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Obituaries: Lawrence E. Payne

Lawrence E. Payne, 1923–2011

Lawrence E. Payne, who received a BS in mechanical engineering in 1946 and an MS and PhD in applied mathematics from Iowa State in 1950 under D.L. Holl, died August 11, 2011, of acute lymphoma. He was born in McCleansboro, Illinois, a small farming community near Carbondale, in 1923. His first position was at the University of Arizona, followed by 14 years at the University of Maryland in the Institute for Fluid Dynamics and Applied Mathematics. He joined the faculty of Cornell University as a full professor in January 1965 and remained there until his retirement in 1994.

Although he had a very distinguished academic career, Larry was fond of telling his friends and students that his first employer was the Merit Shoe Company in Detroit, where he was hired in July 1941 as a shoe salesman. In his usual self-effacing style, he neglected to mention that by the time he left the shoe business in February 1943, he was the assistant manager. He then joined the U.S. Navy, serving a little over three years before returning to Ames for graduate school.

Larry was a recognized international leader in partial differential equations, especially in isoperimetric inequalities, improperly posed problems of mathematical physics, and elasticity.

The classical isoperimetric inequality states that among all planar domains of a given area, the circle has the smallest perimeter. Today, an inequality depending on several physical and geometric quantities is called isoperimetric if it is optimal, in the sense that equality is attained for some domain. The study of such inequalities began in the 19th century. The first results in this direction were inequalities between the torsional rigidity of a cylindrical beam, the fundamental frequencies of fixed membranes and clamped plates, and the area of their underlying domains. These and related inequalities are described in *Isoperimetric Inequalities in Mathematical Physics*, the pioneering book of Pólya and Szegö, which greatly advanced research in the field and which also served Larry as a source of in-spiration for his important contributions to this topic.

Larry's fine instincts for classical analysis and his computational skill combined with geometrical intuition enabled him, mostly in collaboration with Hans Weinberger, to extend significantly some old results, to derive new inequalities, and to discover new relations between physical and geometrical quantities. His papers are gold mines of clever tricks. Among his most quoted is a paper with Weinberger on the optimal Poincaré inequality for convex domains where the second eigenvalue of a free membrane is estimated in terms of the diameter of the domain [14]. Larry described the state of the art as of 1967 in a survey paper, "Isoperimetric Inequalities and their Applications," [10], which appeared in *SIAM Review* in 1967 and for which he received the Steele Prize of the American Mathematical Society.

The Payne nodal line conjecture dates back to that 1967 *SIAM Review* paper. The conjecture states that the nodal line of any eigenfunction that corresponds to the second eigenvalue of the Dirichlet Laplacian in a two-dimensional domain B must contain at least two boundary points. Larry showed this to be true when B intersects any vertical line in a single segment or not at all, and when B is symmetric about one vertical line. The conjecture was shown to be true when B is convex [1,9], and a multiply connected domain was found for which it is false [5], but the question of whether the conjecture is true for all simply connected domains is wide open. A recent review of the subject and its generalizations can be found in [6].

A second important conjecture grew out of joint work by Payne, Pólya, and Weinberger. In their paper [12], they proved the bound $\lambda_{n+1}/\lambda_n \leq 3$ for the ratio of any two consecutive eigenvalues of any two-dimensional fixed vibrating membrane. They conjectured that the ratio λ_{n+1}/λ_n is, in fact, bounded above by the ratio of the second to the first eigenvalues of a circular membrane, so that the inequality is isoperimetric, and that $[\lambda_2 + \lambda_3]/\lambda_1$ is bounded by twice this bound. The first part was shown to be true for $n = 1$, 2, and 3 [3]. Whether the first part is valid for $n \geq 4$ and whether the second part is valid are open questions.

Among the subspecialties in PDEs that attracted Larry's interest and benefited greatly from his attention was that of problems that are improperly or non-well posed. He was a pioneer in this field, which has since grown into an important area of research. A problem is properly or well posed if (i) it has a solution, (ii) the solution is unique, and (iii) the solution depends continuously on the data for the problem. This formulation goes back to Hadamard, who gave as an example the initial boundary value problem for the Laplace equation $u_{xx} + u_{yy} = 0$ in a half strip. Hadamard remarked [4] that such problems were not likely to be of much physical interest, and the subject consequently languished for a number of years. Larry contributed several results that contradicted Hadamard's assertion. This and many other problems are discussed in Larry's seminal monograph [11], a highly readable volume that did for this topic what his 1967 *SIAM Review* article did for isoperimetric inequalities and eigenvalue problems.

Among the ill-posed problems that have interested engineers, scientists, and mathematicians are inverse problems. Larry and his students obtained a wide variety of uniqueness and continuous-data-dependence results for Cauchy problems for elliptic equations, equations of mixed type, systems of elliptic equations, and evolutionary equations, such as the backward heat equation. The latter results rely, at their heart, on the demonstration that some positive, continuous functional of the solution has a convex logarithm.

Another area in PDEs profoundly influenced by Larry concerns questions related to the global existence of solutions of semilinear wave equations. He and Howard Levine observed the following: Suppose that $f(t)$ is a positive, differentiable function whose reciprocal, $1/f(t)$, is concave down, and that $f(0) > 0$. Then the graph of the tangent line to the reciprocal at $t = 0$ has slope $-f(0)/f^2(0)$ and must cross the t axis at $f(0)/f(0)$. Therefore, the graph of $1/f(t)$ must cross the axis at some point to the left of or at $f(0)/f(0)$. Consequently, the graph of $f(t)$ must have a vertical asymptote at this crossing point. Using this observation, they were able to establish simple conditions under which certain types of semilinear waves or semilinear parabolic equations do not possess global solutions [7,8]. Larry and David Sattinger later used potential well theory to address questions of the same type [13]. Many extensions and applications of these results have appeared during the last 40 years [2].

Larry was famously modest about his accomplishments. Asked about his influence in ill-posed problems for PDEs, he attributed it to luck: He got into the field before the easy problems were solved, he said, and he was smart to get out before he had to solve the really hard problems. In reality, he had an uncanny ability to find good problems to work on. His choice of problems, together with the techniques he developed to solve them, are lasting contributions to the fields in which he worked.

He was the author or co-author of nearly 300 articles and two books. He was the primary adviser of 15 PhD students, some of whom came from such faraway places as Iran, South Korea, and South Africa.

Larry was the leader in building the applied mathematics program within the Cornell Mathematics Department and in establishing the Center for Applied Mathematics there. His quiet, persuasive manner enabled him to succeed where those with more dynamic but less politic personalities had failed. The center as well as the group in numerical analysis and partial differential equations came to be considered by many as among the best in the world.

In addition to the 1972 Steele Prize, in 1990 he received an honorary Doctor of Science degree from the National University of Ireland. Friends and colleagues from all over the world attended a three-day conference in his honor held at Cornell in October 1990.

In addition to Larry's numerous scientific achievements, his colleagues the world over held him in the highest esteem as a friend, adviser, and colleague. He gave selflessly of his time and energy and was one of the kindest and most patient people any of us ever met. These are, without a doubt, sentiments with which all who knew him will agree.—*Catherine Bandle, University of Basel; Howard Levine, Iowa State University; Fadil Santosa, University of Minnesota; and Hans Weinberger, University of Minnesota.*

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