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Control Theorists Chart New Waters for Synchronized Swimmers

By Barry A. Cipra

Call it the department chair's dilemma: How do you get a swarm of independently minded minions moving in concert toward some useful goal? The problem may be inherently insoluble in a professorial setting, in part because the notion of academic motion is purely metaphoric. But encouraging signs of progress have emerged for a more literal analog. Researchers in control theory are perfecting a repertoire of micromanagerial algorithms necessary—or at least sufficient—for the task of steering groups of otherwise autonomous vehicles in the service of a common goal.

In an invited presentation at the Sixth SIAM Conference on Control and its Applications, held in New Orleans in July, Naomi Ehrich Leonard of Princeton University described progress she and colleagues have made in the theory of collective motion. Leonard is an expert on geometric mechanics and control, with a particular interest in cooperative control and collective motion of natural and engineered systems, from birds and fish to autonomous underwater vehicles. She was one of the key participants in the Autonomous Ocean Sampling Network II project, which field-tested a fleet of underwater gliders in Monterey Bay, California, in 2003, and she is the principal investigator for the Adaptive Sampling and Prediction (ASAP) project, which will conduct further tests in Monterey Bay next summer.

A central goal of these projects is to develop an autonomous system for observing and predicting ocean dynamics. The fundamental idea is for autonomous vehicles to act as mobile sensor platforms, coordinating their motion to collect as efficiently as possible the most useful data for assimilation into ocean-forecasting models. The oceanographic goal of the 2003 experiment was to study upwelling events. Control theoretically, the objective was to test algorithms that coordinate the movement of independent vehicles.

The researchers found that they could keep three gliders at the vertices of a linearly translating equilateral triangle over a run of approximately 16 hours (see Figure 1). In one experiment the gliders stayed roughly three kilometers apart during the entire run. With an additional control term in the second half of the run, one edge of the triangle was kept perpendicular to the direction of the group's motion—an impressive accomplishment, given that the gliders could communicate only when they surfaced, every couple of hours.

In a second experiment, the researchers successfully reduced the distance between gliders, moving in a triangular formation, from six to three kilometers, despite currents that rivaled the effective speed of the gliders. The results have helped shape plans for the month-long ASAP trials in 2006.

Meanwhile, the control theorists are developing an increasingly sophisticated theory of multi-agent control. In her talk, Leonard highlighted recent work with Rodolphe Sepulchre of l'Université de Liège in Belgium and her student Derek Paley in the department of mechanical and aerospace engineering at Princeton. The trio have studied an idealized problem concerning the stabilization of collective motion in the plane. Their approach provides a set of basic procedures, such as switching from linear to circular trajectories, that can be used in the design of more complicated maneuvers.

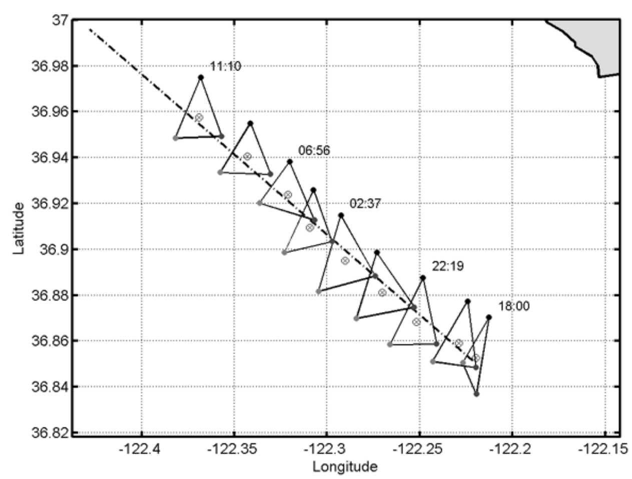

Figure 1. *Paths of three underwater gliders and their center of mass over a 16-hour sea trial in Monterey Bay, August 6, 2003. From E. Fiorelli et al., "Multi-AUV Control and Adaptive Sampling in Monterey Bay," 2004.*

The idealized problem is closely related to the theory of coupled phase oscillators. It posits point particles in the complex plane, moving with constant, unit speed. The use of complex variables is a notational convenience, allowing the velocity vector to be written in the form $e^{i\theta_k}$, where θ_k is the orientation of particle k. The control is a steering rate, $d\theta_k/dt = u_k$, which depends only on the relative positions and orientations of the particles, $r_{jk} = r_j - r_k$ and $\theta_{jk} = \theta_j - \theta_k$. (Getting the group to move in a particular direction requires the inclusion of a reference heading; the simplest way to do so is to couple the reference heading to ust one particle—in effect, to pick a leader and give it a compass.)

The key to the stability analysis begins with some simple algebra. The group's center of mass is the average of the individual position vectors $r = 1/N \Sigma r_k$. Its linear momentum (giving each particle unit mass) is $p = 1/N \Sigma e^{i\theta_k}$; p can also be interpreted as the centroid of the phasors (headings) of the particles. The magnitude of p lies between 0 and 1: It's at a maximum when the particles are traveling in synch, e.g., along parallel lines, and 0 when the particles are conspiring to be "anti-synchronized," e.g., equispaced and moving in a circle.

This suggests that the control be based on the gradient of the "phase potential," $K|p|^2$, where K is a parameter that can be positive or negative. Doing so leads to the phase control $u_k = \omega_0 - K/N \Sigma \sin \theta_{jk}$, where ω_0 is a constant angular rate. If $\omega_0 = 0$, the control produces a steady state consisting of straight-line trajectories—all in the same direction if $K < 0$, and scattered but in a balanced distribution (so that the center of mass stays fixed) if $K > 0$. If $\omega_0 \neq 0$, the steady state has the particles moving in circles—all in phase if $K < 0$ and out of phase if $K > 0$ (see Figure 2).

To get all the particles going around the *same* circle requires a "spacing" control; this drives the center of each particle's circle (defined as if it were controlled with $u_k = \omega_0$) to a common point. Even then, the particles may be scattered around the circle more or less randomly (as long as their center of mass is the center of the circle). Getting them evenly spaced is done by some additional massaging of the phase control.

Leonard and colleagues, in fact, have devised a systematic way to get any kind of symmetric spacing. With 12 particles, for example, they can achieve a "splay" state with all 12 particles evenly spaced, or states with six evenly spaced groups of 2, four groups of 3, three groups of 4, two groups of 6, or even one group of 12, i.e., the synchonized state (see Figure 3). (They have done a similar analysis for the synchronized state with particles traveling in straight lines.)

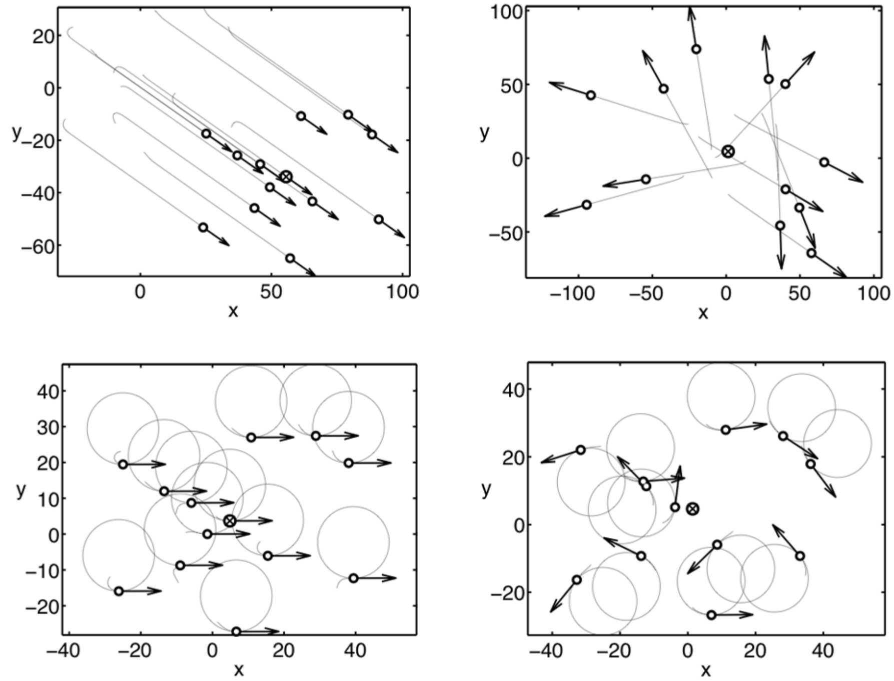

Figure 2. *Steady-state trajectories are straight lines (top) or circles (bottom), depending on whether* ω_0 *is zero or nonzero, and synchronized (left) or anti-synchronized (right), which depends in turn on whether* K *is negative or positive. From N. Leonard et al., "Collective Motion, Sensor Networks and Ocean Sampling," 2005.*

The formulas and analysis are simplest when it is assumed that the particles all "talk" to each other (i.e., particle k knows r_{jk} and θ_{jk} for all j). Leonard and colleagues, however, have also worked out the general case in which communications are more limited; the formulas involve pseudometrics defined with an inner product based on the Laplacian matrix of the communication graph.

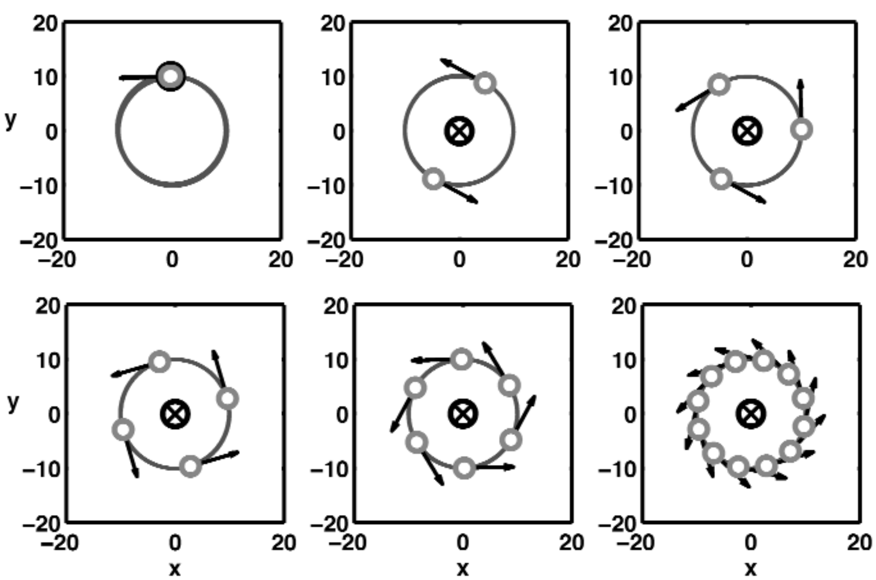

Figure 3. *Six simulations of 12 particles, each corresponding to a different symmetric pattern for the particles moving around a circle. Figure courtesy of Naomi Ehrich Leonard.*

The payoff of all this analysis is that it's possible to systematically stabilize steady patterns chosen from a parametrized family using simple controls. Thus, for example, a group of particles can circle in one location for a while, then zip off to another spot, circle there, expanding or contracting the circle as desired, or they can follow a zigzag course, much as a school of fish might do (see Figure 4). Put into practice with real vehicles, this should give researchers tremendous flexibility when it comes to adaptive sampling. Tourists in Monterey Bay next summer may catch an occasional glimpse of control theory at work: a dozen or more gliders in a data-gathering cavort.

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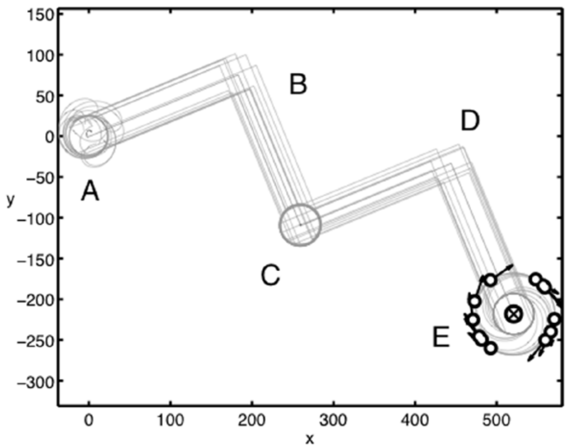

Figure 4. *Twelve point particles starting in random positions near (0,0) first congregate in a circle, then head off in an east–northeast direction (A), shift to the southeast (B), pause in a circular formation before repeating the zigzag (C and D), and end by enlarging the circle (E). From Rodolphe Sepulchre, Derek Paley, and Naomi Ehrich Leonard, "Stabilization of Planar Collective Motion, Part I: All-to-All Communication," 2005.*