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Two Theories of Relativity, All but Identical in Substance

Einstein 1905: The Standard of Greatness. *By John S. Rigden, Harvard University Press, Cambridge, Massachusetts, 2005, 173 pages, $21.95.*

Science and Hypothesis. *By H. Poincaré, Dover Publications, New York, 1952, 244 pages, $11.95.*

BOOK REVIEW

By James Case

According to John Rigden—identified on the dustjacket of *Einstein 1905* as an adjunct professor of physics at Washington University in St. Louis and the author of two previous popular science books—Einstein read Poincaré's 1902 book *La Science et l'Hypothèse* in 1904. If so, he must have read it in the original French, because English and German translations remained unavailable until 1905 and 1906, respectively. *Science and Hypothesis* (S&H) is the 1952 Dover version of the original English translation. In the book, Poincaré discussed a number of unanswered questions in physics, including the three that Einstein was to answer the following year. The questions concern the mysterious way sparks are emitted from the surfaces of metals bathed in bright light, the nature and causes of Brownian motion, and the failure of every attempt—most notably those by Michaelson and Morely—to measure the motion of the Earth relative to the surrounding ether.

Though he did in fact mention all three of the foregoing problems, Poincaré had remarkably little to say about any one of them. In his famous Paris lecture of August 1900, Hilbert had declared:

> "The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics."

Similarly, Poincaré was concerned in S&H primarily with foundational matters. Not until Kolmogorov succeeded in axiomatizing probability theory in 1930 did most interested parties cease to regard the subject as a branch of physics, rather than mathematics. Poincaré even repeated the ancient remark—which he attributed to an unnamed friend—that the (Gaussian) law of error distributions satisfies both experimentalists (who deem it a mathematical theorem) and theoreticians (who imagine it to be a law of nature).

Although Poincaré did not call for the axiomatization of either probability or mechanics, he considered clear presentations of both to be long overdue. "Treatises on mechanics," he complained, "do not clearly distinguish between what is experiment, what is mathematical reasoning, what is convention, and what is hypothesis." He further noted that, whereas the English taught mechanics as an experimental science, continental instructors tended to present it as an exercise in deductive a priori logic, full of definitions, theorems, and proofs. Even the fact that the point masses, rigid bodies, Newtonian fluids, and linearly elastic solids of classical mechanics were presumed to occupy portions of three-dimensional Euclidean space was, in his view, nothing more than a handy convention. The entire subject could be as coherently—if not as conveniently—reformulated in terms of any other geometry.

Alongside the chapters Poincaré devoted to the (then) five main branches of physics—mechanics, optics, electrodynamics, thermodynamics, and probability—the mere sentences about the problems that Einstein would treat seem insignificant. Only if the younger man had already been pondering those very questions—as, we know from his surviving correspondence, he was thinking about Earth's progress through the surrounding ether—do Poincaré's brief remarks seem capable of galvanizing effect. Yet perhaps even the briefest of mentions by someone of Poincaré's stature could have given a youthful Einstein exactly the encouragement he needed to resume dormant investigations. As fate would have it, Einstein had apparently abandoned even the pursuit of his doctorate during 1904, the Zurich faculty having rejected not one but two theses he had submitted for their approval.

Poincaré devoted an entire chapter of S&H to the subject of relative motion, with particular reference to a paper he wrote in 1898, arguing that man has no direct intuition of the simultaneity of two (instantaneous) events taking place at different locations. Indeed, he can but rarely certify that two distinct time intervals are of equal duration, given that observers in motion relative to one another typically disagree on the issue. Elsewhere, he pondered the cosmological implications of non-Euclidean geometries, and wondered how certain it is that light travels in straight lines through interstellar space. All in all, S&H would seem to present as fair, as comprehensive, and as accurate an account as is ever likely to be written of the state of mathematical physics following the discoveries of what Poincaré described as "the cathode rays, the X-rays, uranium and radium rays; in fact a whole new world of which none had suspected the existence."

The first of Einstein's epochal papers was received by the editor of *Annalen der Physik* on March 18, 1905, and published shortly thereafter (in volume 17, pages 132–148). It was the only one of his 1905 papers that he himself regarded as revolutionary, and was the one the Nobel committee cited when awarding him (retroactively in 1922) the 1921 prize in physics. Although the committee members did not yet accept the particulate theory of light and were unalterably opposed to both special and general relativity, they could not deny that Einstein's predictions concerning the photoelectric effect were abundantly confirmed by experiment. Ironically, the fullest confirmation came from the Caltech laboratory of R.A. Millikan, who (until his death in 1953) refused to accept Einstein's hypothesis concerning the existence of photons.

In the paper, Einstein argued that radiation should behave thermodynamically, "as if it were composed of mutually independent energy quanta of magnitude $h\nu$," and proceeded on that assumption to analyze the phenomena known as Stokes's rule, the photoelectric effect, and gaseous ionization. His results concerning the photoelectric effect, being the most detailed, were also the most easily confirmed. By assuming that a light quantum could instantaneously transfer its entire energy to a surface electron in metal, he was able to account for the observed fact that surface electrons are instantly ejected from any metal on which light of sufficiently high frequency is shone. Moreover, for any given metal, the kinetic energy of the ejected electrons should increase in proportion to the frequency of the incident light, and there should be a cutoff frequency, below which the light fails to eject any electrons at all. Millikan confirmed all this in 1916, much to his own chagrin. Not until 1923, through the experiments of Arthur Compton at Washington University in St. Louis, was the correctness of Einstein's light-particle theory demonstrated to the satisfaction of older physicists. It is not clear that Millikan ever fully accepted it.

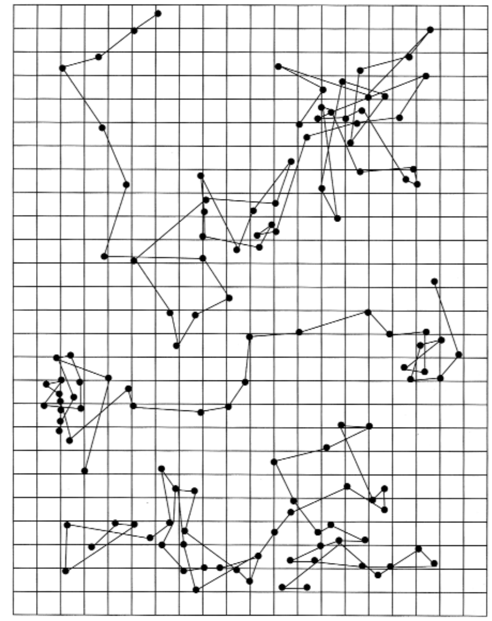

Figure 1. *Brownian motion. The positions of three granules of mastic were recorded every 30 seconds by Jean Perrin. Each 30-second position was connected by a straight line, which reveals the zig-zagging path of the granule. From* Einstein 1905.

The editor of *Annalen der Physik* received Einstein's paper on Brownian motion on May 11, 1905; it appeared later that year (in volume 17, pages 549–560). To some extent, it resembled his doctoral dissertation on molecular dimensions. Though completed in April 1905, the dissertation is not strictly part of Rigden's story because it was not published until the following year. Nevertheless, he has devoted a short chapter to its contents.

Einstein considered the motion of microscopic particles, with diameters on the order of 0.001 mm, suspended in a liquid such as water. Being aware of the dimensions of water molecules, he knew that a collision with just one molecule could not visibly alter the momentum of such a particle in suspension. Yet he predicted that the particles would undergo microscopically observable and statistically predictable motion. His reasons had to do with the fact that neither the density nor the velocity of water molecules would be uniformly distributed throughout the liquid. On the contrary, the microscopic equivalent of mobile high- and low-pressure weather systems would be continually forming and reforming. When struck on one side by a particularly powerful high-pressure system, or attracted on the other by a particularly powerful low-pressure system, a particle in suspension would naturally move away from the former and toward the latter.

As he had in his dissertation, Einstein calculated a diffusion constant to govern the migration rate of suspended particles away from regions of high concentration. He then developed a formula, involving only the temperature T of the suspending liquid, the radii r of the suspended particles, Avogadro's number N, and a constant k, now known as Boltzmann's constant, to specify the horizontal distance a particular particle in suspension might be expected to move during a specified amount of time. From his formula, he predicted that, in water at 17° Celsius, particles of diameter 0.001 mm would move an average distance of 0.006 mm in one minute. He closed by exhorting experimentalists to test this very specific prediction.

By the time he became aware of Einstein's prediction, Jean Baptiste Perrin was already at work on Brownian motion. In 1908, he carried out a series of experiments designed to test Einstein's prediction. In the most refined, he and a colleague took turns watching particles of mastic with radii of 0.0052 mm suspended in water at 17° Celsius through a microscope. Focusing on a particular particle and recording its position every 30 seconds for several minutes, they produced a series of plots like the three shown in Figure 1. The straight-line segments merely connect successive positions of the particle in question, sampled at 30-second intervals. When the experiments were complete and the data analyzed, Perrin concluded that the results could not "leave any doubt as to the exactitude of the formula proposed by Einstein."

Einstein's analysis, together with Perrin's confirmation, did something more. According to Max Born, writing in 1949, those two events "did more than any other work to convince physicists of the reality of atoms and molecules, of the kinetic theory of heat, and of the fundamental part of probability in the natural laws." For physicists, unlike chemists, still tended—at the dawn of the 20th century—to doubt the reality of atoms and molecules. Poincaré capitulated in about 1908, writing that "Atoms are no longer just a useful fiction; we can rightfully claim to see them since we can actually count them." Wilhelm Ostwald, an even more outspoken skeptic, conceded in 1909 that "We have recently come into possession of experimental proof of the discrete or grainy nature of matter." After about 1911, the year of the first Solvay Conference, such doubts were rarely expressed. Max Planck, who died in 1916 without ever conceding, may have been the last reputable scientist to deny the existence of atoms.

Almost any reader of *SIAM News* will have some familiarity with the content of Einstein's third and fourth 1905 papers, in which he announced his special theory of relativity. The fourth—received by the editor of *Annalen der Physik* on September 27, 1905, and published within the year—was little more than an appendix to the third, containing a derivation of what Rigden calls the most famous equation in history: $E = mc^2$. What Rigden does not mention is that, as recounted in [2], Poincaré had already outlined his own theory of relativity to the French Academy of Sciences in June of that year.

As Freeman Dyson explains in a 2003 review of [1] for *The New York Review of Books*, Einstein's and Poincaré's theories of relativity were almost identical in substance. Both agreed with what was known at the time about the behavior of fast-moving particles, and both made the same predictions concerning the results of future experiments. Both were based on the "principle of relativity" enunciated by Poincaré in S&H. Whereas the two theories were all but identical in substance, they differed significantly in form. Poincaré couched his argument in terms of the weightless, colorless "ether" he and his contemporaries had assumed—at least since Maxwell—to permeate all space as an elastic medium through which electromagnetic waves (including light waves) might be transmitted.

As Dyson puts it, Poincaré "loved the ether, and continued to believe in it, even when his own theory showed that it was unobservable." As a result, "His theory of relativity was a patchwork quilt. . . . Einstein, on the other hand, saw the old framework as cumbersome and unnecessary and was delighted to be rid of it." The superior clarity and simplicity of Einstein's argument appealed to his contemporaries, who saw to it that his name—rather than that of Poincaré—is permanently associated with the theory of special relativity.

In S&H Poincaré explained what leaders in the field were thinking and doing around the time at which Einstein became Einstein. Rigden's book explains what others knew at the time, and how Einstein built on the parts of that knowledge he deemed relevant. It provides a handy companion to John Stachel's compilation [2] of the actual papers (in translation) Einstein wrote during that banner year.

References

[1] Peter Galison, *Einstein's Clocks, Poincaré's Maps: Empires of Time*, W.W. Norton, New York, 2003.
[2] John Stachel, ed., *Einstein's Miracle Year: Five Papers That Changed the Face of Physics*,Princeton University Press, Princeton, NJ, 1998.

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