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Matrices, Epidemics, and Olga Taussky Todd

By Barbara Lee Keyfitz

Pauline van den Driessche (right) made the first Olga Taussky Todd lecture a true tribute to Taussky Todd, according an important role to matrix theory in her talk on mathematical models of infectious diseases. Van den Driessche, shown here with Barbara Keyfitz, received an unexpected gift from Frank Uhlig at the close of the lecture: a photograph of Taussky Todd and her husband, the late John Todd.

Much of the mood of ICIAM '07 was celebratory, so perhaps it would be inaccurate to single out the first AWM–EWM–ICIAM Olga Taussky Todd Lecture as a celebration. Nonetheless, the crowd that assembled early Tuesday evening to hear Pauline van den Driessche's talk, "Matrices in Action for Epidemic Models," participated in an emotional as well as an intellectual high point of the Congress. The elegant lecture room in the ETH Hauptgebäude contributed to the mood, with its perfect facilities for a well-prepared talk that attracted many non-mathematician guests as well as enough delegates to snap up the 250 T-shirts contributed by Google.

The idea of a lecture at ICIAM, given by a woman "who has made outstanding contributions in applied mathematics and/or scientific computation" and co-sponsored by the Association for Women in Mathema-tics (AWM) and European Women in Mathematics (EWM), had been discussed for some time. The model is the very successful Emmy Noether Lecture now given at the International Congress of Mathema-ticians. After a warm reception of the idea by the ICIAM Board, Rolf Jeltsch not only gave the lecture a plenary spot at ICIAM '07, but also arranged for Google's sponsorship of the event. (Google also hosted a reception after the talk, where they gave away the famous T-shirts: black with "Google" in bright pink, the second "o" replaced by the female symbol ♀.) It was the ICIAM Board that suggested naming the series after Olga Taussky Todd, who, like Emmy Noether before her, had ties both to Europe and to the U.S., and triumphed over adversity to contribute to the research enterprise at the highest level. Both also became role models for women everywhere.

It took the concentrated effort of many people to turn a good idea into a great event. Marjo Lipponen and Laura Tedeschini of European Women in Mathematics helped to make the lecture a joint award. Fern Hunt set up a committee to write terms of reference, with Joyce McLaughlin as chair. An early decision was that all fields of applied and industrial mathematics, including computational science, would be appropriate for the award, but that special consideration for the first lecture should be given to the fields in which Olga Taussky Todd had worked. This helped, but only slightly, to whittle down the list of excellent nominations received. As chair of the selection committee (Angelika Bunse-Gerstner, Chandler Davis, Robert Tichy, Christine Bessenrodt, Richard Varga), I would also like to acknowledge the many supportive men who contributed time, ideas, and nominations. First among them is the late John Todd, who gave the project his blessing and encouragement. And all involved are grateful to Frank Uhlig, who presented a wonderful framed photograph of the Todds to van den Driessche.

Pauline van den Driessche has earned an international reputation for her research in both linear algebra and mathematical biology. In addition, she has served as an inspiration, and a mentor, to the numerous young researchers who have worked under her supervision as students or postdoctoral fellows. Her degrees are from Imperial Col-lege, London, and the University College of Wales. She was a faculty member at the University of Victoria, in British Columbia, from 1965 until 2006, when she became an emeritus professor and an adjunct professor in the Department of Computer Science. She was a co-recipient of the Bellman Prize (2002–03), and is the 2007 recipient of the Canadian Mathematical Society's Krieger–Nelson Prize. The Canadian MITACS group with which she works on the dynamics of epidemics has had a major impact in mathematical biology. Techniques developed by the group are used in formulating and analyzing models of epidemics such as West Nile virus and HIV/AIDS.

Even before a word is spoken about mathematical models, the subject of infectious diseases is guaranteed to fascinate an audience. Pandemic influenza outbreaks cause millions of deaths worldwide (20 million for the "Spanish Flu" of 1918–19, when the world was much less populated and less crowded than it is today), and these days reports of avian influenza are often in the news. Less commonly known is that public health agencies, including Health Canada and the U.S. Centers for Disease Control, use mathematical models, and are greatly interested in the development of new models, to predict epidemics and to devise control strategies. (When I contacted Pauline to discuss this article, she had just returned from a CDC summer school she helped to conduct.) And although the concepts presented in the talk were greatly simplified to suit a general audience, one suspects that researchers in this field routinely present stripped-down versions of their work to their nonquantitative clients.

Officials at the CDC probably do not get to learn about the legacy of Olga Taussky Todd and the role of matrix theory—and maybe this is not information they need—but van den Driessche's references to linear algebra in the talk at ICIAM went far beyond a mere gracious acknowledgment of the occasion. It does not hurt even advanced students of applied mathematics to be reminded that linear problems—and the language of matrices in which they are expressed—lie at the base of much mathematical modeling, even when realistic models are necessarily nonlinear.

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Van den Driessche focused on one central goal in the construction of models: estimation of $\mathcal{R}_0$, the average number of individuals an infected individual will infect. The simplest (S–I–R) model divides a population, assumed otherwise to be homogeneous and spatially mixed, into three classes (susceptible, infected, recovered), with rules for moving from one class to another. Rates of transmission are found from theoretical principles, empirical rules, and data. The problems get more interesting as they become more realistic, further dividing the population into classes by age, adding latency periods, and incorporating spatial inhomogeneity in some way. Most interesting of all is to include intervention strategies (vaccination or isolation, for example), and yet more classes, to allow for conditions like partial immunity. Mathematical models can quantify qualitative hunches, such as the change of $\mathcal{R}_0$ when travel between regions increases. The final part of the lecture examined nonlinear models more closely. Typically, these contain many equilibria, some ("disease-free") more desirable than others ("endemic"), and the study of their stability provides some elegant applications of bifurcation theory. Open questions include the best way to incorporate spatial effects, which may mean analyzing coupled partial differential equations, and how to model contact networks, which also means understanding better how infectious diseases are really transmitted.

Enthusiastic questioning of the speaker at the end of the talk indicated a desire on the part of the audience to learn more about this area. The following list should get interested readers off to a good start:

R.M. Anderson and R.M. May, *Infectious Diseases of Humans: Dynamics and Control*, Oxford University Press, Oxford, UK, 1991.

J. Arino, F. Brauer, P. van den Driessche, J. Watmough, and J. Wu, *Simple models for containment of a pandemic*, J. Roy. Soc. Interface, 3 (2006), 453–457.

J. Arino, C.C. McCluskey, and P. van den Driessche, *Global results for an epidemic model with vaccination that exhibits backward bifurcation*, SIAM J. Appl. Math., 64 (2003), 260–276.

O. Diekmann and J.A.P. Heesterbeek, *Mathematical Epidemiology of Infectious Diseases: Model Building, Analysis and Interpretation*, John Wiley & Sons, Hoboken, NJ, 2000.

H.W. Hethcote, *The mathematics of infectious diseases*, SIAM Rev., 42 (2000), 599–653.

Y.-H. Hsieh, P. van den Driessche, and L. Wang, *Impact of travel between patches for spatial spread of disease*, Bull. Math. Biol., 69 (2007), 1355–1375.

P. van den Driessche and J. Wat-mough, *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*, Math. Biosciences, 180 (2002), 29–48.

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