From *SIAM News*, Volume 44, Number 4, May 2011

# Modeling and Simulating Tsunamis with an Eye to Hazard Mitigation

*Asked by* SIAM News *for a survey of the current state of mathematical/computational aspects of tsunami modeling and simulation, based mainly on sessions at the SIAM Conference on Mathematical & Computational Issues in the Geosciences (Long Beach, California, March 21–24), Jörn Behrens (University of Hamburg) and Randy LeVeque (University of Washington) quickly responded with the following article, along with some early simulation results (see Figures 1 and 2).*

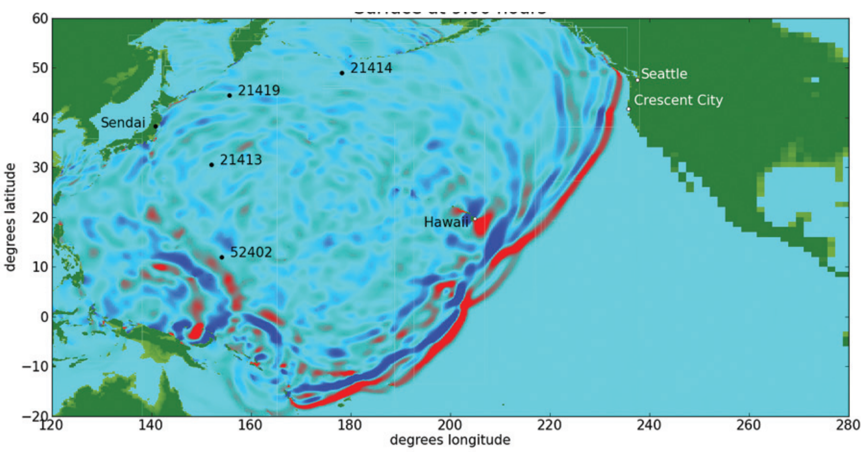
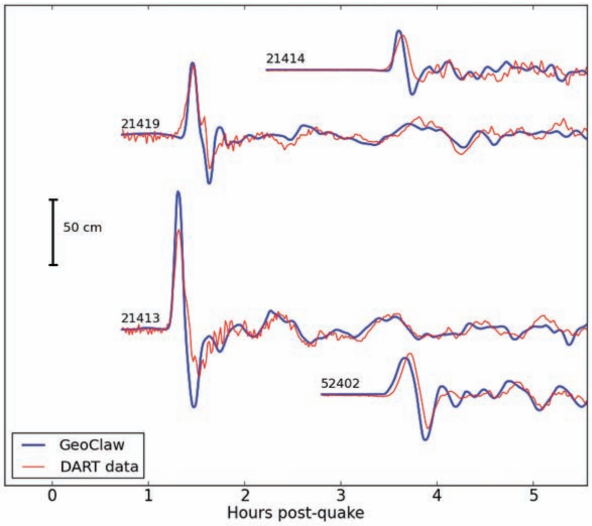


**Figure 1.** *The Great Tohoku tsunami of March 11, 2011, nine hours after the earthquake. This computation was performed with the GeoClaw software [2,4], using high-resolution finite volume methods and adaptive mesh refinement on rectangular grid cells. Dark red indicates a sea surface displacement of about +10 cm, dark blue about –10 cm.*

A Monday-morning invited lecture at this year's SIAM Conference on Mathematical & Computational Issues in the Geosciences could hardly have been more topical. Titled "The role of applied computational mathematics in an end-to-end near-field tsunami early warning system in Indonesia," the talk came just 10 days after the March 11 earthquake and subsequent tsunami on the coast near Sendai, Japan. Of course, the horrific pictures of the earthquake and tsunami are in our thoughts, and we extend our deep sympathy to the victims of this recent natural disaster.

As scientists, we need to focus on what can be learned from the event, which is now known as the "Great Tohoku tsunami," and how this knowledge can be applied to help mitigate future disasters. The Great Sumatra–Andaman tsunami of 2004, following decades without a major tsunami, seemed to be a once-in-a-lifetime event. But the intervening years have illustrated the fickleness of Poisson processes. Many other areas in the world are also potentially at risk, including the west coast of the United States, where a magnitude 9 earthquake along the Cascadia subduction zone causes a catastrophic tsunami every several hundred years.

**Figure 2.** *Surface displacement at selected DART buoys in the Pacific, with measured data shown in red and results from a simulation based on the earthquake source mechanism proposed by the UCSB seismology group shown in blue [5].*



The shock of the 2004 tsunami, claiming 230,000 casualties around the Indian Ocean, led to a resurgence of interest in tsunami modeling and made the research challenges known to a broader scientific audience, including many members of the SIAM community. This fruitful involvement was well illustrated at the SIAM conference, beginning with the lecture mentioned above, in which Jörn Behrens discussed new developments in tsunami modeling stemming from modern methods in applied mathematics and computational science. In the course of the week, a more detailed and comprehensive assessment emerged from at least 22 minisymposium and contributed presentations of tsunami-related work, including a three-part minisymposium, CSE Challenges in Earthquake and Tsunami Simulation.

■■■

Tsunamis are often well modeled by the two-dimensional shallow water equations, with source terms arising from the varying topography or bathymetry (underwater topography) over which the wave propagates. Although the equations are in only two space dimensions, the problem poses abundant algorithmic challenges (as summarized more completely in a article soon to appear in the 2011 issue of *Acta Numerica* [4]). One of the main difficulties is the inherently multiscale character of this phenomenon, with typical length scales ranging from thousands of kilometers (for the extent of an ocean basin) down to tens of meters (the resolution needed for accurate simulation of the general inundation behavior of a wave penetrating coastal regions).

To attempt a realistic assessment of a real event, a simulation method must bridge up to five orders of magnitude in each space and time dimension. Traditionally, this was achieved with distinct models for the different phases of the simulation, with clear separation of scales: A model for the initial source provided input data to the deep-ocean propagation model, which in turn provided data to a detailed (and very local) inundation model. Recently devised adaptive methods render this artificial separation unnecessary, without sacrificing the ability to perform the computations in a short time frame.

Related developments described at the conference include block-structured approaches, unstructured grids, and efficient tree-based data structures.

A whole new range of numerical methods has entered the field. Earlier operational tsunami simulation tools relied mainly on low-order finite difference approaches or classic finite element methods. By contrast, many of the new tools discussed at the conference use finite volume methods based on Riemann solvers or discontinuous Galerkin-type methods to approximate the governing equations for wave propagation. With their avoidance of numerical dispersion, these conservative methods are often very well suited to solving the nonlinear shallow water equations used in tsunami modeling.

An issue considered in several presentations is that of balancing the pressure-gradient term in the momentum equations against the source terms arising from the varying bathymetry or topography. If this is not done properly, spurious waves, orders of magnitude larger than the tsunami, can be generated even in the ocean at rest. Use of so-called well-balanced schemes avoids this serious problem.

Another major issue is the need to model the inundation of coastal regions, often kilometers inland for a large tsunami. Most of the recently developed software tools use some form of "wetting and drying" algorithm that allows grid cells to change dynamically from zero fluid depth (when dry) to positive depth (when flooded). The need to handle complex coastlines whose topology changes as islands or isolated lakes appear makes this approach appealing, preferable to attempting to track a moving fluid boundary, but care is required to avoid nonphysical negative depths.

Even the governing nonlinear shallow water equations are open to question in some contexts. For near-shore detailed simulations, three-dimensional non-hydrostatic effects sometimes play a crucial role, and various non-hydrostatic corrections have been considered. This is particularly true for small-scale modeling of flow–structure interactions, an area of growing interest. Dispersive terms are also thought to be important in some situations, particularly in modeling the propagation of short-wavelength tsunamis, such as those that arise from landslides rather than earthquakes.

The largest unknown in modeling a tsunami is generally the seafloor motion that generates the wave (the "source mechanism"; see Figure 3 for one example). In the case of the March 11 megathrust earthquake on the subduction zone off the coast of Japan, slip of the oceanic plate beneath the continental plate occurred on a collection of fault planes extending from near the sea floor to a depth of roughly 50 km. Different groups of seismologists have proposed several models based on seismic signals recorded at many stations around the world and, in some cases, on tsunami data as well. This is a challenging inverse problem, and the proposed models, when translated into seafloor motion and used to initialize tsunami simulations, often give quantitatively different tsunamis. One role of tsunami modeling is to help constrain the seismic models and determine the actual mechanism of the earthquake.

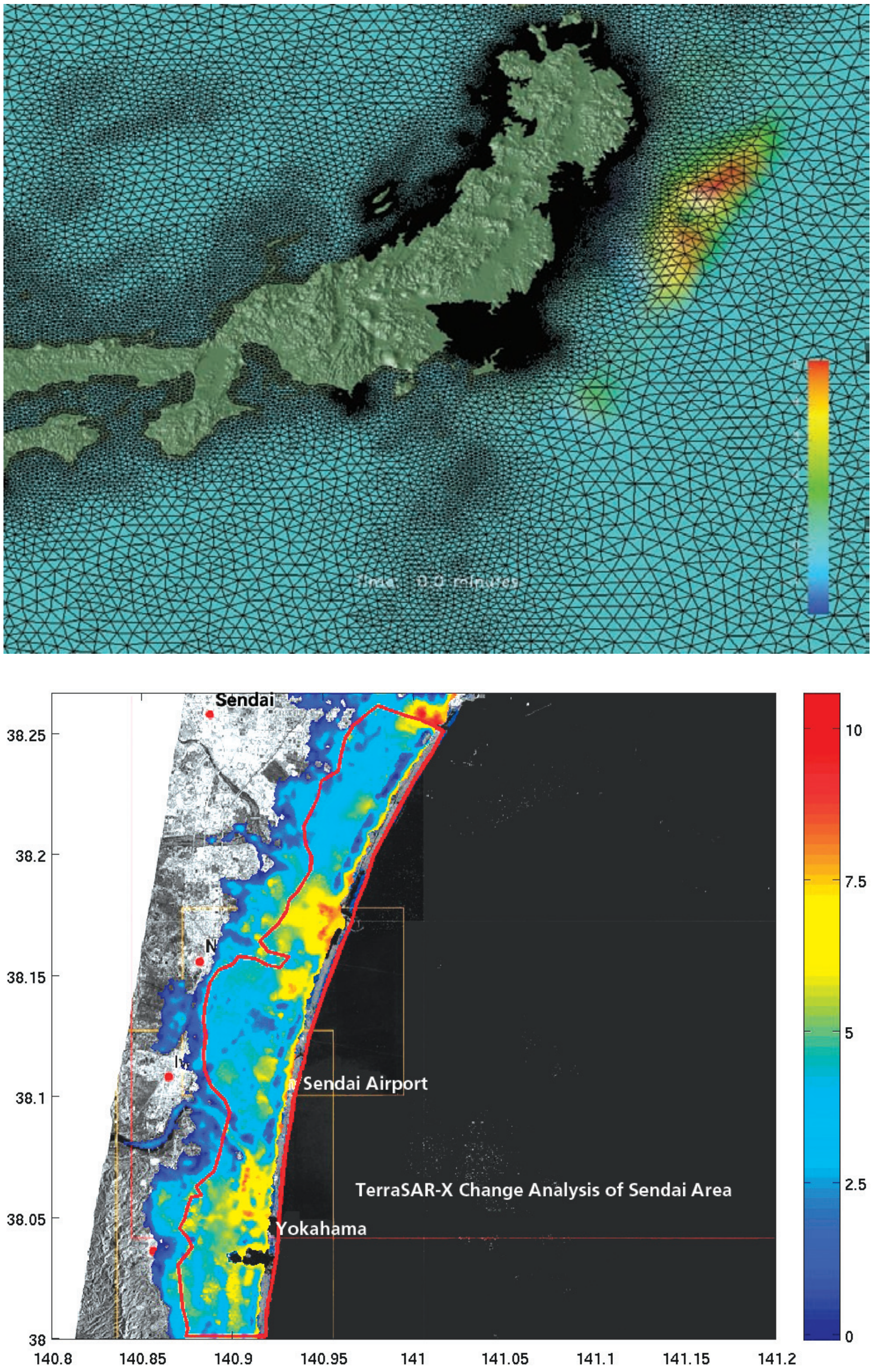


**Figure 3.** *Early simulation results for the Great Tohoku tsunami. Simulations of such an event are subject to uncertainty at every step in the simulation chain. Even though the extensive network of seismic and GPS dislocation sensors in Japan has permitted a good first estimate for the initial uplift (from Gavin Hayes, USGS, [6]) other source solutions have also been proposed. The initial condition together with an unstructured locally refined mesh is shown at the top. The unstructured-mesh operational tsunami code TsunAWI [3] used this uplift in wave propagation and inundation simulations; an inundation result is shown at the bottom. The topographic and bathymetric data, available only from public resources, are the source of additional uncertainties and may explain the overestimation of inundation compared to the red outline marking the flooded area. Uncertainty also arises in determining the inundation area, which was derived from satellite imagery without ground verification.*

■ ■ ■

Construction of complete seismic models often takes days or weeks after an event. The ability to provide rapid estimates of tsunami magnitude while a wave is still propagating, a current research aim, would assist in the issuing of tsunami warnings. Many early warning systems are now in place. For example, NOAA has positioned DART (Deep-Ocean Assessment and Reporting of Tsunamis) systems at many points around the Pacific rim. Pressure sensors on the ocean floor can measure the water pressure accurately enough to sense a tsunami passing by; the sensors are tethered to buoys that transmit real-time data (available on the web at http://www.ndbc.noaa.gov/dart.shtml).

The location of several DART buoys, along with a simulation of the Great Tohoku tsunami, is shown in Figure 1 (page 1); Figure 2 shows the time history of the surface at these locations, along with computational results from a seismic model constructed at UC Santa Barbara several days after the earthquake [5]. NOAA scientists approached the difficult inverse problem of estimating the source in real time from measured data by performing a least squares fit of the DART data obtained so far to a linear combination of precomputed responses from "unit source" events along the subduction zone.

A novel approach to tsunami early warning in the context of the Indonesian (near-field) Tsunami Early Warning System (InaTEWS) was presented in the early invited talk at the SIAM conference. In order to reduce the uncertainty within the first few moments of a tsunamogenic earthquake, diverse independent measurements (earthquake parameters, sea level information from deep-ocean and coastal gauges, and earth-crust deformation vectors taken from high-precision differential GPS stations on land) are used in combination. Certain quantities, such as the mismatch of precomputed scenario data to measurement data, and reliability and skill indices, can be derived from a simple but efficient and robust uncertainty propagation model that has been developed for an analog forecasting system [1]. With these quantities, it is possible to produce an accurate near-field tsunami hazard assessment within seconds, as demonstrated for several recent incidents along the Indonesian Indian Ocean coast.

In addition to modeling past or currently unfolding events, tsunami modeling can play an important role in preparing for future tsunamis, both in predicting worst-case scenarios and in performing probabilistic hazard assessments. While it is impossible to predict individual earthquakes, seismologists are often able to provide probability distributions for likely scenarios. Techniques from uncertainty quantification can be used to propagate these probabilities through tsunami simulations, producing probabilistic inundation maps that can be used by policymakers and emergency planners to assist in determining the communities most at risk, along with the best evacuation plans.

New momentum was injected into computational tsunami science after the 2004 Sumatra–Andaman tsunami, and significant achievements have been realized: New discretization schemes entered the field, new equation sets were introduced, novel approaches to high-performance computing emerged, and techniques for uncertainty propagation and quantification helped improve forecasts. These advances have led to more accurate planning tools that could help mitigate future events. As the devastating Great Tohoku tsunami reminds us, however, none of these tools can prevent disaster. The primary goal is to save as many lives as possible, which can be realized only through a combination of good science, good preparation, public education, and well-maintained warning systems that operate constantly.

## Acknowledgments

The authors would like to thank the Tsunami Modeling Group at Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, and in particular Sven Harig for providing some of the results presented.

## References

[1] J. Behrens, A. Androsov, A.Y. Babeyko, S. Harig, F. Klaschka, and L. Mentrup, *A new multi-sensor approach to simulation assisted tsunami early warning*, Nat. Hazards Earth Syst. Sci., 10 (2010), 1085–1100; doi:10.5194/nhess-10-1085-2010.

[2] M.J. Berger, D.L. George, R.J. LeVeque, and K.T. Mandli, *The GeoClaw software for depth-averaged flows with adaptive refinement*, Advances in Water Resources, to appear, 2011; http://arxiv.org/abs/1008.0455.

[3] S. Harig, C. Chaeroni, W.S. Pranowo, and J. Behrens, *Tsunami simulations on several scales: Comparison of approaches with unstructured meshes and nested grids*, Ocean Dynamics, 58 (2008), 429–440; doi:10.1007/s10236-008-0162-5.

[4] R.J. LeVeque, D.L. George, and M.J. Berger, *Tsunami modeling with adaptively refined finite volume methods*, Acta Numer., to appear, 2011; http://www.clawpack.org/links/an11/.

[5] Preliminary model III from http://www.geol.ucsb.edu/faculty/ji/big_earthquakes/2011/03/0311/Honshu_main.html.

[6] USGS, http://earthquake.usgs.gov/earthquakes/eqinthenews/2011/usc0001xgp/finite_fault.php.