of work by Howard and Trotter.

Dwight Duffus  
Emory University  
Mathematics and Computer Science Department  
dwight@mathcs.emory.edu

Bill Sands  
University of Calgary  
sands@ucalgary.ca

## MS9

### Families of Maximal Chains

Fix integers  $n$  and  $k$  with  $n \geq k \geq 3$ . Duffus and Sands proved that if  $P$  is a finite poset and  $n \leq |C| \leq n + (n - k)/(k - 2)$  for every maximal chain in  $P$ , then  $P$  must contain  $k$  pairwise disjoint maximal antichains. They also constructed a family of examples to show that these inequalities are tight. These examples are 2-dimensional which suggests that the dual statement may also hold. In this paper, we show that this is correct. Specifically, we show

that if  $P$  is a finite poset and  $n \leq |A| \leq n + (n - k)/(k - 2)$  for every maximal antichain in  $P$ , then  $P$  has  $k$  pairwise disjoint maximal chains. Our argument actually proves a somewhat stronger result, and we are able to show that an analogous result holds for antichains.

David Howard

Georgia Institute of Technology  
dmh@math.gatech.edu

William T. Trotter

School of Mathematics  
Georgia Institute of Technology  
trotter@math.gatech.edu

### MS9

#### Lattices and Polytopes from Graphs

Abstract not available at time of publication.

Kolja Knauer

TU-Berlin  
knauer@math.TU-Berlin.de

### MS9

#### Chains in Normalized Matching Posets

Given a poset  $P$  with  $n$  elements and a partition  $\mu$  of the integer  $n$ , can we find a partition of  $P$  into chains such that the sizes of the chains are given by the non-zero parts of  $\mu$ ? In the case when  $P$  is a normalized matching (aka LYM) poset, combining a number of 25 to 35 year old conjectures by Griggs, Füredi, and others, we conjecture that the answer is yes if and only if  $\mu$  is majorized (or dominated) by the partition of  $n$  coming from a nested chain decomposition of  $P$ . In this talk, we will survey the evidence for this conjecture.

Shahriar Shahriari

Pomona College  
SShahriari@pomona.edu

### MS10

#### Flows in Oriented Matroids

Extensions of flow number to matroids via definitions of the circular flow number of a graph,  $\phi_c(G)$ , are known to work for certain classes for example orientation classes of orientable matroids and  $\sqrt[k]{I}$ -matroids. Dually, we obtain notions of circular chromatic number for the same classes. We will mention some results on this topic and discuss the idea of *Eulerian matroid* thus obtained, as time allows.

Laura Chavez

UNIVERSIDAD AUTONOMA  
METROPOLITANA  
laurachav@gmail.com

Luis A. Goddyn

Department of Mathematics  
Simon Fraser University  
goddyn@math.sfu.ca

### MS10

#### Overview of Flows in Graphs and Matroids

As an approach to studying the chromatic number, Tutte introduced the notion of (group-valued) flows in graphs. In

the past 50 years, flows in graphs has been connected to a wide variety of graph problems involving cycles, spanning trees, orientations, embeddings, edge colorings and graph polynomials. More recently, the notion of flows has been extended to various classes of matroids, such as binary, ternary, orientable and sixth-root-of-unity matroids. I will give an overview of this subject, emphasizing what I consider to be the important questions, and touching on topics to be presented by the other speakers of this minisymposium.

Luis Goddyn

Simon Fraser University  
goddyn@sfu.ca

### MS10

#### Representations of Even Cut Matroids.

Even cut matroids are a minor closed class of matroids that are represented by grafts. A difficulty when dealing with even cut matroids arises from the fact that the same even cut matroid may be represented by grafts which are not related by simple operations. We define equivalence classes on the set of representations of even cut matroids and we study how equivalence classes behave under a single element extension or co-extension. The goal is to show that the set of representations of any even cut matroid can be covered by a constant number of equivalence classes.

Irene Pivotto, Bertrand Guenin

University of Waterloo  
ipivotto@math.uwaterloo.ca, bguenin@math.uwaterloo.ca

### MS10

#### Thin Trees, Nowhere-zero Flows, and the Asymmetric Traveling Salesman Problem

I will talk about recent results on designing approximation algorithms for the Asymmetric Traveling Salesman Problem (ATSP). Our approach is based on constructing a “thin” spanning tree from the solution of a classical linear programming relaxation of the problem and augmenting the tree to an Eulerian subgraph. I will talk about Goddyn’s conjecture on the existence of such trees and its relationship to nowhere-zero flows.

Amin Saberi

Management Science and Engineering  
Stanford University  
saberi@stanford.edu

### MS10

#### The Lattice of Integer Flows of a Regular Matroid

Abstract not available at time of publication.

David Wagner

University of Waterloo, Canada  
Dept. of Combin. and Opt.  
dgwagner@uwaterloo.ca

### MS11

#### A Solution to Havel’s Problem

In 70’s, Havel posed the following problem: does there exist a constant  $C$  such that every planar graph in that the distance between any two triangles is at least  $C$  is 3-colorable?

We answer this question in affirmative.

Zdenek Dvorak

Department of Applied Mathematics  
Charles University, Prague  
rakdver@kam.mff.cuni.cz

Daniel Kral

Charles University  
kral@kam.mff.cuni.cz

Robin Thomas

Georgia Tech  
thomas@math.gatech.edu

**MS11**

### Doubly Critical Graphs

A graph  $G$  is *doubly critical* if the removal of any pair of adjacent vertices yields a graph whose chromatic number is two less than that of  $G$ . A long-standing conjecture of Lovász states that the only connected doubly critical graphs are the complete graphs. We will give an overview of results and problems related to this conjecture and discuss some recent developments.

Tomas Kaiser

University of West Bohemia  
kaisert@kma.zcu.cz

**MS11**

### Maximum Number of $q$ -colorings

We consider an old problem of Linial and Wilf to determine the structure of graphs which allow the maximum number of  $q$ -colorings among graphs with  $n$  vertices and  $m$  edges. We prove that if  $n$  is large compared to  $q$ ,  $m = n^2/4$  and  $q$  is even, then the maximum is achieved by a complete balanced bipartite graph. This partially confirms a conjecture of Lazebnik. Our proof builds on methods of Loh, Pikhurko and Sudakov, which reduce the problem to a quadratic program.

Sergey Norin

Princeton University  
snorin@math.princeton.edu

**MS11**

### Fano Colorings of Cubic Graphs and a Conjecture of Fan and Raspaud

A Fano coloring is a coloring of the edges of a cubic graph by points of the Fano plane such that the colors around every vertex form a line. Macajova and the speaker proved (2005) that six lines suffice to color every bridgeless cubic graph, and conjectured that four lines would do. We show that this conjecture is true for cubic graphs with a 2-factor having exactly two odd circuits.

Martin Skoviera

Comenius University, Bratislava, Slovakia  
skoviera@dcs.fmph.uniba.sk

Edita Macajova

Department of Computer Science  
Comenius University, Bratislava, Slovakia  
macajova@dcs.fmph.uniba.sk

**MS11**

### Color-critical Graphs on Surfaces

A fundamental question in topological graph theory is as follows: Given a surface  $\Sigma$  and an integer  $t \geq 0$ , which graphs drawn in  $\Sigma$  are  $t$ -colorable? We say that a graph is  $(t+1)$ -critical if it is not  $t$ -colorable, but every proper subgraph is. In 1993, Carsten Thomassen showed that there are only finitely many six-critical graphs on a fixed surface with Euler genus  $g$ . In this talk, I will describe a new short proof of this fact. In addition, I will describe some structural lemmas that were useful to the proof and describe a list-coloring extension that is helpful to ongoing work that there are finitely many six-list-critical graphs on a fixed surface. This is a joint project with Ken-ichi Kawarabayashi of National Institute of Informatics, Tokyo.

Carl Yerger

School of Mathematics  
Georgia Institute of Technology  
cyerger@math.gatech.edu

**MS12**

### Packing Hamilton Cycles in Random and Pseudo-random Hypergraphs

We say that a  $k$ -uniform hypergraph  $C$  is a Hamilton cycle of type  $\ell$ , for some  $1 \leq \ell \leq k$ , if there exists a cyclic ordering of the vertices of  $C$  such that every edge consists of  $k$  consecutive vertices and for every pair of consecutive edges  $E_{i-1}, E_i$  in  $C$  (in the natural ordering of the edges) we have  $|E_{i-1} \cap E_i| = \ell$ . We prove that for  $\ell \leq k < 2\ell$ , with high probability almost all edges of a random  $k$ -uniform hypergraph  $H(n, p, k)$  with  $p(n) \gg \log^2 n/n$  can be decomposed into edge disjoint type  $\ell$  Hamilton cycles. We also provide sufficient conditions for decomposing almost all edges of a pseudo-random  $k$ -uniform hypergraph into type  $\ell$  Hamilton cycles, for  $\ell \leq k \leq 2\ell$ .

Michael Krivelevich

Tel Aviv University  
krivelev@post.tau.ac.il

Alan Frieze

Carnegie Mellon University  
alan@random.math.cmu.edu

**MS12**

### Peer-to-Peer Clustering Protocols

Suppose  $n$  individuals (clusters of size 1) need to rapidly coalesce into a single cluster, where the elementary operation is to merge two existing clusters. In this talk, we analyze a randomized peer-to-peer clustering algorithm proposed by the Distributed Systems group at Microsoft Research, which empirically appeared to achieve the asymptotically optimal running time. We provide the first rigorous bounds on the performance of this and a related algorithm, extending an approach of Oded Schramm.

Po-Shen Loh

Department of Mathematical Sciences  
Carnegie Mellon University  
ploh@cmu.edu

Eyal Lubetzky

Microsoft Research  
eyal@microsoft.com

MS12

**Spectra of Lifted Ramanujan Graphs**

Let  $G$  be a  $d$ -regular graph with all nontrivial eigenvalues at most  $\lambda$  in absolute value and let  $\rho$  be the spectral-radius of its universal cover. We show that with high probability the absolute value of every nontrivial eigenvalue of an  $n$ -lift of  $G$  is  $O(\max\{\lambda, \rho\} \log \rho)$ , translating to  $O(\sqrt{d} \log d)$  for a Ramanujan base-graph. This result is thus tight up to a logarithmic factor, and for  $\lambda \leq d^{2/3-\epsilon}$  it substantially improves the previously known upper bounds of Friedman (2003) and of Linial and Puder (2008). In particular, it implies that a typical  $n$ -lift of a Ramanujan graph is nearly Ramanujan.

Eyal Lubetzky  
Microsoft Research  
eyal@microsoft.com

Benny Sudakov  
Department of Mathematics  
UCLA  
bsudakov@math.ucla.edu

Van Vu  
Rutgers University  
vanvu@math.rutgers.edu

MS12

**Expansion, Relaxation and Approximation**

*Small set expansion* in a graph refers to the isoperimetric question of bounding the edge expansion when restricted to sets of vertices of small size, while *spectral profile* serves as its corresponding (functional) relaxation. Both notions have been of much interest lately, due to connections to mixing time in Markov chains and optimization over sparse subspaces. Further motivated by the computational complexity of the small set expansion problem, in joint work with D. Steurer and P. Raghavendra, we consider a semi-definite relaxation of the spectral profile and obtain logarithmic factor approximations for both the spectral profile and the isoperimetric profile. I hope to describe the relaxation and the basic ideas behind our approximation.

Prasad Tetali  
School of Mathematics, Georgia Institute of Technology  
Atlanta, Ga 30332-0160  
tetali@math.gatech.edu

MS12

**Graphs Which are Hard to Make Triangle-free Can be Packed with Many Triangles**

It is known that a graph can be made triangle-free by removing (slightly less than) half its edges. We call a graph triangle-bound if (essentially) half of its edges must be deleted in order to make it triangle-free. It is conjectured that the edges of a dense triangle-bound graph can be packed with (almost)  $|E(G)|/3$  triangles. It is known that there is always a packing of size (almost)  $|E(G)|/4$ . We prove a result in this direction. For any  $\beta > 0$  there is  $\alpha > 0$ , so that if  $G$  is a regular triangle-bound graph with degree  $\beta n$  then it has  $|E(G)|(1/4 + \alpha)$  edge-disjoint triangles.

Raphael Yuster  
Department of Mathematics  
University of Haifa

raphy@math.haifa.ac.il

MS13

**The Hopf Monoid of Generalized Permutahedra**

Joyal's notion of species constitutes a good framework for the study of certain algebraic structures associated to combinatorial objects. We discuss the notion of "Hopf monoid" in the category of species and illustrate it with several examples. We introduce the Hopf monoid of generalized permutahedra (the latter are certain polytopes recently studied by Postnikov, Reiner, and Williams, among others). Our main result is an explicit antipode formula for this Hopf monoid. We explain how reciprocity theorems of Stanley on graphs and of Billera, Jia and Reiner on matroids can be deduced from this result.

Federico Ardila  
San Francisco State University  
federico@math.sfsu.edu

Marcelo Aguiar  
Texas A&M  
maguiar@math.tamu.edu

MS13

**On Noncommutative Combinatorial Inverse System**

We introduce a notion of a combinatorial inverse system in non-commutative variables. We present two important examples, some conjectures and results. These conjectures and results were suggested and supported by computer investigations.

Nantel Bergeron  
York University, Canada  
bergeron@mathstat.yorku.ca

MS13

**Valuative Invariants for Polymatroids**

Many important invariants of matroids and polymatroids, such as the Tutte polynomial, the Billera-Jia-Reiner quasi-symmetric function, and Derksen's invariant  $\mathcal{G}$ , are valuative. We construct the  $\mathbf{Z}$ -modules of all  $\mathbf{Z}$ -valued valuative functions and of valuative invariants for matroids and polymatroids on a fixed ground set. We give Hopf algebra structures for these objects and explicit formulas for their ranks.

Alex Fink  
University of California at Berkeley  
finka@math.berkeley.edu

Harm Derksen  
University of Michigan  
hderksen@umich.edu

MS13

**Multigraded Combinatorial Hopf Algebras and Refinements of Odd and Even Subalgebras**

We develop a theory of multigraded combinatorial Hopf algebras modeled on the theory of graded combinatorial Hopf algebras developed by Aguiar, Bergeron, and Sottile [Compos. Math. 142 (2006), 1–30]. In particular we in-

roduce the notion of canonical  $\mathbf{k}$ -odd and  $\mathbf{k}$ -even subalgebras associated with any multigraded combinatorial Hopf algebra, extending simultaneously the work of Aguiar et al. and Ehrenborg. Among our results are specific categorical results for higher level quasisymmetric functions, several basis change formulas, and a generalization of the descents-to-peaks map.

Samuel K. Hsiao  
Bard College  
hsiao@bard.edu

Gizem Karaali  
Pomona College  
gizem.karaali@pomona.edu

### MS13

#### Noncrossing Partition Hopf Algebra

We introduce and study the incidence Hopf algebra  $H$  of the family of noncrossing partition lattices of finite linearly ordered sets. After presenting some basic combinatorial results on noncrossing partitions, we give two formulas for the antipode of  $H$ . The first of these is given as an alternating sum indexed by certain dissections of polygons and is cancellation-free. The second gives values on the canonical set of generators in terms of a second set of generators, related to the first by Lagrange inversion; this formula is not only cancellation-free but is in fact sign-free. Finally, we use an edge labeling of noncrossing partition lattices, defined by Richard Stanley, to construct an isomorphism between  $H$  and the Hopf algebra of symmetric functions and an embedding of  $H$  in the Hopf algebra of parking functions. These maps have sign-free expressions in terms of the canonical bases of these Hopf algebras.

William Schmitt  
George Washington University  
wschmitt@gwu.edu

Hillary Einziger  
The George Washington University  
hillaryre@gmail.com

### MS14

#### Pairs of Elements in Unavoidable Minors of 3-connected Binary Matroids

Ding, Oporowski, Oxley, and Vertigan proved that every sufficiently large 3-connected, binary matroid has a large minor isomorphic to a binary spike,  $M(K_{3,n})$ ,  $M^*(K_{3,n})$ , or the  $n$ -spoke wheel. We will show that any element of a sufficiently large, 3-connected, binary matroid,  $M$ , is contained in one of six large minors related to these four, and we discuss the corresponding result for a pair of elements of  $M$ .

Deborah Chun  
Louisiana State University  
dchun@math.lsu.edu

### MS14

#### The Excluded Minors for the Class of Matroids that are Binary or Ternary

Suppose that  $\mathcal{M}$  and  $\mathcal{N}$  are minor-closed classes of matroids. Then  $\mathcal{M} \cup \mathcal{N}$  is also minor-closed. If we know the excluded minors for  $\mathcal{M}$  and  $\mathcal{N}$ , we would naturally want to

characterize the excluded minors for  $\mathcal{M} \cup \mathcal{N}$ . This is often a very difficult problem. Vertigan showed that  $\mathcal{M} \cup \mathcal{N}$  need not have a finite number of excluded minors, even in the case that  $\mathcal{M}$  and  $\mathcal{N}$  themselves have only a single excluded minor each. However, it may be the case that if  $\mathcal{M}$  and  $\mathcal{N}$  are, respectively, the classes of matroids representable over two finite fields, then  $\mathcal{M} \cup \mathcal{N}$  always has finitely many excluded minors. We show that this is true in the case that  $\mathcal{M}$  and  $\mathcal{N}$  are respectively the classes of GF(2)- and GF(3)-representable matroids.

Dillon Mayhew  
Victoria University of Wellington  
New Zealand  
dillon.mayhew@msor.vuw.ac.nz

Bogdan Oporowski, James Oxley  
Louisiana State University  
bogdan@math.lsu.edu, oxley@math.lsu.edu

Geoff Whittle  
Victoria University of Wellington  
New Zealand  
geoff.whittle@msor.vuw.ac.nz

### MS14

#### On the Number of Points in a Matroid with No $n$ -point Line as a Minor

We give a tight upper bound for the number of elements in a simple rank- $r$  matroid with no  $n$ -point line as a minor when  $r$  is sufficiently large relative to  $n$ , and show that the matroids attaining the bound are projective geometries. This result settles a conjecture made by Joseph Kung in 1993.

Peter Nelson  
University of Waterloo  
Canada  
apnelson@math.uwaterloo.ca

Jim Geelen  
University of Waterloo  
jfggeelen@math.uwaterloo.ca

### MS14

#### Some Minor-minimal Unbalanced Matroids

Balanced matroids were introduced by Feder and Mihail in 1992, in connection with their study of random walks on basis-exchange graphs of matroids. We present a family which contains most known examples of minor-minimal unbalanced matroids, and determine exactly which matroids in this family are minor-minimally unbalanced. We conclude with a summary of what (little) is known about the set of excluded minors for balanced matroids.

David Wagner  
University of Waterloo  
Canada  
dgwagner@math.uwaterloo.ca

### MS14

#### Stability, Fragility, and Rota's Conjecture

Rota conjectured that, for each finite field  $F$ , the class of  $F$ -representable matroids has finitely many excluded minors. A matroid is  $N$ -fragile if for each element either the

deletion or the contraction has no  $N$ -minor. The *Bounded Canopy Conjecture* is that all  $\text{GF}(q)$ -representable  $N$ -fragile matroids have bounded branch width. We prove a result which implies that for  $\text{GF}(5)$  Rota's Conjecture reduces to the Bounded Canopy Conjecture.

Stefan van Zwam

Centrum Wiskunde & Informatica, The Netherlands  
University of Waterloo, Canada  
s.h.m.van.zwam@cw.nl

Dillon Mayhew, Geoff Whittle  
Victoria University of Wellington  
New Zealand  
dillon.mayhew@msor.vuw.ac.nz, geoff.whittle@vuw.ac.nz

**MS15**

**Hamiltonicity and Traceability of Strong  $k$ -traceable Oriented Graphs**

In this talk the hamiltonicity and traceability of strong  $k$ -traceable oriented graphs are considered. Also, the strong component structure of  $k$ -traceable oriented graphs that are not traceable is investigated.

Jean Dunbar

Converse College  
jean.dunbar@converse.edu

**MS15**

**Further Progress Towards the Traceability Conjecture**

We show that for every integer  $k \geq 2$  there exists an integer  $t(k)$  ( $< 2k^2$ ) such that every  $k$ -traceable oriented graph of order at least  $t(k)$  is traceable. Furthermore, we establish an upper bound on the order of  $k$ -traceable oriented graphs with girth  $g > 3$  and apply our findings to prove that the PPC holds for 1-deficient oriented graphs with girth at least 6.

Marietjie Frick

University of South Africa  
marietjie.frick@gmail.com

**MS15**

**Independent Sets Which Meet All Longest Paths**

The case  $a = 1$  of the PPC may be stated as "Every digraph contains an independent set which meets every longest path". In this talk I survey sufficient conditions for a digraph to have an independent set which meets all longest paths.

Hortensia Galeana-Sanchez

Universidad Nacional Autonoma de Mexico  
hgaleana@matem.unam.mx

**MS15**

**An Introduction to the Path Partition Conjecture and Traceability Conjecture**

A digraph  $D$  of order  $n \geq k \geq 2$  is  $k$ -traceable if every  $k$  vertices of  $D$  induce a traceable subdigraph. The Traceability Conjecture (TC) states that every  $k$ -traceable oriented graph of order at least  $2k - 1$  is traceable.  $D$  is 1-deficient if  $\lambda(D) = n - 1$ . In this talk we introduce background material on the PPC and the TC and establish a

connection between the TC and the PPC for 1-deficient oriented graphs.

Ortrud R. Oellermann

The University of Winnipeg  
o.oellermann@uwinnipeg.ca

**MS15**

**Hypotraceable Oriented Graphs and the Traceability Conjecture**

A digraph  $D$  is hypotraceable if  $D$  is not traceable but  $D - v$  is traceable for every  $v \in V(D)$ . Every nontraceable  $k$ -traceable oriented graph of order  $n$  contains an induced hypotraceable oriented graph of order  $h$  for some  $k < h \leq n$ . We investigate the order and structure of hypotraceable oriented graphs and show that for  $2 \leq k \leq 6$  every  $k$ -traceable oriented graph is traceable. This implies that the PPC holds for 1-deficient oriented graphs of order at most 12.

Susan van Aardt

University of South Africa  
vaardsa@unisa.ac.za

**MS16**

**Introduction to Linear Series on Tropical Curves**

We will give an overview of Riemann-Roch and Abel-Jacobi theory on metric graphs and tropical curves.

Matthew Baker

Georgia Institute of Technology  
mbaker@math.gatech.edu

**MS16**

**A Tropical Proof of the Brill-Noether Theorem**

We produce Brill-Noether general graphs in every genus, confirming a conjecture of Baker and giving a new proof of the Brill-Noether Theorem, due to Griffiths and Harris. Our proof provides an explicit criterion for a curve to be Brill-Noether general over discretely valued fields of arbitrary characteristic.

Filip Cools

K.U. Leuven  
filip.cools@wis.kuleuven.be

Jan Draisma

T.U.Eindhoven, The Netherlands  
j.draisma@tue.nl

Sam Payne, Elina Robeva

Stanford  
spayne@stanford.edu, erobeva@stanford.edu

**MS16**

**Rank-determining Sets of Tropical Curves**

The rank of a divisor is a concept appearing in the Riemann-Roch theorem for finite graphs or tropical curves. A "rank-determining set" of a tropical curve  $\Gamma$  is defined to be a subset  $A$  of  $\Gamma$  such that the rank of a divisor  $D$  on  $\Gamma$  is always equal to the rank of  $D$  restricted on  $A$ . I will present a criterion for rank-determining sets and show constructively that there exist finite rank-determining sets. We can compute the rank of an arbitrary divisor on any

tropical curve based on finite rank-determining sets. In addition, I will discuss general properties and further applications of rank-determining sets.

Ye Luo

Georgia Institute of Technology  
yl92@math.gatech.edu

**MS16**

### Linear Systems on Tropical Curves

A tropical curve is a metric graph with possibly unbounded edges, and tropical rational functions are continuous piecewise linear functions with integer slopes. We define the complete linear system  $|D|$  of a divisor  $D$  on a tropical curve analogously to the classical counterpart. Due to work of Baker and Norine, there is a rank function  $r(D)$  on such linear systems, as well a canonical divisor  $K$ . Completely analogous to the classical case, this rank function satisfies Riemann-Roch and analogues of Riemann-Hurwitz. This talk will describe joint work with Josephine Yu and Christian Haase investigating the structure of  $|D|$  as a cell complex. Among other results, we show that linear systems are quotients of tropical modules, finitely generated by vertices of the cell complex.

Gregg Musiker

MIT  
musiker@math.mit.edu

**MS16**

### Projective Embeddings of Tropical Curves

We study embeddings of a tropical curve into a projective space given by a finite set of tropical rational functions in a linear system of a divisor  $D$ . The image can be modified to a tropical curve of degree equal to  $\deg(D)$ . The tropical convex hull of the image realizes the linear system as a polyhedral complex. We also show that curves for which the canonical divisor is not very ample are hyperelliptic.

Josephine Yu

Georgia Institute of Technology  
josephine.yu@gmail.com

**MS17**

### Nowhere-Harmonic Colorings of Graphs

By studying the vertex Laplacian of a graph, we introduce nowhere-harmonic colorings along with analogues of the chromatic polynomial and Stanley's theorem relating negative evaluations of the chromatic polynomial to acyclic orientations. Our primary tool for these investigations is the theory of "inside-out polytopes," developed by M. Beck and T. Zaslavsky, and the theory of Ehrhart quasi-polynomials for rational polytopes.

Benjamin Braun

University of Kentucky  
braun@ms.uky.edu

Matthias Beck

San Francisco State University  
beck@math.sfsu.edu

**MS17**

### Ehrhart Polynomials of Non-integral Polytopes

An active area of research is the characterization of the Ehrhart polynomials of polytopes. However, due to the phenomenon of quasi-period collapse, not all Ehrhart polynomials come from *integral* polytopes. Results regarding integral polytopes do not always generalize to arbitrary Ehrhart polynomials. We discuss recent progress in this more general context. In particular, we consider non-integral analogues of reflexive polytopes and the generalization of Scott's bound (Scott 1978) to the non-integral case.

Tyrrell B. McAllister

University of Wyoming  
tmcallis@uwyo.edu

**MS17**

### Kneser's Theorem and Inequalities in Ehrhart Theory

We demonstrate how additive number theory can be used to produce new classes of inequalities in Ehrhart theory. More specifically, we use a classical result of Kneser to produce new inequalities between the coefficients of the Ehrhart  $h^*$ -vector of a lattice polytope. As an application, we deduce all possible 'balanced' inequalities between the coefficients of the Ehrhart  $h^*$ -vector of a lattice polytope containing an interior lattice point, in dimension at most 6.

Alan Stapledon

University of British Columbia  
astaplnd@gmail.com

**MS17**

### Counting With Rational Generating Functions

Consider the Ehrhart quasi-polynomial,  $f(t)$ , of a rational polytope,  $P$  (that is,  $f(t)$  counts the number of integer points in  $tP$ ). One often considers the (algebraically-manipulable) generating function  $\sum_{t \geq 0} f(t)x^t$ . Alternatively,  $f(t)$  can be written as a "step-polynomial" (an explicit function built from floor functions). Fortunately, we do not have to choose between nimble generating functions and concrete step-polynomials for this and other counting problems, as one can convert back and forth between them in polynomial time (in fixed dimension). This is joint work with Sven Verdoolaege.

Kevin Woods

Oberlin College  
kevin.woods@oberlin.edu

**MS18**

### Some Highly Arc Transitive Digraphs

Highly arc transitive digraphs are digraphs with the property that any two directed walks of equal length are equivalent under the automorphism group. These graphs were introduced by Cameron Praeger and Wormald, and have proved to be interesting objects both in combinatorics and group theory. Here we construct some highly arc transitive digraphs which resolve two conjectures from this original paper. This is joint work with Bojan Mohar and Robert

Samal.

Matthew Devos, Bojan Mohar  
Simon Fraser University  
mdevos@sfu.ca, mohar@sfu.ca

Robert Samal  
Simon Fraser University, Burnaby  
rsamal@sfu.ca

### MS18

#### Graphs with No Octahedron Minor

We characterize graphs that don't contain the Octahedron as a minor. Related problems will also be discussed.

Guoli Ding  
Louisiana State University  
ding@math.lsu.edu

### MS18

#### The $K$ Edge-Disjoint Paths Problem in Digraphs with Bounded Independence Number

In 1980, Fortune, Hopcroft, and Wyllie showed that the following algorithmic problem ( $k$ -EDP) is NP-complete with  $k = 2$ :

$k$  Edge-Disjoint Paths ( $k$ -EDP)

Instance: A digraph  $G$ , and  $k$  pairs  $(s_1, t_1), \dots, (s_k, t_k)$  of vertices of  $G$ .

Question: Do there exist directed paths  $P_1, \dots, P_k$  of  $G$ , mutually edge-disjoint, such that  $P_i$  is from  $s_i$  to  $t_i$  for  $i = 1, \dots, k$ ?

In this talk we will present a polynomial time algorithm to solve  $k$ -EDP for fixed  $k$  in digraphs with bounded independence number. This is joint work with Paul Seymour.

Alexandra Fradkin  
Princeton  
aovetsky@princeton.edu

Paul Seymour  
Princeton University  
pds@math.princeton.edu

### MS18

#### Pairs of Signed Graphs with the Same Even Cycles

We investigate the following question: Given two signed graphs with the same set of even cycles what is the relation between these signed graphs? We provide two answers to this question. We first show that any pair of such signed graphs are related by a sequence of operations that preserve even cycles at each step. We also show that for any family of edge sets  $S$ : we can partition, the set of all signed graphs whose even cycles are exactly  $S$ , into a constant number of equivalence classes. Moreover, the relation between any two signed graphs in the same equivalence class is much simpler than the relation between an arbitrary pair of signed graphs which have the same even cycles.

Bertrand Guenin, Irene Pivotto  
University of Waterloo

bguenin@math.uwaterloo.ca, ipivotto@math.uwaterloo.ca

Paul Wollan  
Department of Mathematics  
University of Hamburg  
paul.wollan@gmail.com

### MS18

#### On a Quadratic Upper Bound for the Number of Vertices in 6-Critical Graph

Thomassen proved that there are only finitely many 6-critical graphs embeddable in any fixed surface. We prove that if a graph  $G$  is a  $C$ -6-critical plane graph with outer cycle  $C$ , then  $G$  has at most  $O(|C|^2)$  vertices. We then use discharging to show that there only  $O(g^2)$  vertices in a 6-critical graph which embeds on a surface of genus  $g$ .

Luke Postle  
Georgia Tech  
ljpostle@math.gatech.edu

Zdenek Dvorak  
Department of Applied Mathematics  
Charles University, Prague  
rakdver@kam.mff.cuni.cz

### MS20

#### Okounkov Bodies and Toric Degenerations of Schubert Varieties

We show how the Okounkov body construction developed recently by Kaveh-Khovanskii and Lazarsfeld-Mustața gives rise to flat degenerations of a variety to a toric variety, and apply this perspective to construct toric degenerations of Schubert varieties which are compatible with degenerations of their Bott-Samelson resolutions.

Dave Anderson  
University of Washington  
dandersn@math.washington.edu

### MS20

#### Affine Schubert Positivity, Total Positivity, and Geometric Satake

Let  $G$  be a complex simple simply-connected algebraic group. A theorem proved independently by Ginzburg and Peterson states that the homology  $H_*(Gr_G)$  of the affine Grassmannian of  $G$  is isomorphic to the ring of functions on the centralizer  $X$  of a principal nilpotent in the Langlands dual  $G^V$ . There is a notion of total positivity on  $X$ , using Lusztig's general definitions, and there is also a notion of Schubert positivity, using Schubert classes of  $Gr_G$ . We connect the two notions using the geometric Satake correspondence. In addition, we give an explicit parametrization of the positive points of  $X$ . This is joint work with Konstanze Rietsch, generalizing work of hers in type  $A$ .

Thomas Lam  
University of Michigan  
tfylam@umich.edu

### MS20

#### A Chevalley Formula in the Equivariant $K$ -Theory

### of Kac-Moody Flag Varieties

In previous work with Postnikov, we gave a Chevalley formula in the equivariant  $K$ -theory of finite type flag varieties in terms of our alcove model. I will present a generalization of this formula to Kac-Moody flag varieties using an extension of the alcove model; there is another version, based on Lakshmibai-Seshadri paths, similar to the one of Pittie-Ram. For the type  $A$  affine Grassmannian, the formula can be phrased using the Misra-Miwa partition model.

Cristian Lenart

State University of New York at Albany  
lenart@albany.edu

Mark Shimozono

Virginia Polytechnic Institute and State University  
mshimo@vt.edu

### MS20

#### Double Schubert Polynomials for Classical Groups

A classical problem in Schubert Calculus is to find polynomial representatives for Schubert classes in the flag manifolds. In type  $A$ , Lascoux and Schutzenberger's Schubert polynomials are canonical such representatives. The situation is more subtle in the other classical types. Using Schur's  $P$  and  $Q$  functions, Billey and Haiman constructed a canonical family of polynomials, which are solutions of certain divided-difference equations. In joint work with T. Ikeda and H. Naruse we use localization techniques, and the factorial  $P$  and  $Q$ -Schur functions of Ivanov, to extend Billey and Haiman's construction to equivariant cohomology. The resulting polynomials possess quite pleasant combinatorial properties: stability, positivity, symmetry.

Leonardo C. Mihalcea

Baylor University  
Leonardo\_Mihalcea@baylor.edu

Takeshi Ikeda, Hiroshi Naruse

Okayama University  
ike@xmath.ous.ac.jp, rdcv1654@cc.okayama-u.ac.jp

### MS20

#### Circle Actions and Equivariant Cohomology of Peterson Varieties

A well-known algorithm called GKM theory computes equivariant cohomology for a large class of varieties with torus actions from purely combinatorial data. GKM theory works perfectly for generalized flag varieties, so it has been used widely in modern Schubert calculus. We generalize this theory, showing that a kind of GKM theory holds for a broader class of varieties having one-dimensional torus actions. As an application, we give a simple, combinatorial computation of equivariant and ordinary Schubert calculus of Peterson varieties.

Julianna Tymoczko

University of Iowa  
tymoczko@divms.uiowa.edu

### MS21

#### Viewing Market Price Discovery as an Algorithmic Process

Self-organizing behavior can often be viewed as arising

from a distributed process. It is natural to ask when and why it occurs. Our thesis is that an algorithmic perspective may be helpful. One instance of such a distributed process is pricing in markets. A basic tenet of well-functioning markets is that they discover (converge toward) prices that simultaneously balance supplies and demands of all goods; these are called equilibrium prices. Further, the markets are self-stabilizing, meaning that they converge toward new equilibria as conditions change. This talk will seek to explain why this could happen. More specifically, we describe the setting of Ongoing Markets (by contrast with the classic Fisher and Exchange markets). An Ongoing Market allows trade at non-equilibrium prices, and, as its name suggests, continues over time. The main task remaining is to specify and analyze a suitable price update rule. We consider a (tatonnement-style) rule with the following characteristics: 1. There is a separate instance of the (price update) procedure for each good. 2. The procedure is distributed: (i) the instances are independent, and (ii) each instance uses limited "local" information. 3. It is simple. 4. It is asynchronous: price updates do not have to be simultaneous. And for appropriate markets the rule enables: 5. Fast convergence. 6. Robustness in the face of (somewhat) inaccurate data. This talk is based on joint works with Lisa Fleischer and Ashish Rastogi.

Richard Cole

NYU  
cole@cs.nyu.edu

### MS21

#### New Convex Programs and Distributed Algorithms for Fisher Markets with Linear and Spending Constraint Utilities

We shed new light on convex programs and distributed algorithms for Fisher markets with linear and spending constraint utilities. We give a new convex program for the linear utilities case of Fisher markets. This program easily extends to the case of spending constraint utilities as well, thus resolving an open question raised by Vazirani. We show that the gradient descent algorithm with respect to a Bregman divergence converges with rate  $O(1/t)$  under a condition that is weaker than having Lipschitz continuous gradient (which is the usual assumption in the optimization literature for obtaining the same rate). We show that the Proportional Response dynamics recently introduced by Zhang is equivalent to a gradient descent algorithm for solving the new convex program. This insight also gives us better convergence rates, and helps us generalize it to spending constraint utilities.

Nikhil R. Devanur

Microsoft Research, Redmond  
nikdev@microsoft.com

### MS21

#### Coordination Mechanisms for Selfish Scheduling and Routing

We investigate the influence of different algorithmic choices on the approximation ratio in selfish scheduling, and survey the recently developed results about coordination mechanisms for machine scheduling. Our goal is to design local policies that minimize the inefficiency of resulting equilibria. In particular, we present the first coordination mechanisms achieving a logarithmic bound for makespan over unrelated machines, and show strong lower bounds for a

class of coordination mechanisms.

Vahab Mirrokni  
Google Inc.  
mirrokni@google.com

#### MS21

##### **On Scheduling Mechanisms: Theory, Practice and Pricing**

Markets of computing resources typically consist of a Cloud and Jobs that arrive over time and request computing resources in exchange for payment. A Mechanism in this context consists of a scheduling algorithm and a pricing policy. We start with a theoretical impossibility result showing the Inapproximability of Randomized Dominant-Strategy Mechanisms for a natural scheduling problem on Unrelated-Machines (formulated as a mechanism design problem in the seminal paper of Nisan and Ronen). We then briefly model the dynamic interaction between the provider and the consumers as a simple one-shot game. Using a simulation-based approach we show that market stability in the form of symmetric Nash-Equilibrium is likely to be achieved (under the assumption of small risk aggressive group of users). If time permits, we shall briefly discuss Individually-Fair scheduling mechanisms. Joint work with: Lior Amar, Amnon Barak, Michael Schapira and Sergei Shudler.

Ahuva Mu'alem  
California Institute of Technology  
ahumu@yahoo.com

#### MS21

##### **Aggregation and Manipulation in Prediction Markets**

Prediction markets are markets established primarily to aggregate distributed information and forecast future events. In this talk, I will discuss recent results on strategic behavior and information aggregation in prediction markets, focusing on the market scoring rule (MSR) prediction markets introduced by Hanson (2003). MSR markets have an attractive incentive property: a trader trading only once optimizes her expected profit by trading honestly. We analyze strategic behavior in two realistic situations: when traders may trade multiple times, and when traders have a future payoff that creates a conflict of interest. In these situations, we characterize information distributions in which honest trading is optimal, as well as distributions in which there is bluffing or withholding of information in equilibrium. In the latter case, I also present a simple modification of the MSR that improves the rate of information aggregation in equilibrium.

Rahul Sami  
UMich  
rsami@umich.edu

#### MS22

##### **Distributive Lattices of Two-Rowed Standard Young Tableaux, with Applications to Symmetric Function Inequalities**

In joint work with A. Cuttler and M. Skandera, the speaker has investigated partial orders defined on various families of symmetric functions, defined by the relation  $f \leq g$  iff  $f(X) \leq g(X)$  for every substitution of nonnegative variables  $X$ . This partial order includes many classical inequalities (e.g.

AGM, Muirhead) as well as many new ones. In subsequent work (some of it joint with J. Lima) the speaker has investigated a stronger attribute, called Y-positivity, from which the original inequalities follow trivially. In one case, the arguments involve an interesting lattice defined on Standard Young Tableaux. When the tableaux have two rows, the lattice is distributive, and a key step in the proof of Y-positivity is based on the FKG inequality. We will describe these lattices and discuss some possible extensions of this work.

Curtis Greene  
Haverford College  
cgreene@haverford.edu

#### MS22

##### **Diamond-free Posets**

Abstract not available at time of publication.

Jerrold R. Griggs  
Univ. of S. Carolina, Columbia  
griggs@math.sc.edu

#### MS22

##### **Acyclic Sets in k-majority Tournaments**

Given a set  $S$  of linear orders of a ground set  $X$ , the *majority digraph* of  $S$  is the directed graph on  $X$  where there is an edge from  $u$  to  $v$  when a majority of the orders in  $S$  rank  $u$  above  $v$ . For odd  $k$ , a *k-majority tournament* is a tournament that arises as the majority digraph of a set of  $k$  orders. When the orders in  $S$  are interpreted as a ranking of preferences among a set of alternatives  $X$ , acyclic sets in the majority tournament can be viewed as a consensus ranking of a subset of  $X$ . We study the maximum size of an acyclic set of vertices in  $k$ -majority tournaments. Every  $n$ -vertex 3-majority tournament contains an acyclic set of size  $n^{1/2}$ ; we present a family of 3-majority tournaments which have no acyclic sets of size larger than  $2n^{1/2}$ . We show that every  $n$ -vertex 5-majority tournament contains an acyclic set of size  $n^{1/4}$ . For general  $k$ , every  $k$ -majority tournament contains an acyclic set of size  $n^{f(k)}$ , where  $f(k) = 3^{-(k-1)/2}$ . On the other hand, there are  $k$ -majority tournaments in which every acyclic set has size at most  $n^{g(k)}$ , where  $g(k) = O(\log \log k / \log k)$ . This is joint work with Dan Schreiber and Douglas B. West.

Kevin Milans  
University of Illinois  
Department of Mathematics  
milans@math.uiuc.edu

Daniel Schreiber  
University of Illinois at Urbana-Champaign  
Department of Computer Science  
dschrei2@uiuc.edu

Douglas B. West  
University of Illinois at Urbana-Champaign  
Department of Mathematics  
west@math.uiuc.edu

#### MS22

##### **On Maximal F-free Subsets of Finite Posets**

Say that a finite poset  $F$  has the *maximal antichain prop-*

erty (MAP) if every maximal  $F$ -free subset of every finite poset  $P$  contains a maximal antichain of  $P$ . Analogously we define the *maximal chain property* (MCP) and the *maximal element property* (MEP). We characterize all posets with the MAP, the MCP, and the MEP. We also give an application of our characterization of the MEP to a colouring problem for posets. Some of this work is joint with Jia Shen.

Bill Sands

University of Calgary  
sands@ucalgary.ca

## MS22

### Submodular Functions and Packing of Steiner Trees in Graphs

Nash-Williams proved that every  $2k$ -edge-connected graph has  $k$  edge-disjoint spanning trees. Kriesell conjectured more generally that if a set  $S$  in  $V(G)$  is  $2k$ -edge-connected in  $G$ , then  $G$  has  $k$  edge-disjoint trees that contain  $S$  (a set is  $j$ -edge-connected in  $G$  if it cannot be separated by deleting fewer than  $j$  edges from  $G$ ). Lap-Chi Lau proved that it suffices for  $S$  to be  $24k$ -edge-connected in  $G$ . We prove that it suffices for  $S$  to be  $6.5k$ -edge-connected in  $G$ . Our result follows from a common generalization of the Tree Packing Theorem and Hakimi's criterion for orientations with specified outdegrees. We prove the general theorem using submodular functions and the Matroid Union Theorem.

Hehui Wu

University of Illinois at Urbana-Champaign  
hehuiwu2@illinois.edu

Douglas B. West

University of Illinois, Urbana  
Department of Mathematics  
west@math.uiuc.edu

## MS23

### On Rogers's Proof of Identifiability for the GTR + Gamma + I Model in Phylogenetics

In 2001, Rogers's gave a proof of identifiability for the popular general time reversible (GTR) Markov model with Gamma distributed rates mixed with invariable (I) sites for DNA evolution along a phylogeny. Recently, Allman, Ane, and Rhodes have pointed out an error in Rogers's proof and provided a proof using three-way species comparisons to show that the model without invariable sites is identifiable. We will discuss Rogers's approach and provide the proof of the missing link for the model with invariable sites using only pairwise species comparisons. There are a few exceptional cases that our method cannot handle, mainly the Jukes-Cantor model for DNA evolution. We will discuss what is known about that situation to date.

Elizabeth Housworth

Indiana University  
Department of Mathematics  
ehouswor@indiana.edu

## MS23

### Generalizing the Four Gamete Condition and Splits Equivalence Theorem: Local Conditions For Per-

### fect Phylogeny

For binary input, the classical four gamete condition/splits equivalence theorem gives a concise necessary and sufficient condition for the existence of a perfect phylogeny and is the building block for many theoretical results and practical algorithms. In this talk, we discuss recent work to obtain local conditions that generalize the four gamete condition to multiple-state perfect phylogeny. In the special case of three-state perfect phylogeny, we further show that there is a set of four subpatterns such that any three-state characters not allowing a perfect phylogeny must contain one of these four subpatterns. This generalizes the four gamete condition which states that a pair of characters not allowing a binary perfect phylogeny must contain the subpattern of all four binary combinations. (Joint work with Dan Gusfield and Srinath Sridhar)

Fumei Lam

Computer Science  
UC Davis  
flam@cs.ucdavis.edu

## MS23

### Icosahedral Viral Assembly Pathway Enumeration Using Tree Orbits Under Permutation Group Action

We use combinatorics and group theory to answer questions about the assembly of icosahedral viral shells, or any symmetric macromolecular assembly. Although the final geometric structure of such assemblies is fairly well understood in terms of the constituent subunits, the assembly pathways are not. Specifically, it is of interest to obtain a probability distribution over valid assembly pathways. In this talk, the capsid is modeled by a polyhedron whose facets represent the monomers. The assembly is modeled by a rooted tree, the leaves representing the facets of the polyhedron, the root representing the assembled polyhedron, and the internal vertices representing intermediate stages of assembly (subsets of facets). The probability of an assembly pathway is influenced by the size of the orbit of a representative assembly tree under the icosahedral group action. Besides its virological motivation, the enumeration of orbits of trees under the action of a finite group is of independent mathematical interest. If  $G$  is a finite group acting on a finite set  $X$ , then there is a natural induced action of  $G$  on the set  $\mathcal{T}_X$  of trees whose leaves are bijectively labeled by the elements of  $X$ . If  $G$  acts simply on  $X$ , then  $|X| := |\mathcal{X}_n| = n \cdot |G|$ , where  $n$  is the number of  $G$ -orbits in  $X$ . The basic combinatorial results in this paper are (1) a formula for the number of orbits of each size in the action of  $G$  on  $\mathcal{T}_{X_n}$ , for every  $n$ , and (2) a simple algorithm to find the stabilizer of a tree  $\tau \in \mathcal{T}_X$  in  $G$  that runs in linear time and does not need memory in addition to its input tree. Joint work with Miklos Bóna and Andy Vince

Meera Sitharam

Computer and Information Science  
University of Florida  
sitharam@cise.ufl.edu

## MS23

### The Shape of Phylogenetic Treespace

Distances between trees are often NP-hard to compute. In this talk, we discuss recent work on efficiently approximating distances between trees. We focus on biological inspired ways to compare trees (SPR, TBR, geodesic dis-

tance in treespace, and tanglegrams), as well as rotation distance for ordered trees.

Katherine St. John

Computer Science  
City University of New York  
stjohn@lehman.cuny.edu

### MS23

#### Bayes Estimators for Phylogenetic Reconstruction

Tree reconstruction methods are often judged by their accuracy, measured by how close they get to the true tree. Yet most reconstruction methods like ML do not explicitly maximize this accuracy. To address this problem, we propose a Bayesian solution. Given tree samples, we propose finding the tree estimate which is closest on average to the samples. This “median” tree is known as the Bayes estimator (BE). The BE literally maximizes posterior expected accuracy, measured in terms of closeness (distance) to the true tree. We discuss a unified framework of BE trees, focusing especially on tree distances which are expressible as squared euclidean distances. Notable examples include Robinson–Foulds distance, quartet distance, and squared path difference. Using simulated data, we show Bayes estimators can be efficiently computed in practice by hill climbing. We also show that Bayes estimators achieve higher accuracy, compared to maximum likelihood and neighbor joining.

Ruriko Yoshida

Statistics  
University of Kentucky  
ruriko@polytopes.net

### MS24

#### The Cycle Switching Graph of the Steiner Triple Systems of Order 19 is Connected

Switching is a local transformation that when applied to a combinatorial object gives another object with the same parameters. It is here shown that the cycle switching graph of the 11,084,874,829 isomorphism classes of Steiner triple systems of order 19 as well as the cycle switching graph of the 1,348,410,350,618,155,344,199,680,000 labeled such designs are connected.

Petteri Kaski, Veli Mäkinen

Helsinki Institute for Information Technology HIIT  
CS Dept., University of Helsinki  
petteri.kaski@cs.helsinki.fi, vmakinen@cs.helsinki.fi

Patric R. J. stergård

Department of Communications and Networking  
Helsinki University of Technology TKK  
patric.ostergard@tkk.fi

### MS24

#### The Degree Sequence Problem for Partial Steiner Triple Systems

A partial Steiner triple system is a collection of triples on a given set of vertices (or points) such that every pair of vertices is in at most one triple. The degree of a vertex in such a system is the number of triples that contain it. The degree sequence of a partial Steiner triple system is the list of its vertex degrees. The problem is when given a list of positive integers to determine if there is a partial Steiner

triple system with this list as its degree sequence. Recent progress on this problem is presented.

Donald Kreher

Department of Mathematical Sciences  
Michigan Technological University  
kreher@mtu.edu

### MS24

#### Colouring Block Designs

A block design with point set  $V$  and block set  $\mathcal{B}$  is said to be  $c$ -colourable if the points of  $V$  can be partitioned into  $c$  sets called colour classes such that no block of  $\mathcal{B}$  has all of its points in a single colour class. A design is said to be  $c$ -chromatic if it is  $c$ -colourable but not  $(c-1)$ -colourable. For all integers  $c \geq 2$ ,  $k \geq 6$  and  $\lambda \geq 1$ , we show that for sufficiently large  $v$  the obvious necessary conditions for the existence of a BIBD( $v, k, \lambda$ ) are sufficient for the existence of a  $c$ -chromatic BIBD( $v, k, \lambda$ ).

Daniel Horsley, David Pike

Memorial University of Newfoundland  
danhorsley@gmail.com, dapike@mun.ca

### MS24

#### Spanning Trees with Specified Differences in Cayley Graphs

Let  $G$  be a finite group of order  $n$  and  $L$  a multiset of  $n-1$  group elements with  $L_s$  its underlying set. Let  $C(G, L_s)$  be the undirected Cayley graph on  $G$  with generating set  $L_s$ . If there exists a tree in  $C(G, L_s)$  whose multiset of edge labels is precisely  $L$  then it is necessary that  $L$  contain at most  $n(1 - 1/(|G:H|))$  for any  $H < G$ . We show that this condition is also sufficient.

Brett C. Stevens

Carleton University  
School of Mathematics and Statistics  
brett@math.carleton.ca

Ben Seamone

School of Mathematics and Statistics  
Carleton University  
bseamone@connect.carleton.ca

### MS24

#### Transversals and Orthogonal Latin Squares

A transversal of a latin square is a selection of entries that hits each row, column and symbol exactly once. We can construct latin squares whose transversals are constrained in various ways. For orders that are not twice a prime, these constructions yield 2-maxMOLS, that is, pairs of orthogonal latin squares that cannot be extended to a triple of MOLS. If only Euclid’s theorem was false, we’d have nearly solved the 2-maxMOLS problem!

Peter Danziger

Ryerson University  
danziger@ryerson.ca

Ian Wanless

School of Mathematical Sciences  
Monash University  
ian.wanless@sci.monash.edu.au

Bridget Webb  
Open University, UK  
b.s.webb@open.ac.uk

**MS25****Connectivity and Forbidden Families for Hamiltonian Properties**

We consider a number of new results dealing with hamiltonian properties in 3 or 4-connected claw-free graphs. We are especially interested in the relationship between connectivity and the family of pairs of forbidden subgraphs sufficient to ensure the property of interest. Properties of interest include being Hamiltonian, pancyclic and hamiltonian connected.

Ronald Gould

Dept. of Mathematics and Computer Science  
Emory University  
rg@mathcs.emory.edu

**MS25****Square Paths and Cycles**

A square cycle (path) is a cycle (path) together with all of its 2-chords. I will survey results on square paths and cycles, and other related substructures.

H. A. Kierstead

Arizona State University  
kierstead@asu.edu

**MS25****Closures and Forbidden Subgraphs for Paths and Cycles in Graphs**

The closure concept for hamiltonicity in claw-free graphs has proved to be a useful tool for studying hamiltonian problems in claw-free graphs. However, its applicability is restricted only to properties P that are stable, i.e. such that G has P if and only if the closure of G has P. A well-known example of an unstable property is Hamilton-connectedness. In the talk we survey recent results on variations of the closure that are applicable to some other cycle and path properties (Hamilton-connectedness, 2-factor). As an application, we show that every 7-connected claw-free graph is Hamilton-connected, we obtain a degree condition for Hamilton-connectedness, we simplify the characterization of forbidden pairs for 2-factors, and we present recent progress in forbidden pairs for Hamilton-connectedness and discuss remaining open cases.

Zdenek Ryjacek

University of West Bohemia, Pilsen, Czech Republic  
ryjacek@kma.zcu.cz

**MS25****Circumference of 3-connected Claw-free Graphs and Large Eulerian Subgraphs of 3-edge-connected Graphs**

The circumference of a graph is the length of its longest cycles. Results of Jackson, and Jackson and Wormald, imply that the circumference of a 3-connected cubic  $n$ -vertex graph is  $\Omega(n^{\log_2(1+\sqrt{5})-1})$ , and the circumference of a 3-connected claw-free graph is  $\Omega(n^{\log_{150} 2})$ . We generalise the first result by showing that every 3-edge-

connected graph with  $m$  edges has an Eulerian subgraph with  $\Omega(m^{\log_2(1+\sqrt{5})-1})$  edges. We use this result together with the Ryjáček closure operation to improve the lower bound on the circumference of a 3-connected claw-free graph to  $\Omega(n^{\log_2(1+\sqrt{5})-1})$ .

Xingxing Yu

School of Mathematics  
Georgia Tech  
yu@math.gatech.edu

Mark Bilinski

Department of Mathematic  
Louisiana State University  
bilinski@math.lsu.edu

Bill Jackson

School of Mathematical Sciences  
Queen Mary University of London  
b.jackson@qmul.ac.uk

Jie Ma

School of Mathematics  
Georgia Institute of Technology  
jiema@math.gatech.edu

**MS26****Reduced Divisors and Embeddings of Tropical Curves**

Given a divisor  $D$  on a tropical curve  $\Gamma$ , we show that reduced divisors define an integer affine map from the tropical curve to the complete linear system  $|D|$ . We consider the cases where this map defines an embedding of the curve into the linear system, and in this way classify all the tropical curves with a very ample canonical divisor. As an application of the reduced-divisor map, we show the existence of Weierstrass points on tropical curves of genus at least two.

Omid Amini

CNRS / DMA-École Normale Supérieure Paris  
omid.amini@ens.fr

**MS26****The 4x4 Minors of a 5xn Matrix are a Tropical Basis**

We compute the space of  $5 \times 5$  matrices of tropical rank at most 3 and show that it coincides with the space of  $5 \times 5$  matrices of Kapranov rank at most 3, that is, the space of five labeled coplanar points in  $TP^4$ . We then prove that the Kapranov rank of every  $5n$  matrix equals its tropical rank; equivalently, that the 44 minors of a  $5n$  matrix of variables form a tropical basis. This answers a question asked by Develin, Santos, and Sturmfels. Joint work with Anders Jensen and Elena Rubei.

Melody Chan

UC Berkeley  
mtchan@math.berkeley.edu

**MS26****Lifting Tropical Curves**

Tropical Geometry is an area of mathematics that associates polyhedral complexes to algebraic varieties by a process called *tropicalization*. The combinatorial properties of

the complex reflects the geometry of the variety. Under tropicalization, an algebraic curve becomes a very special graph embedded in space. We give a combinatorial obstruction for a graph to be a tropicalization of a curve. This obstruction is phrased in the language of linear systems on tropical curves.

Eric Katz

University of Texas Austin  
eekatz@math.utexas.edu

## MS26

### Harmonic Morphisms of Graphs

Harmonic morphisms of graphs are natural discrete analogues of holomorphic maps between Riemann surfaces. We will talk about applications of harmonic morphisms to the analysis of the structure of graph Jacobians. We will also discuss a possible discrete analogue of the correspondence lemma by Castelnuovo and Severi. Based partially on joint work with Matthew Baker and Peter Whalen.

Sergey Norin

Princeton University  
snorin@math.princeton.edu

## MS26

### Computing Reduced Divisors on Graphs

It is known that, relative to any fixed vertex  $q$  of a finite graph, there exists a unique  $q$ -reduced divisor in each linear equivalence class of divisors. We give an efficient algorithm for finding such reduced divisors. We also give applications, including a new and completely algebraic algorithm for generating random spanning trees. Other applications include algorithms related to chip-firing games and sand-pile model, as well as certain algorithmic problems about the Riemann-Roch theory on graphs.

Farbod Shokrieh

Georgia Institute of Technology  
shokrieh@math.gatech.edu

## MS27

### Structure and Enumeration of One-face Maps

A *one-face map*, or *polygon gluing*, is a graph embedded on a topological surface such that its complement is a topological disk. I will describe a bijective decomposition that relates one-face maps on a fixed surface to plane trees with distinguished vertices. In particular, this bijection gives a (long awaited) combinatorial interpretation to the fact that for each orientable surface, one-face maps are enumerated by the product of a polynomial and a Catalan number.

Guillaume Chapuy

PIMS/Simon Fraser University  
Canada  
guillaume.chapuy@lix.polytechnique.fr

## MS27

### $K_{1,1,2}$ -linkage for Planar Triangulations

Given graphs  $G$  and  $H$ , we say  $G$  is  $H$ -linked if for every injective mapping  $f : V(H) \rightarrow V(G)$  there is a subgraph  $H'$  of  $G$  that is a subdivision of  $H$ , with  $f(v)$  being the vertex of  $H'$  corresponding to each vertex  $v$  of  $H$ . Goddard showed that 4-connected planar triangulations are  $C_4$ -

linked; we strengthen this by showing that they are  $K_{1,1,2}$ -linked ( $K_{1,1,2}$  is  $K_4$  with one edge deleted).

Mark Ellingham, Michael D. Plummer

Department of Mathematics  
Vanderbilt University  
mark.ellingham@vanderbilt.edu,  
michael.d.plummer@vanderbilt.edu

Gexin Yu

Department of Mathematics  
The College of William and Mary  
gyu@wm.edu

## MS27

### On the Structure of Crossing Critical Graphs

A graph  $G$  is said to be crossing critical if the removal of any edge of  $G$  decreases its crossing number. In the talk we shall discuss the structure of crossing critical graphs.

Bojan Mohar

Simon Fraser University  
mohar@math.sfu.ca

Zdenek Dvorak

Department of Applied Mathematics  
Charles University, Prague  
rakdver@kam.mff.cuni.cz

## MS27

### Vertex-face Curves for Bipartite Quadrangulations on the Torus

A *vertex-face  $m$ -curve* for a map  $G$  on a surface  $S$  is a set of pairwise disjoint  $m$  simple closed curves on  $S$  which visit all vertices of  $G$  exactly once, all faces of  $G$  exactly once, but cross no edges of  $G$ . We shall prove that every bipartite quadrangulation on the torus admits a vertex-face 1-curve and a vertex-face 2-curve. Applying them, we can get results on Hamiltonicity and book embedding for torus quadrangulations.

Atsuhiko Nakamoto

Yokohama National University  
Japan  
nakamoto@edhs.ynu.ac.jp

## MS27

### Symmetric Maps and Residual Finiteness

Automorphism groups of tessellations of the infinite plane are known to be residually finite, which means that for any non-identity element the group contains a subgroup of finite index avoiding that element. It is also well known that maps on surfaces, and 'highly symmetric' maps in particular, are quotients of such tessellations. Residual finiteness may therefore help in proving results for highly symmetric maps. We briefly outline the corresponding theory and present illustration examples.

Josef Siran

Open University  
j.siran@open.ac.uk

MS28

**Limiting the Rate of Locally Testable Codes**

Motivated by the question of whether an asymptotically good family of locally testable codes (LTCs) exists, we show three results that have a common theme. They all give subconstant upper bounds on the rate of certain families of LTCs, showing these families are not asymptotically good. Based on joint works with Venkatesan Guruswami, Tali Kaufman, Madhu Sudan and Michael Viderman.

Eli Ben-SassonIsrael Institute of Technology  
eli@cs.technion.ac.il

MS28

**Succinct Representation of Codes with Applications to Testing**

Motivated by questions in property testing, we search for linear error-correcting codes that have the ‘single local orbit’ property: i.e., they are specified by a single local constraint and its translations under the symmetry group of the code. We show that the dual of every ‘sparse’ binary code whose coordinates are indexed by elements of  $F_{2^n}$  for prime  $n$ , and whose symmetry group includes the non-singular affine transformations of  $F_{2^n}$ , has the single local orbit property.

Elena GrigorescuMassachusetts Institute of Technology  
elena\_g@mit.edu

Tali Kaufman

MIT  
kaufmant@mit.edu

Madhu Sudan

Microsoft Research  
madhu@microsoft.com

MS28

**Error-free List Decoding Algorithms with Linear Complexity for Binary Reed-Muller Codes**

In contrary to randomized algorithms we consider deterministic list decoding algorithms which being error-free cannot have complexity smaller than linear for any family of codes with constant relative distance, in particular, for Reed-Muller codes of any fixed order. It is well known that such RM codes can be list decoded with linear complexity for the decoding radius not exceeding half of the code distance. For RM codes of first order such radius allowing error-free list decoding with linear complexity can be increased up to the code distance but for general RM codes we can prove it only for the decoding radius not exceeding the Johnson bound. Based on joint works with Ilya Dumer and Cedric Tvernier.

Grigory KabatianskyRussian Academy of Sciences  
kaba@iitp.ru

MS28

**Testing and Decoding Sparse Random Linear Codes from High Error**

We show that sparse random linear codes are locally

testable and locally list-decodable from  $(\frac{1}{2} - \epsilon)$ -fraction errors, for every constant  $\epsilon > 0$ . More precisely, we show that any linear code  $C \subseteq F_2^n$  which is:

- sparse (i.e., has only  $\text{poly}(n)$  codewords)
- unbiased (i.e., each nonzero codeword has Hamming weight  $\in [1/2 - n^{-\gamma}, 1/2 + n^{-\gamma}]$  for some constant  $\gamma > 0$ )

can be locally tested and locally list decoded from  $(\frac{1}{2} - \epsilon)$ -fraction errors using only  $\text{poly}(1/\epsilon)$  queries to the received word. This generalizes a result of Kaufman and Sudan, who gave a local tester and local (unique) decoder for such codes from some constant fraction of errors. For a particularly prominent family of sparse, unbiased codes, the dual-BCH codes, we show that these local testing and list-decoding algorithms can be implemented in time polylogarithmic in the length of the codeword.

Swastik Kopparty, Shubhangi SarafMassachusetts Institute of Technology  
swastik@mit.edu, shibs@mit.edu

MS28

**The Extended Norm-trace Function Field and Applications**

The extended norm-trace function field is a generalization of the Hermitian and norm-trace function fields which are of importance in coding theory. In this talk, we provide explicit bases for certain Riemann-Roch spaces on the extended norm-trace function field. These bases provide explicit generator and parity check matrices for certain algebraic geometry codes on the extended norm-trace function field. This includes certain one-point codes and multipoint codes.

Gretchen Matthews, Justin PeacheyClemson University  
gmatthe@clemson.edu, jpeache@clemson.edu

MS28

**Matching Vector Codes**

An  $r$ -query locally decodable code encodes a message to a codeword, such that every message bit can be recovered with a high probability, by a randomized decoding procedure that reads only  $r$  bits, even if the codeword is corrupted in up to  $\delta$  fraction of locations. Recently a new class of locally decodable codes, based on families of vectors with restricted dot products has been discovered. We refer to those codes as Matching Vector (MV) codes. In this work we develop a new view of matching vector codes and uncover certain similarities between MV codes and classical Reed Muller codes. Our view allows us to obtain a deeper insight into power and limitations of MV codes. (Joint work with Zeev Dvir and Parikshit Gopalan).

Sergey YekhaninMicrosoft Research  
yekhanin@microsoft.com

MS29

**Online Linear Discrepancy**

The linear discrepancy of a poset  $\mathbf{P}$  is the least  $k$  for which there is a linear extension  $L$  of  $\mathbf{P}$  such that if  $x$  and  $y$  are incomparable in  $\mathbf{P}$ , then  $|h_L(x) - h_L(y)| \leq k$ , where  $h_L(x)$  is the height of  $x$  in  $L$ . In this paper, we consider linear discrepancy in an online setting and devise an on-

line algorithm that constructs a linear extension  $L$  of a poset  $\mathbf{P}$  so that  $|h_L(x) - h_L(y)| \leq 3k - 1$ , when the linear discrepancy of  $P$  is  $k$ . This inequality is best possible, even for the class of interval orders. Furthermore, if the poset  $\mathbf{P}$  is a semiorder, then the inequality is improved to  $|h_L(x) - h_L(y)| \leq 2k$ . Again, this result is best possible.

Mitchel T. Keller

Georgia Institute of Technology  
keller@math.gatech.edu

Noah Streib, William T. Trotter  
School of Mathematics  
Georgia Institute of Technology  
nstreib3@math.gatech.edu, trotter@math.gatech.edu

### MS29

#### The Subexponential Upper Bound for On-line Chain Partitioning Problem

Abstract not available at time of publication.

Tomasz Krawczyk

Jagiellonian University  
krawczyk@tcs.uj.edu.pl

Bartłomiej Bosek

Jagellion University, Poland  
bosek@tcs.uj.edu.pl

### MS29

#### On-line Chain Partitions of Orders: Recent Results and Open Problems

Abstract not available at time of publication.

Bartłomiej Bosek

Jagellion University, Poland  
bosek@tcs.uj.edu.pl

Stefan Felsner

Technical University of Berlin  
Institute for Mathematics  
felsner@math.tu-berlin.de

Kamil Kloch

University of Passau  
kamil.kloch@uni-passau.de

Tomasz Krawczyk, Grzegorz Matecki

Jagiellonian University  
krawczyk@tcs.uj.edu.pl, grzegorz.matecki@tcs.uj.edu.pl

Piotr Micek

Jagellionian University  
piotr.Micek@tcs.uj.edu.pl

### MS29

#### First Fit Coloring of Interval Graphs

Kierstead/Trotter's online coloring algorithm uses at most  $3k - 2$  colors on  $k$ -chromatic interval graphs, best possible. Kierstead showed that first-fit uses at most  $40k$  colors on  $k$ -chromatic interval graphs. Pemmaraju/Raman/Varadarajan improved this to  $10k$ . This can be lowered to  $8k$ . Chrobak/Ślusarek showed that first-fit uses asymptotically  $4.45k$  colors on some  $k$ -chromatic

interval graphs. We raise this to  $5k$ .

David A. Smith, H. A. Kierstead

Arizona State University  
dsmith@mathpost.la.asu.edu, kierstead@math.la.asu.edu

W. T. Trotter

Georgia Institute of Technology  
trotter@gatech.edu

### MS29

#### The Total Linear Discrepancy of a Poset

In this talk we discuss the *total linear discrepancy* of a poset. If  $L$  is a linear extension of a poset  $P$ , and  $x, y$  is an incomparable pair in  $P$ , the height difference between  $x$  and  $y$  in  $L$  is  $|L(x) - L(y)|$ . The total linear discrepancy of  $P$  in  $L$  is the sum over all incomparable pairs of these height differences. The total linear discrepancy of  $P$  is the minimum of this sum taken over all linear extensions  $L$  of  $P$ . While the decision problem of determining whether the (ordinary) linear discrepancy of a poset is at most  $k$  is NP-complete, the total linear discrepancy can be computed in polynomial time. In this talk we characterize those linear extensions that are optimal for total linear discrepancy. The characterization provides an easy way to count the number of optimal linear extensions.

Ann N. Trenk

Wellesley College  
atrenk@wellesley.edu

David Howard

Georgia Institute of Technology  
dmh@math.gatech.edu

Randy Shull

Department of Computer Science  
Wellesley College  
rshull@wellesley.edu

Noah Streib

Georgia Institute of Technology  
nstreib3@gatech.edu

### MS30

#### Strings, Trees, and RNA Folding

Understanding the folding of RNA sequences into three-dimensional structures is one of the fundamental challenges in molecular biology. In this talk, we focus on understanding how an RNA viral genome can fold into the dodecahedral cage known from experimental data. Using strings and trees as a combinatorial model of RNA folding, we give mathematical results which yield insight into RNA structure formation and suggest new directions in viral capsid assembly.

Christine E. Heitsch

School of Mathematics  
Georgia Tech  
heitsch@math.gatech.edu

### MS30

#### Averaging Metric Trees

The space of metric phylogenetic trees is a polyhedral complex, and as constructed by Billera, Holmes, and Vogtmann

(2001), also non- positively curved. This additional property ensures a well-defined notion of an average or mean tree for a given set of trees. In this talk, I will describe this mean tree and how to compute it, as well as discuss what it represents and some applications.

Megan Owen  
Mathematics  
North Carolina State University  
maowen@ncsu.edu

**MS30**  
**Computational Problems in Cancer Genome Sequencing**

Cancer is a disease driven by somatic mutations that accumulate in the genome during an individual's lifetime. These somatic mutations occupy a continuum of scales ranging from single nucleotide mutations through structural rearrangements that include deletions, insertions, inversions, and translocations of large blocks of DNA sequence. Recent improvements in DNA sequencing technology are enabling the measurement of numerous cancer genomes of diverse types. I will describe solutions to several computational problems that arise in the analysis of cancer genome sequence data. These include: (i) a framework for classifying and comparing structural variants measured using a variety of DNA sequencing technologies; (ii) combinatorial algorithms to find the most parsimonious sequence of rearrangements and duplications that transform a normal genome into a cancer genome; (iii) a technique to identify groups of interacting genes, or pathways, that are mutated at significant frequency in specific cancer types. I will illustrate applications of these approaches to data from The Cancer Genome Atlas.

Ben Raphael  
Brown University  
braphael@cs.brown.edu

**MS30**  
**Simultaneous Alignment and Phylogenetic Tree Estimation**

Molecular sequences evolve under processes that include substitutions, insertions, and deletions (jointly called "indels"), as well as other mechanisms (e.g., duplications and rearrangements). The inference of the evolutionary history of these sequences has thus been performed in two stages: the first estimates the alignment on the sequences, and the second estimates the tree given that alignment. While such methods seem to work well on relatively small datasets, these two-stage approaches can produce highly incorrect trees and alignments when applied to large datasets, or ones that evolve with many indels. In this talk, I will present a new method, SATe, that my lab has been developing that uses maximum likelihood to estimate the alignment and tree at the same time, and that can be used to analyze datasets with up to 1000 sequences on a desktop in 24 hours. Our study, using both real and simulated data, shows that this method produces much more accurate trees than the current best methods. Joint work with Kevin Liu, Sindhu Raghavan, Serita Nelesen, and Randy Linder.

Tandy Warnow  
Computer Science  
University of Texas, Austin  
tandy@cs.utexas.edu

**MS31**  
**Existence Conditions for a Stable Set Meeting all Maximum Cliques**

Rabern recently proved that any graph with  $\omega \geq \frac{3}{4}(\Delta + 1)$  contains a stable set meeting all maximum cliques. We strengthen this result, proving that such a stable set exists for any graph with  $\omega > \frac{2}{3}(\Delta + 1)$ . The proof of this result uses a newly observed existence condition for independent transversals over sets of unequal size, which we extend to a result on weighted fractional total colourings.

Andrew D. King  
Columbia University  
andrew.d.king@gmail.com

**MS31**  
**Perfect Matchings in Claw-free Cubic Graphs**

Lovász and Plummer conjectured that there exists a fixed positive constant  $c$  such that every cubic  $n$ -vertex graph with no claw has at least  $2^{cn}$  perfect matchings. Their conjecture has been verified for bipartite graphs by Voorhoeve and planar graphs by Chudnovsky and Seymour. We prove that every claw-free cubic  $n$ -vertex graph with no claw has more than  $2^{n/12}$  perfect matchings, thus verifying the conjecture for claw-free graphs.

Sang-Il Oum  
KAIST  
sangil@kaist.edu

**MS31**  
**Discrepancy in Graphs and Hypergraphs**

How uniformly can the edges of a graph or hypergraph be distributed? How much or little can two graphs or hypergraphs be made to overlap when placed on the same set of vertices? I will discuss some substantial extensions of classical results of Erdős and Spencer and of Erdős, Goldberg, Pach and Spencer on the first problem, and some surprising new results on the (closely related) second problem. This is joint work with Béla Bollobás.

Alex Scott  
Oxford  
scott@maths.ox.ac.uk

**MS31**  
**The Edge-disjoint Paths Problem in Four-edge-connected graphs**

Suppose we are given  $k$  pairs of vertices of a graph, and want to test whether there exist  $k$  vertex-disjoint paths linking the pairs. For fixed  $k$ , Robertson and the speaker gave a polynomial-time algorithm to answer this; but the proof of the correctness of the algorithm was complicated, and used the Graph Minors structure theorem. What if we only ask that the paths be pairwise edge-disjoint? One way to solve this is to apply the algorithm for vertex-disjoint paths to the line graph. But it turns out that for four-edge-connected graphs there is also a much easier algorithm, which is explained in this talk.

Maria Chudnovsky  
Columbia  
mchudnov@columbia.edu

Paul Seymour  
Princeton University  
pds@math.princeton.edu

**MS31****Co-strongly Perfect Clawfree Graphs. Fractional and Integral Version.**

Strongly perfect graphs have been studied by several authors (e.g. Berge, Duchet, Ravindra, Wang). This talk deals with a fractional relaxation of strong perfection. Motivated by a wireless networking problem, we consider claw-free graphs that are fractionally strongly perfect in the complement. We obtain a forbidden induced subgraph characterization and display graph-theoretic properties of such graphs. It turns out that the forbidden induced subgraphs that characterize claw-free fractionally co-strongly perfect graphs are precisely the cycle of length 6, all cycles of length at least 8, four particular graphs, and a collection of graphs that are constructed by taking two graphs, each a copy of one of three particular graphs, and joining them by a path of arbitrary length in a certain way. Wang gave a characterization of strongly perfect claw-free graph. As a corollary we obtain a characterization of claw-free graphs whose complements are strongly perfect.

Yori Zwols  
Columbia University  
yz2198@columbia.edu

Maria Chudnovsky  
Columbia  
mchudnov@columbia.edu

Bernard Ries  
University of Warwick  
bernard.ries@wbs.ac.uk

**MS32****Tree Reconstruction and a Waring-type Problem on Partitions**

The “line graph” of a graph  $G$  is a new graph  $L(G)$  whose vertices are the edges of  $G$ , with a new edge in  $L(G)$  from  $e$  to  $f$  if  $e$  and  $f$  were incident in  $G$ . Graham’s Tree Reconstruction Conjecture says that, if  $T$  is a tree (a connected, acyclic graph), then the sequence of sizes of the iterated line graphs of  $T$  uniquely determine  $T$ . That is,  $T$  can be reconstructed from  $\{|L^{(j)}(G)|\}_{j=0}^{\infty}$ , where  $L^{(0)}(G) = G$  and  $L^{(j+1)}(G) = L(L^{(j)}(G))$ . Call two trees equivalent if they yield the same sequence; we call the resulting equivalence classes “Graham classes.” Clearly, the conjecture is equivalent to the statement that the number of Graham classes of  $n$ -vertex trees is equal to the number of isomorphism classes of such trees, which is known to be about  $2.955765^n$ . We show that the number of Graham classes is at least superpolynomial in  $n$  (namely,  $\exp(c \log n^{3/2})$ ) by converting the question into the following Waring-type problem on partitions. For a partition  $\lambda = \{\lambda_1, \dots, \lambda_k\}$  of the integer  $n$  and a degree  $d$  polynomial  $f \in \mathbf{R}[\mathbf{x}]$ , define  $f(\lambda) = \sum_{j=1}^k f(\lambda_j)$ . We show that the range of  $f(\lambda)$  over all partitions  $\lambda$  of  $n$  grows as  $\Omega(n^{d-1})$ . The proof employs a well-known family of solutions to the Prouhet-Tarry-Escott problem. Evidence suggests the conjecture that the size of the range is actually  $\Theta(n^d)$ .

Joshua Cooper  
University of South Carolina

Department of Mathematics  
cooper@math.sc.edu

**MS32****The Number of Shi Regions with a Given Separating Wall**

For an irreducible crystallographic root system  $\Phi$  spanning a Euclidean vector space  $V$  and a positive integer  $m$ , the extended Shi arrangement is the collection of hyperplanes  $H_{\alpha,k} = \{x \in V | (x, \alpha) = k\}$  for  $\alpha \in \Phi$  and  $k = 1, \dots, m$ . This arrangement dissects the dominant chamber into regions. Athanasiadis generalized the Narayana numbers by enumerating the regions which have  $k$  walls of the form  $H_{\alpha,m}$  which separate them from the origin. In this talk, we discuss the enumeration of these regions which have  $H_{\alpha,m}$ , for a fixed  $\alpha \in \Phi$ , as a separating wall when  $\Phi = A_{n-1}$ . This is joint work with E. Tzanaki and M. Vazirani.

Susanna Fishel  
Department of Mathematics  
Arizona State University  
fishel@math.asu.edu

**MS32****Affine K-theoretic Tableaux**

Among many other applications, Young tableaux can be used to define Schur functions and to encode the cohomology structure of the Grassmannian variety by way of Pieri and Littlewood-Richardson rules. The study of K-theory of affine Grassmannians led us to discover affine set-valued tableaux. We prove that these tableaux define certain affine Grothendieck polynomials and encode their associated Pieri rules. This extends our earlier work showing that tableaux tied to the type-A affine Weyl group give a natural approach to the quantum cohomology of Grassmannians, (co)homology of affine Grassmannians, Gromov-Witten invariants, and Macdonald polynomials.

Jennifer Morse  
Department of Mathematics  
Drexel University  
morsej@math.drexel.edu

**MS32****Reduced Kronecker Coefficients**

The *reduced Kronecker coefficients* is a family of positive integers that until recently has been overlooked. In this talk we aim to introduce and present several results on the reduced Kronecker coefficients as well as to illustrate their applicability in the understanding of the Kronecker coefficients (structure coefficients for the internal product of Schur functions). We also show that the reduced Kronecker coefficients are related to the Heisenberg product on symmetric functions as defined by Aguiar, Ferrer and Moreira.

Emmanuel Briand  
University of Sevilla  
mbriand@us.es

Rosa Orellana  
Department of Mathematics  
Dartmouth College  
rosa.c.orellana@dartmouth.edu

Mercedes Rosas  
University of Sevilla  
mrosas@us.es

**MS32****Rational Classes of Permutations**

A permutation class is a set of permutations closed under the natural combinatorial notion of subpermutation. It is commonly believed that most permutation classes have very complicated, in fact, non-holonomic, generating functions. Yet some permutation classes possess rational generating functions. I will discuss the ongoing problem of characterizing such classes.

Michael Albert, Mike Atkinson  
Department of Computer Science  
University of Otago  
malbert@cs.otago.ac.nz, mike@cs.otago.ac.nz

Vincent Vatter  
Department of Mathematics  
Dartmouth  
vincent.vatter@dartmouth.edu

**MS33****K-theory of Cominuscule Grassmannians**

The K-theoretic Schubert structure constants of a homogeneous space  $G/P$  are known to have signs that alternate with codimension by a result of Brion. For Grassmannians of type A, these constants are computed by a generalization of the classical Littlewood-Richardson rule that counts set-valued tableaux. I will report on recent results on the K-theory of Lagrangian and maximal orthogonal Grassmannians, including a Pieri rule (with Vijay Ravikumar) for multiplying an arbitrary Schubert class with a special Schubert class.

Anders Buch  
Rutgers University  
asbuch@math.rutgers.edu

**MS33****Towards Generalizing Schubert Calculus in Symplectic Category**

We extend some of the ideas from Schubert calculus to the more general setting of Hamiltonian torus actions on compact symplectic manifolds with isolated fixed points. Given a generic component  $\Phi$  of the moment map, which is a Morse function, we define a canonical class  $\alpha_p$  in the equivariant cohomology of the manifold  $M$  for each fixed point  $p \in M$ . When they exist, canonical classes form a natural basis of the equivariant cohomology of  $M$ ; in particular, when  $M$  is a flag variety, these classes are the equivariant Schubert classes. We show that the restriction of a canonical class  $\alpha_p$  to a fixed point  $q$  can be calculated by a rational function which depends only on the value of the moment map, and the restriction of other canonical classes to points of index exactly two higher. Therefore, the structure constants can be calculated by a similar rational function. Our restriction formula is manifestly positive in many cases, including when  $M$  is a flag manifold. We also prove the existence of integral canonical classes in the case that  $M$  is a GKM space (after Goresky, Kottwitz and MacPherson) and  $\Phi$  is index increasing. In this case, our restriction formula specializes to an easily computable

rational sum which depends only on the GKM graph. This is joint with Susan Tolman.

Rebecca Goldin  
George Mason University  
rgoldin@math.gmu.edu

Susan Tolman  
University of IL, Champaign-Urbana  
stolman@math.uiuc.edu

**MS33****Wronskians and Schubert Calculus**

I will talk about some of the consequences of the Mukhin-Tarasov-Varchenko Theorem (formerly the Shapiro-Shapiro conjecture), which states that if the Wronskian of  $d$  polynomials has only real roots, then the vector space spanned by these polynomials has a real basis.

Kevin Purbhoo  
University of Waterloo  
kpurbhoo@math.uwaterloo.ca

**MS33****A Littlewood-Richardson Rule for K-theory of Orthogonal Grassmannians**

We present a Littlewood-Richardson rule for the K-theory of the odd orthogonal Grassmannian  $OG(n, 2n + 1)$ . We conjectured this rule in previous work; it is now proved by a combination of (i) a Pieri rule due to Buch-Ravikumar, (ii) a result of Feigenbaum-Sergel showing that the Buch-Ravikumar Pieri rule agrees with our conjecture, and (iii) new combinatorial results showing that our conjectural rule gives rise to an associative product.

Hugh Thomas  
University of New Brunswick  
hthomas@unb.ca

Alexander Yong  
University of Illinois, Urbana-Champaign  
ayong@uiuc.edu

**MS34****Dependent Randomized Rounding in Matroid Polytopes and Applications**

We describe algorithms for randomly rounding a fractional solution in a matroid (base) polytope to an integral one. We consider the pipage rounding technique and also present a new technique, randomized swap rounding. Our main technical results are concentration bounds for functions of random variables arising from these rounding techniques. We prove Chernoff-type concentration bounds for linear functions of random variables arising from both techniques, and also a lower-tail exponential bound for monotone submodular functions of variables arising from randomized swap rounding. The rounding schemes have several applications that we plan to briefly discuss.

Chandra Chekuri  
University of Illinois  
chekuri@cs.illinois.edu

Jan Vondrak  
IBM Almaden Research Center

jvondrak@gmail.com

Rico Zenklusen  
EPFL Lausanne  
rico.zenklusen@gmail.com

#### MS34

##### Submodular Function Minimization and Approximation

In this talk, we will review recent developments in submodular optimization. In particular, we will focus on combinatorial algorithms for submodular function minimization. We also discuss approximating submodular functions everywhere, which provides a generic method to design approximation algorithms for combinatorial optimization problems with submodular cost functions.

Satoru Iwata  
Kyoto University  
iwata@kurims.kyoto-u.ac.jp

#### MS34

##### Non-monotone Submodular Maximization: Randomized Local Search and

In this talk, we will discuss two results for non-monotone submodular maximization. In one part, we give a randomized local search 0.4-approximation algorithm for non-negative functions. In the second part, we present the PASS (Parametrize As a Structure of the Solution) approximation framework, and give a tight approximation algorithm for maximizing a monotone submodular function minus an additive function.

Vahab Mirrokni  
Google Research  
mirrokni@gmail.com

#### MS34

##### Local Search Algorithms for Submodular Maximization Problems

Submodular-function maximization is a central problem in combinatorial optimization, generalizing many important NP-hard problems including Max Cut in digraphs, graphs and hypergraphs, certain constraint satisfaction problems, maximum-entropy sampling, and maximum facility-location problems. Local search algorithms are among most popular when it comes to practical algorithms. We briefly survey the previous work on the analysis of the performance of such algorithms for various variants of submodular maximization problem. We will also discuss recent results on maximizing a submodular functions under multiple matroid or knapsack constraints. We conclude with the list of open problems.

Maxim Sviridenko  
IBM T. J. Watson Research Center  
svir@us.ibm.com

#### MS34

##### PTAS for Matroid Matching

We consider the classical matroid matching problem, which can be solved optimally for linear matroids [Lovasz '80] but not for general matroids in the oracle model. We present a PTAS for every matroid; the algorithm is a simple local

search. More generally, we prove that local search achieves a  $(k/2 + \epsilon)$ -approximation for matroid matching in  $k$ -uniform hypergraphs, which is a problem generalizing both  $k$ -set packing and intersection of  $k$  matroids. In contrast, we show that known linear-programming approaches including the Sherali-Adams hierarchy do not achieve any non-trivial approximation.

Jon Lee  
IBM T.J. Watson Research Center  
jonlee@us.ibm.com

Maxim Sviridenko  
IBM T. J. Watson Research Center  
svir@us.ibm.com

Jan Vondrak  
IBM Almaden Research Center  
jvondrak@gmail.com

#### MS35

##### Bicriticality for Independent Domination and Total Domination

A graph is independent domination bicritical if the removal of any two vertices reduces the independent domination number. Likewise a graph is total domination bicritical if the removal of any two vertices that do not produce an isolated vertex lowers the total domination number. Structural properties and construction techniques for independent domination bicritical and total domination bicritical graphs will be presented.

Michelle Edwards, Gary Macgillivray  
Mathematics and Statistics  
University of Victoria  
michelle@math.uvic.ca, gmacgill@math.uvic.ca

#### MS35

##### Eternal Domination (Gamma Forever!)

At each time interval, a vertex of  $G$  is attacked and a subset of the guards positioned on vertices must move to adjacent vertices so that a guard is on the vertex that was attacked. In eternal domination, the guards must be prepared for an infinite series of attacks. We will discuss some variations and give a characterisation of when  $m$  defenders are sufficient to defend  $G$  by examining the reduced canonical form of a related game.

Ste Finbow  
Math, Stats, and CS  
St. Francis Xavier University  
sfinbow@stfx.ca

Serge Gaspers  
CMM - Universidad de Chile  
sgaspers@dim.uchile.cl

M. E. Messinger  
Ryerson  
messinger@ryerson.ca

Paul Ottaway  
UBC  
paul.ottaway@gmail.com

**MS35****Saturation Point: A Two Player Game**

A  $k$ -limited packing  $P$  in a graph is a subset of the vertices with the property that the closed neighbourhood of any vertex in the graph contains at most  $k$  members of  $P$ . This could model, for instance, the wish to limit the number of undesirable facilities in one's backyard. We consider the following two player game based on this concept. The players alternate choosing a vertex in a graph. The only restriction is that at most  $k$  vertices can be selected in the closed neighbourhood of any vertex. Some observations from this investigation will be outlined.

Bert Hartnell  
Math and CS  
St. Mary's University  
hartnell@smu.ca

Art Finbow, Natasha Ching, Rucha Lingras  
Saint Mary's University  
finbow@smu.ca, natasha.ching@smu.ca,  
ruha.lingras@smu.ca

Rob Gallant  
Sir Wilfred Grenfell College  
rpgallant@swgc.mun.ca

**MS35****The Game Domination Number of Graphs**

The domination game on a graph  $G$  consists of two players, Dominator and Staller, who alternate choosing a vertex from  $G$ . If  $C$  denotes the set of vertices already chosen, then the next player may choose vertex  $w$  if  $N[C \cup \{w\}] - N[C]$  is nonempty. Dominator pursues a strategy designed to end the game in the fewest steps while Staller plays in such a way as to prolong the game as much as possible. We consider several invariants that arise from this game and their relationship to ordinary domination.

Douglas F. Rall  
Furman University  
doug.rall@furman.edu

Bostjan Bresar  
University of Maribor  
bostjan.bresar@uni-mb.si

Sandi Klavzar  
University of Ljubljana  
sandi.klavzar@fmf.uni-lj.si

**MS36****Cellular Automorphisms and Self-Dual Embeddings for the Torus and Klein Bottle**

We classify all cellular automorphisms of the torus and Klein bottle, and explain how these automorphisms can be used to construct all self-dual graph-embeddings in these surfaces. Our approach follows that of Archdeacon-Richter (1992) and Archdeacon-Negami (1994).

Lowell Abrams  
Department of Mathematics  
The George Washington University  
labrams@gwu.edu

Daniel Slilaty  
Department of Mathematics and Statistics  
Wright State University  
daniel.slilaty@wright.edu

**MS36****Finding Shortest Non-trivial Cycles in Directed Graphs on Surfaces**

Let  $D$  be a weighted directed graph cellularly embedded in a surface of genus  $g$ . We describe an algorithm to compute a shortest non-contractible and a shortest surface non-separating cycle of  $D$  in time  $O(\min\{n^2 \log n, \sqrt{g} n^{3/2} \log n\})$ , where  $n$  is the total number of vertices and arcs of  $D$ . Previous results only dealt with undirected graphs. Our algorithm uses a divide-and-conquer technique that simplifies the graph while preserving the topological properties of its cycles.

Sergio Cabello  
University of Ljubljana  
Ljubljana, Slovenia  
sergio.cabello@fmf.uni-lj.si

Éric Colin de Verdière  
École normale supérieure, Paris, France, and CNRS  
eric.colin.de.verdiere@ens.fr

Francis Lazarus  
GIPSA-Lab, CNRS  
Grenoble, France  
francis.lazarus@gipsa-lab.grenoble-inp.fr

**MS36****Shortest Cut Graph of a Surface with Prescribed Vertex Set**

A *cut graph* is a graph embedded on a surface that splits it into a topological disk. Erickson and Whittlesey [SODA 2005] found a very nice greedy algorithm to compute the shortest one-vertex cut graph of a combinatorial surface. This talk will show how their algorithm extends to the computation of a shortest cut graph with given vertex set. Moreover, a simpler proof will be given, revealing that the algorithm actually computes a minimum-weight basis of some matroid.

Éric Colin de Verdière  
École normale supérieure, Paris, France, and CNRS  
eric.colin.de.verdiere@ens.fr

**MS36****Shortest Non-Crossing Walks in the Plane**

Let  $G$  be a plane graph with non-negative edge weights, and let  $k$  terminal pairs be specified on  $h$  face boundaries. We present an algorithm to find  $k$  non-crossing walks in  $G$  of minimum total length that connect all terminal pairs, if any such walks exist, in  $2^{O(h^2)} n \log k$  time. The computed walks may overlap but may not cross each other or themselves. Our algorithm generalizes a result of Takahashi, Suzuki, and Nikizeki [Algorithmica, 1996] for the special case  $h \leq 2$ . We also describe an algorithm for the corresponding geometric problem, where the terminal points lie on the boundary of  $h$  polygonal obstacles of total complexity  $n$ , again in  $2^{O(h^2)} n$  time, generalizing an algorithm of Papadopoulos [Int. J. Comput. Geom. Appl. 1999] for the

special case  $h \leq 2$ . In both settings, shortest non-crossing walks can have complexity exponential in  $h$ . We also describe algorithms to determine in  $O(n)$  time whether the terminal pairs can be connected by *any* non-crossing walks.

Jeff Erickson  
Computer Science Department  
University of Illinois, Urbana-Champaign  
jeffe@cs.uiuc.edu

### MS36

#### Title Not Available at Time of Publication

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Ken-ichi Kawarabayashi  
National Institute of Informatics, Japan  
k\_keniti@nii.ac.jp

### MS37

#### On Mixed Ramsey Numbers

For graphs  $G$  and  $H$ , an edge-coloring of  $K_n$  is  $(G, H)$ -good if has neither a monochromatic copy of  $G$  nor a totally multicolored copy of  $H$ . Let  $S(n, G, H)$  be the set of number of colors used in some  $(G, H)$ -good coloring of  $K_n$ . We prove asymptotically tight bound for  $\max S(n, G, H)$ ,  $\min S(n, G, H)$  for large classes of graphs, and investigate the graphs for which  $S(n, G, H)$  is an interval.

Maria Axenovich  
Iowa State University  
axenovic@iastate.edu

### MS37

#### Typical Structure of Combinatorial Structures

Erdős, Kleitman and Rothschild in 1976 proved that almost all triangle-free graphs are bipartite. This initiated much research on approximating "typical" member of a complicated set with a simple structure. In this work we study the structure of the largest sum-free subsets of random subset of groups. It is joint work with Morris and Samotij.

Jozsef Balogh  
Department of Mathematics  
University of California at San Diego  
jbalog@math.ucsd.edu

### MS37

#### Giant Components in Random Subgraphs of General Graphs

Erdős and Rényi observed that a curious phase transition in the size of the largest component in a random graph  $G(n, p)$ : If  $pn < 1$ , then all components have size  $O(\log n)$ , while if  $pn > 1$  there exists a unique component of size  $\Theta(n)$ . Similar transitions can be seen to exist in so called  $(n, d, \lambda)$  graphs (Frieze, Krivelevich and Martin), dense graphs (Bollobás et. al) and several other special classes of graphs. Here we consider the story for graphs which are sparser and irregular. In this regime, the answer will depend on our definition of a 'giant component'; but we will show a phase transition for graphs satisfying a mild spectral condition. In particular, we present some results which supersede our earlier results in that they have weaker hypotheses and (in some sense) prove stronger results. Addi-

tionally, we construct some examples showing the necessity of our new hypothesis.

Paul Horn  
Emory University  
phorn@mathcs.emory.edu

### MS37

#### The Structure of Typical Hypergraphs with Local Constraints

We prove hypergraph versions of the well-known result of Erdos-Kleitman-Rothschild that almost all triangle-free graphs with vertex set  $[n]$  are bipartite. Our main tool is the Frankl-Rodl hypergraph regularity lemma.

Dhruv Mubayi  
Department of Mathematics Statistics and Computer Science  
University of Illinois at Chicago  
mubayi@math.uic.edu

Jozsef Balogh  
Department of Mathematics  
University of California at San Diego  
jbalog@math.ucsd.edu

### MS37

#### Multipartite Graph Packing

Given two graphs  $H$  and  $G$ , a perfect  $H$ -packing of  $G$  is a spanning subgraph of  $G$  which consists of vertex disjoint copies of  $H$ . The graph packing problem is a generalization of the matching problem and has been widely studied. In this talk we study the minimum degree threshold for  $G$  containing a perfect  $H$ -packing when  $G$  and  $H$  are both  $r$ -partite graphs for  $r \leq 4$ . Some results are joint work with Albert Bush and Ryan Martin.

Yi Zhao  
Georgia State University  
yzhao6@gsu.edu

### MS38

#### The Number of Numerical Semigroups of a Given Genus

A numerical semigroup is a subset of the non-negative integers that contains 0, is closed under addition, and has a finite complement. The size of the complement is called the genus of the semigroup. In this talk I will describe some lower and upper bounds on the number  $n_g$  of numerical semigroups of genus  $g$ . Starting from a known construction of a tree of all numerical semigroups, we approximate it by simpler generating trees whose nodes are labeled by certain parameters of the semigroups. The succession rules of these trees are then translated into functional equations for the generating functions that enumerate their nodes, which are solved to obtain the bounds.

Sergi Elizalde  
Department of Mathematics  
Dartmouth  
sergi.elizalde@dartmouth.edu

MS38

**Ribbon Graphs and Twisted Duality**

We consider two operations on the edge of an embedded (*i.e.* ribbon) graph: giving a half-twist to the edge and taking the partial dual with respect to the edge. These two operations give rise to an action of  $S_3^{e(G)}$ , the *ribbon group* of  $G$ , on  $G$ . We show that this ribbon group action gives a complete characterization of duality in that if  $G$  is any cellularly embedded graph with medial graph  $G_m$ , then the orbit of  $G$  under the group action is precisely the set of all graphs with medial graphs isomorphic (as abstract graphs) to  $G_m$ . We then show how the ribbon group action leads to a deeper understanding of the properties of, and relationships among, various graph polynomials such as the generalized transition polynomial, an extension of the Penrose polynomial to embedded graphs, and the topological Tutte polynomials of Las Vergnas and also Bollobás and Riordan, as well as various knot and link invariants.

Joanna Ellis-Monaghan  
Saint Michaels College  
Department of Mathematics  
jellis-monaghan@smcvt.edu

Iain Moffatt  
University of South Alabama  
imoffatt@jaguar1.usouthal.edu

MS38

**Rectangulations and Baxter Permutations**

Baxter permutations are a class of permutations described by a simple pattern-avoidance condition, and having a pleasant enumeration formula. A closely related class of pattern-avoiding permutations, dubbed the twisted Baxter permutations, arose (via lattice theory) as a natural basis for a sub Hopf algebra of the Malvenuto-Reutenauer Hopf algebra of permutations. Using generating trees, West verified that the two classes of permutations share the same enumeration. The starting point of this research is the project of finding an intrinsic description of the Hopf algebra of twisted Baxter permutations in terms of a set of combinatorial objects in bijection with Baxter permutations. This was accomplished using the (diagonal) rectangulations studied by Ackerman, Barequet, and Pinter, which are closely related to the twin binary trees of Dulucq and Guibert. In addition to the Hopf-theoretic results, we obtain a combinatorial characterization of the natural lattice structure on rectangulations, analogous to the Tamari lattice of triangulations, and gain some insight into the graph, analogous to the associahedron, whose vertices are rectangulations, and whose edges are given by certain pivot moves. We also find a new explicit bijection between Baxter permutations and twisted Baxter permutations. The main tools include a surjective map from permutations to diagonal rectangulations, as well as combinatorial and lattice-theoretic results on a related family of maps from permutations to triangulations. This is joint work with Shirley Law.

Shirley Law, Nathan Reading  
Department of Mathematics  
North Carolina State University  
, nreadin@ncsu.edu

MS38

**Combinatorial and Colorful Proofs of Cyclic Siev-****ing Phenomena**

Let  $S$  be a set which admits an action of the cyclic group  $C_n$  of order  $n$ . Let  $\omega_d$  denote a root of unity of order  $d$  in the group of roots of unity. Finally, let  $f(q)$  be a polynomial in  $q$ . Usually  $f(q)$  will be the generating function for some statistic on  $S$ . We say that the triple  $(S, C_n, f(q))$  exhibits the *cyclic sieving phenomenon (CSP)* if, for every  $c \in C_n$ , we have

$$f(\omega_d) = \text{the number of element of } S \text{ fixed by } c,$$

where  $d$  is the order of  $c$  in  $C_n$ . This concept was first introduced and studied by Reiner, Stanton and White, in part as a generalization of Stembridge's  $q = -1$  phenomenon which is the case  $n = 2$ . It is quite amazing that plugging a root of unity into a generating function would produce a nonnegative integer, much less that these integers would count something. But it appears that the CSP is quite wide spread and there is a growing literature on the subject. Most proofs that a triple exhibits the CSP use either algebraic manipulations involving roots of unity or representation theory. We will present the first completely combinatorial proof of such a result. We will also discuss colored versions of some known examples of the CSP involving triangulations of a convex polygon.

Bruce Sagan  
Michigan State University  
Department of Mathematics  
sagan@math.msu.edu

Yuval Roichman  
Bar Ilan University  
yuvalr@math.biu.ac.il

MS38

**Alternating Permutations and q-Euler Numbers**

The Euler number  $E_n$  counts the number of alternating permutations on the set  $[n]$ . It is well known that its exponential generating function equals  $\tan z + \sec z$ . For this reason,  $E_{2n}$  and  $E_{2n+1}$  are called secant numbers and tangent numbers, respectively. Certain polynomials arising in series expansions for zeros of generalized Rogers-Ramanujan functions provide a  $q$ -analog of the tangent numbers, which is part of a wider class of polynomials with similar combinatorial interpretations. In this talk, we will discuss various  $q$ -Euler numbers. This is joint work with Tim Huber from the University of Texas-Pan American

Ae Ja Yee  
Department of Mathematics  
Pennsylvania State University  
yee@math.psu.edu

Tim Huber  
Department of Mathematics  
University of Texas-Pan American  
hubertj@utpa.edu

MS39

**Learning Submodular Functions**

We study learning submodular functions in a distributional setting. A problem instance consists of a distribution and a non-negative, monotone, and submodular over  $\{0, 1\}^n$ . We are given  $\text{poly}(n)$  samples from this distribution, along with the values of the function at those sample points. The task is to approximate the value of the function to within a

multiplicative factor on points drawn from the distribution, with high probability. We prove several results for this problem.

Maria-Florina Balcan  
Georgia Institute of Technology  
ninamf@cc.gatech.edu

### MS39

#### Algorithm for Partitioning Hypergraphs and Submodular Systems

A  $k$ -cut of a hypergraph is a set of hyperedges whose removal divides the hypergraph into  $k$  connected components. The hypergraph  $k$ -cut problem is one of computing a minimum capacity  $k$ -cut. The submodular system  $k$ -partition problem is a problem of partitioning a given finite set  $V$  into  $k$  non-empty subsets  $V_1, V_2, \dots, V_k$  so that  $f(V_1) + f(V_2) + \dots + f(V_k)$  is minimized where  $f$  is a non-negative submodular function on  $V$ . In this talk, we review recent progress on algorithms for these problems.

Takuro Fukunaga  
Kyoto University  
takuro@amp.i.kyoto-u.ac.jp

### MS39

#### Submodularity in Combinatorial Optimization and Mechanism Design

Submodularity is a central phenomenon in many real-world world applications related to auctions and combinatorial optimization since it captures the economies of scale and the diminishing returns property. In this talk, I will first describe our results on the approximability of various covering problems under submodular cost functions, and then I will present our results on a submodular mechanism design problem that has applications in TV ad auctions.

Gagan Goel  
Georgia Institute of Technology  
gagang@cc.gatech.edu

### MS39

#### Matroids from Lossless Expander Graphs

A set function is partially defined by specifying some sets to take a very large value and some other sets to take a very small value. Can we choose values for the remaining sets so that the resulting function is non-negative and submodular? We show that this is possible if the specified sets are chosen using a lossless expander graph. There are several applications of this construction.

Nicholas Harvey  
University of Waterloo  
harvey@math.uwaterloo.ca

### MS39

#### Submodular Optimization Based on Combinatorial Convex Structures

It is known that a set function defined on the power set of a finite set is submodular if and only if the so-called Lovasz extension of that function is convex. In this talk, we consider constrained submodular minimization and maximization problems, and design approximation algorithms or practical heuristics based on the discrete convexity of

the submodular function.

Kiyohito Nagano  
Tokyo Institute of Technology  
nagano@is.titech.ac.jp

### MS40

#### Analyzing Social Networks of Zebras, Humans, and other Animals

Interactions among individuals are often modeled as social networks, individuals being nodes and interactions becoming edges. The traditional model is static; it aggregates interactions over time and discards all information about the timing and ordering of interactions. We have extended computational methods for social network analysis to explicitly address the dynamic nature of interactions among individuals. We will present our approach and demonstrate its applicability by analyzing social behavior of animals and its ecological implications.

Tanya Y. Berger-Wolf  
University of Illinois, Chicago  
tanyabw@uic.edu

### MS40

#### Robotics-Inspired and Dimension Reduction Methods for the Analysis of Protein Flexibility and Function

Proteins are involved either directly or indirectly in all biological processes in living organisms. It is now widely accepted that modeling protein flexibility is key to understanding protein function. This talk will present recent computational work on modeling the flexibility of proteins using a robotics-inspired approach. It will also discuss the development of non-linear dimension reduction methods tailored to proteins and the impact they can have in the analysis of protein flexibility.

Lydia Kavradi  
Rice University  
kavraki@rice.edu

### MS40

#### Predicting Evolutionary Trajectories in Principle and Practice

The discrete nature of DNA defines a large, albeit enumerable number of mutational trajectories between any two sequence variants. This raises the question of whether or not all such trajectories between given endpoints are also interchangeable in the eyes of natural selection. We demonstrate that for the evolution of an enzyme conferring heightened bacterial resistance to a widely used antibiotic, the answer is no: only a very small fraction of these many mutational trajectories are selectively accessible. We conclude by showing that a principled consideration of protein biology and biochemistry implies that such evolutionary constraints on enzyme evolution may be widespread.

Daniel Weinreich  
Brown University  
daniel\_weinreich@brown.edu

MS40

**Models for Teardrop Spots in 2-DE Gels**

Spot detection and spot matching are essential first steps for proteomics investigations based on two-dimensional gel electrophoresis (2-DE). Many 2-DE gels exhibit teardrop spots that cannot be modeled by current analysis software packages as those assume a Gaussian spot model. We investigated different spot models for these teardrop spots and developed a parametric algorithm that uses a variety of different models for spot intensity distribution.

Carola Wenk

University of Texas at San Antonio  
carola@cs.utsa.edu

MS40

**Fast Hash-Based Algorithms for Analyzing Tens of Thousands of Evolutionary Trees**

Evolutionary trees represent the genealogical relationships among a collection of organisms. Evolutionary trees have been used to improve global agriculture and understand disease transmission. Current techniques to reconstruct the evolutionary tree for a group of organisms can produce tens of thousands of candidate trees. We present a family of fast hash-based algorithms to quickly store and retrieve the genealogical relationships among large collections of evolutionary trees. Extensive experimental results show the effectiveness of our algorithms.

Tiffani Williams

Texas A&M University  
tlw@cse.tamu.edu

MS41

**Graphs without Subdivisions**

Graphs without a subdivision of a big graph do not behave as well as those without a minor of a big graph. This is partially because we do not really know what the graphs without a subdivision of a big graph look like. We shall discuss this issue. In particular, assuming some moderate connectivity, we can say something, which will be presented in this talk.

Ken-ichi Kawarabayashi

National Institute of Informatics, Japan  
k\_keniti@nii.ac.jp

MS41

**Algorithmic Metatheorems for Classes of Sparse Graphs**

Building on structural results for classes of sparse graphs, we design a linear time algorithm for deciding first-order logic (FOL) properties in classes of graphs with bounded expansion, which include proper minor-closed classes of graphs, and an almost linear time algorithm for deciding FOL properties in classes of graphs with locally bounded expansion, which include classes of graphs with locally bounded tree-width or locally excluding a minor. Our results also translate to corresponding classes of relational structures.

Zdenek Dvorak

Department of Applied Mathematics  
Charles University, Prague  
rakdver@kam.mff.cuni.cz

Daniel Kral

Charles University  
kral@kam.mff.cuni.cz

Robin Thomas

Georgia Tech  
thomas@math.gatech.edu

MS41

**Packing Minors Half-integrally**

Given graphs  $G$  and  $H$ , a  $k$ -half integral packing of  $H$ -minors in  $G$  is a collection of subgraphs  $G_1, G_2, \dots, G_k$  of  $G$  such that each vertex of  $G$  belongs to at most two of them, and each  $G_i$  contains  $H$  as a minor. We prove a conjecture of Thomas, showing that the Erdős-Pósa property holds for half-integral packing of  $H$ -minors. That is, for every graph  $H$  there exists a function  $f_H(k)$  such that every graph  $G$  either contains a half-integral packing of  $H$  minors or a set  $X$  of at most  $f_H(k)$  vertices such that  $G-X$  has no  $H$  minor.

Sergey Norin

Princeton University  
snorin@math.princeton.edu

MS41

**Open Questions on Well-quasi-orders**

A well-quasi-order is a reflexive and transitive relation where all descending chains and anti-chains are finite. There have been extensive advances in well-quasi-order theory for graphs and matroids over the past thirty years. Still, the area has many interesting concrete open problems and the natural progression toward testing the observation of Crispin Nash-Williams that all natural well-quasi-orders are better-quasi-orders has only a few highlights, in works of Igor Kriz and Robin Thomas, now nearly 20 years old. This short lecture will describe what seem to me to be the central questions and describe work by two of my former students, respectively, Yared Nigussie and Christian Altomare, toward transforming WQO into BQO, and formulating a conjecture including Richard Laver's theorem about scattered total orders and the graph minor WQO theorem.

Neil Robertson

Mathematics Department  
Ohio State University  
robertso@math.ohio-state.edu

MS41

**Toroidal Triangulations with few Odd-degree Vertices, Structure and Coloring**

The only known families of orientable triangulations with unbounded facewidth which are not 4-vertex colorable, have exactly two odd vertices and an edge joining them. They are possible counterexamples to Grünbaum conjecture: any simple triangulation of an orientable surface can be 3-edge-colored such that three colors appear in the boundary of each face. The conjecture was disproved for surfaces of genus 5 and higher. We will classify all triangulations of the torus with exactly two odd vertices and show the existence of Grünbaum coloring for each class.

Maryam Verdian Rizi

Simon Fraser University

mverdian@sfu.ca

Luis A. Goddyn  
Department of Mathematics  
Simon Fraser University  
goddyn@math.sfu.ca

#### MS42

##### Characterizations of Finite Geometries as Extremal Graphs

It is a classical problem in extremal graph theory to look for those graphs that maximize the number of copies of a subgraph  $H$  and are  $F$ -free; the Turan problem being the most well known example of such problem. In this talk I will discuss how the incidence graphs of some of the most important finite geometries, the so-called generalized polygons, arise as extremal graphs in the above sense. After an overview of some of the older results, I will address more recent results on characterizations of generalized triangles, i.e. projective planes. I will also show how these characterizations lead to some very interesting and intriguing geometric questions.

Stefaan De Winter

University of California, San Diego  
sdewinter@ucsd.edu

#### MS42

##### Computing the Edit Distance Function

We will describe the so-called edit distance function, a function of a hereditary property  $\mathcal{H}$  and of  $p$ , which measures the maximum proportion of edges in a density- $p$  graph that need to be inserted/deleted in order to transform it into a member of  $\mathcal{H}$ . We will describe a technique for computing this function and apply it to several hereditary properties  $\mathcal{H}$ , including those with no induced cycle of length  $\ell$ , for  $\ell \leq 9$ .

Ryan Martin

Iowa State University  
rymartin@iastate.edu

#### MS42

##### Extremal Multigraphs for Edge-colouring

For a multigraph  $G$ , the chromatic index  $\chi'$  of  $G$  is the minimum number of colours needed to colour the edges of  $G$  such that no two edges sharing a vertex have the same colour. There are many well-known upper bounds for  $\chi'$ , including bounds by Shannon, Vizing, Goldberg and Steffen. In this talk we explore the question of which multigraphs actually achieve these bounds. As part of the discussion we present a new partial characterization of those multigraphs achieving Vizing's upper bound, a result obtained jointly with P. Haxell.

Jessica McDonald

Simon Fraser University  
mcdonald.jessica@gmail.com

#### MS42

##### An Extremal Problem for a Constant Number of 1-factors

A simple result of Hetyei states that an  $n$ -vertex graph with a unique 1-factor has at most  $n^2/4$  edges. We extend this

result to when the graph has a small fixed constant number of 1-factors.

John Schmitt

Middlebury College  
Middlebury, VT  
jschmitt@middlebury.edu

#### MS43

##### Crystal Graphs and Dual Equivalence Graphs

In this talk, we present connections between crystal graphs for classical groups and dual equivalence graphs for the corresponding Weyl groups using tableaux combinatorics and local characterizations of the graphs.

Sami Assaf

MIT  
sassaf@math.mit.edu

#### MS43

##### Masks for Kazhdan–Lusztig Polynomials

The Iwahori–Hecke algebra is a deformation of the group algebra of a Coxeter group. In 1979, Kazhdan and Lusztig constructed a basis for this algebra that has found fascinating applications in geometry and representation theory. Unfortunately, the Kazhdan–Lusztig polynomials used to define this basis are given recursively, and no simple manifestly positive description is known for them, even in the symmetric group case. In this talk, we describe a framework developed by Deodhar that gives formulas for Kazhdan–Lusztig bases in terms of combinatorial objects called masks. We explain how to interpret a formula of Lascoux and Schützenberger for Kazhdan–Lusztig polynomials associated to co-Grassmannian permutations in terms of masks. This is joint work with Alex Woo.

Brant Jones

University of California at Davis  
brant@math.ucdavis.edu

#### MS43

##### Loop Groups, R-matrices, and a Birational Action of the Symmetric Group

I will talk about a birational action of the symmetric group on a polynomial ring, and in particular the invariants of this action. We encountered this action studying total positivity in loop groups, but the action also occurs as an R-matrix in Berenstein–Kazhdan's theory of geometric crystals, and also in the theory of discrete Painlevé dynamical systems as studied by Noumi–Yamada. This is joint work with Pavlo Pylyavskyy.

Thomas Lam

University of Michigan  
tflam@umich.edu

#### MS43

##### From Macdonald Polynomials to a Charge Statistic in Classical Types

The charge is an intricate statistic on words, due to Lascoux and Schützenberger, which gives positive combinatorial formulas for Lusztig's  $q$ -analogue of weight multiplicities and the energy function on affine crystals, both of type  $A$ . It has been a long-standing problem to generalize

charge to all classical types. I present a method to address this problem based on the recent Ram-Yip formula for Macdonald polynomials and the quantum Bruhat order on the corresponding Weyl group.

Cristian Lenart  
State University of New York at Albany  
lenart@albany.edu

#### MS43

##### Universal Characters for Spinor Representations

We introduce a family of symmetric functions with coefficients in the ring of integers adjoining a new element  $e$  with the property  $e^2 = 1$ , and investigate their properties. These symmetric functions can be used to describe the structure of the representation ring involving spinor representations of the Pin groups.

Soichi Okada  
Nagoya University  
okada@math.nagoya-u.ac.jp

#### MS44

##### List Colorings of $K_5$ -minor-free Graphs with Special List Assignments

A *list assignment*  $L$  of  $G$  is a function that assigns to every vertex  $v$  of  $G$  a set (list)  $L(v)$  of colors. The graph  $G$  is called  *$L$ -list colorable* if there is a coloring  $\varphi$  of the vertices of  $G$  such that  $\varphi(v) \in L(v)$  for all  $v \in V(G)$  and  $\varphi(v) \neq \varphi(w)$  for all  $vw \in E(G)$ . Joan Hutchinson mentioned the following question asked by Bruce Richter, where  $d(v)$  denotes the degree of  $v$  in  $G$ : Let  $G$  be a planar, 3-connected graph that is not a complete graph. Is  $G$   $L$ -list colorable for every list assignment  $L$  with  $|L(v)| = \min\{d(v), 6\}$  for all  $v \in V$ ? More generally, we ask for which pairs  $(r, k)$  the following question is answered in the affirmative. Let  $r$  and  $k$  be integers and let  $G$  be a  $K_5$ -minor-free  $r$ -connected graph that is not a Gallai tree. Is  $G$   $L$ -list colorable for every list assignment  $L$  with  $|L(v)| = \min\{d(v), k\}$ ? Recall that a *Gallai tree* is a graph  $G$  such that every block of  $G$  is either a complete graph or an odd cycle. We investigate this question by considering the components of  $G[S_k]$ , where  $S_k := \{v \in V(G) \mid d(v) < k\}$  is the set of vertices with small degree in  $G$ . We are especially interested in the minimum distance  $d(S_k)$  in  $G$  between the components of  $G[S_k]$ .

Daniel Cranston  
Virginia Commonwealth University  
Department of Mathematics and Applied Mathematics  
dcranston@vcu.edu

Joan P. Hutchinson  
Macalester College  
Department of Mathematics  
hutchinson@macalester.edu

Anja Pruchnewski  
Technical University of Ilmenau  
anja.pruchnewski@tu-ilmenau.de

Michael Stiebitz  
Institute of Mathematics  
TU Ilmenau  
Michael.Stiebitz@tu-ilmenau.de

Zsolt Tuza

Hungarian Academy of Sciences  
Budapest, Hungary  
tuza@sztaki.hu

Margit Voigt  
University of Applied Sciences  
Dresden  
mvoigt@informatik.htw-dresden.de

#### MS44

##### Choosing a Central Location for Multiple Deliveries

A delivery person must leave the central location of the business, deliver packages at a number of addresses, and then return. Naturally, he/she wishes to reduce costs by finding the most efficient route. This motivated the definition of  $(k-1)$ -stop-return distance by Gadzinski, Sanders, and Xiong, which has since become  $k$ -circuit distance. Given a set of  $k$  distinct vertices  $\mathcal{S} = \{x_1, x_2, \dots, x_k\}$  in a simple graph  $G$ , the  $k$ -circuit-distance of set  $\mathcal{S}$  is defined to be

$$d_k(\mathcal{S}) = \min_{\theta \in \mathcal{P}(\mathcal{S})} \left( d(\theta(x_1), \theta(x_2)) + d(\theta(x_2), \theta(x_3)) + \dots + d(\theta(x_k), \theta(x_1)) \right),$$

where  $\mathcal{P}(\mathcal{S})$  is the set of all permutations from  $\mathcal{S}$  onto  $\mathcal{S}$ . In other words,  $d_k(x_1, \dots, x_k)$  is the length of the shortest circuit through the vertices  $\{x_1, \dots, x_k\}$ . The 2-circuit distance is twice the standard distance between two vertices. We present results about the  $k$ -circuit radius,  $k$ -circuit diameter,  $k$ -circuit center and  $k$ -circuit periphery, with particular attention to the case  $k = 3$ . We also note some relationships between  $k$ -circuit distance and Steiner distance. Recall that the Steiner distance of a set of vertices is the minimum number of edges in a connected subgraph containing those vertices.

Linda Eroh  
Department of Mathematics  
University of Wisconsin at Oshkosh  
eroh@uwosh.edu

Raluca M. Gera  
Naval Postgraduate School  
rgera@nps.edu

Garry Johns  
Saginaw Valley State University  
glj@svsu.edu

Steven J. Winters  
University of Wisconsin Oshkosh  
winters@uwosh.edu

#### MS44

##### Triangular Line Graphs: Tools for Word Ambiguity Detection in Automated Search

One of the chief concerns of linguists is the pervasive ambiguity of natural language. At the lexical level, this manifests in the existence of the multiplicity of senses that a word may have. A natural representation for word relationships is an undirected graph  $G = (V, E)$ , where  $V$  is the vocabulary and vertices are adjacent in  $G$  if and only if the words they represent co-occur in a relevant pattern in the text. Ideally, the words in the same semantic field give rise to the vertices of a component of the graph. However, when words that have multiple senses are part of the graph,

the distinct semantic fields will not be given by the components of  $G$ . In response, Dorow *et al.* provided a tool

that transforms a graph showing word relationships into a new graph for which, generally, each individual component contains only one meaning of the polysemous words. They introduced the *link graph* of a graph whose construction is similar to the construction of line graphs. The link graph is identical to the triangular line graph, a special case of the H-line graph introduced by Chartrand *et al.*: the triangular line graph of  $G$ , denoted by  $T(G)$ , is the graph with vertex set  $E(G)$ , with two distinct vertices  $v_e$  and  $v_f$  adjacent in  $T(G)$  if and only if there exists a subgraph  $H \cong K_3$  of  $G$  with  $e, f \in E(H)$ . The properties of the  $T$  transformation have been studied by Jarrett for  $K_n$  and Dorrough for arbitrary  $G$ , with emphasis on stabilization of iterations of  $T(K_n)$  and  $T(G)$ , respectively. This presentation summarizes the main known results, and examines how the structural properties of triangular line graphs can aid predictions of the curvature metric on the triangular line graph, thereby helping to identify polysemous words.

Raluca M. Gera  
Naval Postgraduate School  
rgera@nps.edu

Pranav Anand  
Linguistics Department  
University of California Santa Cruz  
panand@ucsc.edu

Henry Escudro  
Department of Mathematics  
Juniata College  
escudro@juniata.edu

Craig Martell  
Computer Science Department  
Naval Postgraduate School  
cmartell@nps.edu

#### MS44 Generalization of the Friendship Theorem

The Friendship Theorem states that if any two people in a party have exactly one common friend, then there exists a politician who is a friend of everybody. In this paper, we prove the following generalization of the Friendship Theorem. If every pair of strangers in a party has exactly one common friend, then either 1) there exists a politician who is a friend of everybody; or 2) everyone has exactly the same number of friends and every pair of friends has exactly the same number of common friends; or 3) there exist two fixed numbers  $d$  and  $r$  such that each person has either  $d$  friends or  $r$  friends. (The corresponding graphs for Cases 2 and 3 are strongly regular graphs and bi-regular graphs, respectively.)

Jian Shen, Eugene Curtin  
Texas State University  
email: js48@txstate.edu, ec01@txstate.edu

Yong Jiang, Rong Qiu  
University of Science and Technology of China  
yjjiang@ustc.edu.cn, rqh@ustc.edu.cn

#### MS44 Domination in Functigraphs

Let  $G_1$  and  $G_2$  be copies of a graph  $G$ , and let  $f : V(G_1) \rightarrow V(G_2)$  be a function. Then a *functigraph*  $C(G, f) = (V, E)$  is a generalization of a permutation graph, where  $V = V(G_1) \cup V(G_2)$  and  $E = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2), v = f(u)\}$ . We study domination in functigraphs.

Linda Eroh  
Department of Mathematics  
University of Wisconsin at Oshkosh  
eroh@uwosh.edu

Raluca M. Gera  
Naval Postgraduate School  
rgera@nps.edu

Cong Kang  
Texas A&M University at Galveston  
kangc@tamug.edu

Craig E. Larson  
Virginia Commonwealth University  
clarson@vcu.edu

Eunjeong Yi  
Texas A&M University at Galveston  
yie@tamug.edu