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A Wake-up Call for Squirrel Dynamics

By Barry A. Cipra

Sleep hath its own world,
And a wide realm of wild reality
—Lord Byron, *The Dream*

Evolution's incessant interplay of genetic roulette and harsh, selective reality has produced a vast panoply of Rube Goldberg-worthy designs and not a few apparent paradoxes of the sort so adored by creation pseudo-scientists. An example of the latter is the squirrelly hibernal behavior of *Spermophilus tridecemlineatus* and some of its mammalian kith. Like bears, *S. tridecem.*, commonly known as the thirteen-lined ground squirrel (*spermophilus* = "seed lover," an epithet that won't surprise anyone who's tried to keep squirrels from a backyard bird feeder), rides out the winter by lowering its body temperature and reduc-ing its oxygen consumption and heart rate. Unlike bears, though, the ground squirrel doesn't sleep soundly; rather, it raises its body temperature and stirs for a few hours every week or two. It's unclear what benefit is derived from these energy-intensive oscillations, technically known as "interbout arousals" (IBAs). The mechanism that regulates them is also unknown. Thanks to a mathematical model, however, researchers now know that the regulatory mechanism need not be overly baroque.

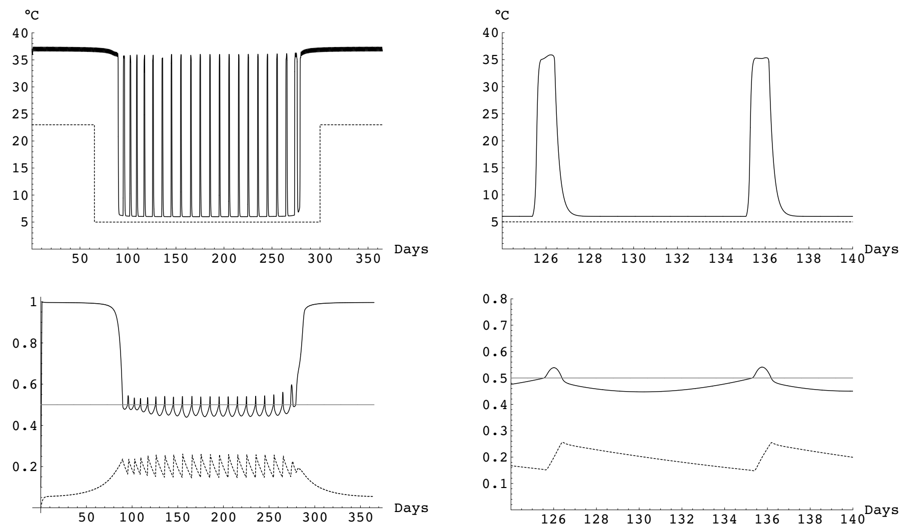

Figure 1. Left: *The solid, comb-like curve (top) shows the oscillations in core body temperature in a model of interbout arousals. (The dashed step function is the model's simplified version of ambient temperature.) The solid, scalloped curve and dashed, sawtooth curve (bottom) are the oscillations in hypothetical proteins B_p and A, respectively. The horizontal line at 0.5 is the switch point above which B_p induces arousal.* **Right:** *Blowups showing a pair of IBAs. Figure courtesy of Marshall Hampton and Matthew Andrews, from "A Simple Molecular Model of Mammalian Hibernation,"* Journal of Theoretical Biology, *Vol. 247, No. 2, 2007, 297–302.*

Biologist Matthew Andrews and mathematician Marshall Hampton of the University of Minnesota Duluth have shown that a relatively simple dynamical system based on the interaction of two hypothetical proteins can account for IBAs. Hampton presented their results in a poster session in Snowbird, at SIAM's 2007 Conference on Applications of Dynamical Systems. The poster received one of the five James Yorke "red sock awards" given at the meeting.

Hibernation is precipitated by a lowering of the body's steady-state "set" temperature. During the summer months, squirrels have roughly the same set temperature as humans, namely 37 degrees C (a.k.a. 98.6 degrees F). Come winter, though, the squirrel's set temperature drops like a rock, and its core body temperature follows, much like your house when you lower the thermostat as you go to bed. *S. tridecem.* winds up with body temperatures between 2 and 10 degrees C. The Arctic ground squirrel, *S. parryii*, takes things to an extreme, with measured core body temperatures as low as –3 degrees C. (Asked about the value of studying hiberation, researchers cite potential applications in cryosurgery and organ-transplant technology.) Arousal occurs when the set temperature rises. The central question is, What governs these changes?

James Yorke of the University of Maryland (far left) presented "red sock awards," a highlight of recent SIAM dynamical systems conferences, to 2007 poster presenters (from left) Daniele Avitabile (University of Surrey), Alethea Barbaro (UC Santa Barbara), and Marshall Hampton (University of Minnesota Duluth), whose work is discussed in the accompanying article. Other recipients were Jo Mason (University of Bristol) and Jue Wang (University of Wisconsin, Madison). Photograph by Hinke Osinga.

The heart of the Andrews–Hampton model is a pair of substances—presumably proteins—that interact in a temperature-dependent way, expressed in formulas of the form $A' = f(T,A)$ and $B' = g(T,A,B)$, where A and B are the substances. Roughly speaking, temperature-inspired production of A spurs the conversion of B to an altered form B_p (with $B + B_p = 1$ in normalized units), and it's B_p that "wakes" the critter by raising its set temperature. The variable T abbreviates a somewhat complicated combination of set, body, and ambient temperatures, along with a smattering of temperature-related parameters. The production of A is propelled by the onset of cold weather, but limited by the presence of A itself. This sets up the oscillations in the B/B_p complex and thereby the oscillations in body temperature (see Figure 1).

The model's body temperature curve is qualitatively similar to those from experimental measurements in various species. Just what substances play the roles of components A and B—if, indeed, things are as simple as the model imagines—remains unknown, although there are a couple of candidates. Knowing exactly how arousals arise should help evolutionary biologists understand how they arose in the first place.

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