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A Subversive Model of Particle Physics

By Barry A. Cipra

In a fitting homage to "an extraordinary man," Alan Newell of the University of Arizona gave the inaugural Martin D. Kruskal Prize Lecture at this year's SIAM Conference on Nonlinear Waves and Coherent Structures, held June 13–16 at the University of Washington, Seattle. Newell described some of his ideas for an alternative to the Standard Model in particle physics, based on phase transitions in pattern-forming systems, taking seriously the slogan on a T-shirt his longtime friend and colleague was known to wear: "Subvert the Dominant Paradigm."

"Martin Kruskal was one of kind," New-ell says. "He would always encourage doing slightly unusual things." Dogged determination was another of his attributes, Newell recalls. "Once a problem gripped him, he never let go."

Both traits were key to Kruskal's signature achievement: the discovery of solitons (see sidebar). The particle-like behavior he and Norman Zabusky observed in numerical solutions to a classic wave equation inspired a paradigm shift in mathematical physics. Fifty years ago, integrable systems of differential equations were virtually synonymous with linearity; within a decade, integrable systems of nonlinear equations had taken center stage, a position the nonlinear theory has occupied ever since. "Linear theory," Newell quips, "is a rest home for applied mathematicians."

Newell makes no claim that his own observations will shift any paradigms. But if they do, it will be due in part to the example of a man who was a role model for subversive thinking.

Whence Symmetry

The Standard Model is the outrageously successful theory that accounts for three-fourths of physics. (It unifies the strong, weak, and electromagnetic forces; only gravity escapes its embrace.) In the broadest of outlines, it posits quantum fields based on certain unitary and special unitary groups, from which spring the bosons and fermions of the observed (and sometimes unobserved) universe. Its most recent tour de force is the apparent spotting of the long elusive Higgs boson, which purports to explain why some particles have inertia or mass—the ratio of momentum to speed, or energy to speed squared.

A key word in the Standard Model is *symmetry*. In accord with the principle discovered by Emmy Noether nearly a hundred years ago, conserved properties stem from symmetries (put very loosely: Things that don't change depend on changes that effect no change). The Standard Model sports a global spacetime playground based on translational and rotational symmetries, dotted with the swings, seesaws, and monkey bars of the symmetry groups SU(3), SU(2), and U(1). These symmetries (along with a batch of "accidental" ones that seemingly come for free) enforce a physics of conserved energy and momenta (both straight and angled), spin and charge (electric, color, and weak hyper). In particular, the unitary groups give rise to "fractional" spin and charge: quantities that require a full two or three turns to remain invariant.

To Newell, those groups seem somewhat jerry-rigged: They're posited precisely to give the results the theory needs. Accordingly, he set out to see if it would be possible to start with nothing more than the symmetries of translation and rotation and "stress" them into producing objects with fractional invariants. The Standard Model already makes use of symmetry breaking, of course. (It's part of how the Higgs boson accomplishes its massive undertaking.) But Newell's aim is to squeeze the local gauge symmetries out of the global symmetry of spacetime.

It doesn't take a degree in theoretical physics to imagine that it might be possible. Evidence for spatial symmetry breaking is as plain as the ridges on your fingertips. "We have such systems all over the place," Newell says. "They're called pattern-forming systems."

The "grand-daddy" of pattern-forming systems, Newell notes, is Rayleigh–Bénard convection, with its roiling regularity. A thin layer of fluid, heated from

New (and Old) Wave Math

Martin Kruskal (1925–2006) worked on a wide range of problems in pure and applied mathematics, but is best known for the discovery of solitons. Made over the course of a decade in collaboration with Norman Zabusky, Robert Miura, John Greene, and Clifford Gardner, the discovery began with an analysis of the now-famous Fermi–Pasta–Ulam problem. In a numerical experiment, the researchers sought to gauge the effect of weak nonlinearity in a system of coupled harmonic oscillators. They had expected the system to "thermalize," with energy in the lowest Fourier mode flowing irreversibly into higher modes—indeed, the original idea for the experiment was to study the *rate* of thermalization. Instead, they found that the energy never went beyond the first few modes, and reverted to the first mode in almost periodic fashion.

Kruskal and Zabusky found that the continuum limit of the FPU system led to the Korteweg–de Vries equation, already familiar from the theory of uni-directional shallow water waves. The KdV equation had originally been introduced to account for the existence of "solitary" waves, famously first observed in 1834 by the Scottish engineer John Scott Russell. It has a simple solitary wave solution in the form of a hyperbolic secant whose speed varies with its amplitude: The taller the wave, the faster it goes. But Kruskal and Zabusky found additional gold in the KdV equation.

In a four-page paper published in *Physical Review Letters* in 1965, they reported the results of their own numerical experiments, in which they found that a single-crested cosine wave (in a domain with periodic boundary conditions) quickly decomposed into a train of solitary waves of different heights and, hence, different speeds. (Technically speaking, the hyperbolic secant is not a solution of the KdV equation on a finite interval with periodic boundary conditions, but only the greatest of sticklers—like Kruskal—ever sweats the exponentially small stuff.) Because of their varying heights, the solitary waves—there were seven of them in the numerical experiment—traveled at different speeds, and thus had to interact as they traveled around and around the periodic domain. That's where the real surprise popped up and solitons earned their name: Instead of merging like raindrops or shattering like glass balls, the solitary waves emerged from the collisions intact.

It's easy to be blasé these days about nonlinear waves that interact like particles, but it was an eye-opener in the 1960s. In their write-up, Kruskal and Zabusky underlined the key observation: "Here we have a nonlinear physical process in which interacting localized pulses do not scatter irreversibly."

Kruskal realized that conserved quantities had to be lurking within the KdV equation. In fact, there are infinitely many. The KdV equation, moreover, turned out to be prototypical in this regard: Kruskal and colleagues started finding integrable systems under virtually every nonlinear rock they examined. The theory grew from a cottage industry to one of the main themes of modern mathematical physics.—*BAC* below, becomes a rhythmic sea of stripes, with hot fluid forcing its way up along one edge of each stripe and denser, cool fluid diving down along the other edge. The width of the stripes (or honeycombs, also a frequently observed pattern) is determined by the physics, but the orientation is a more or less random choice. Indeed, different portions of the fluid may opt for different orientations. The upshot is that, in striped patterns, "phase grain boundaries" are widely seen, along with point defects known as concave and convex disclinations (see Figure 1). On your fingertips, these point defects are known as triradii and loops.

For these two-dimensional disclinations, an imaginary two-headed arrow perpendicular to a stripe, when transported continuously in a circle around the point defect, turns only halfway around, a natural analog to the fermion spin of 1/2. Newell and colleagues have worked out an extensive 2-D theory of "phase diffusion" to account for the stable patterns observed in phase grain boundaries. When the theory is taken into higher dimensions, the analog of concave disclinations leads to loops that twist by multiples of $2\pi/3$. In one case, the result is a defect with index $\pm 2/3$, which Newell calls a "pattern up quark"; in another case, the result has index $\pm 1/3$, for a "pattern down quark." In the convex case, the 3-D analog is a "pattern lepton" with index ± 1.

Figure 1. *What goes around comes around. Could point defects known as concave (left) and convex (right) disclinations, which arise in pattern-forming systems, account for fractional charges and spin in the Standard Model of particle physics?*

Despite the suggestive names and results, "this is *not* an attempt to replace the Standard Model," Newell says. It's "just a little game" that's "more than likely destined for the dustbins of history." Nonetheless, it's interesting to see what happens to the simplest symmetries when a system is stressed far from equilibrium, he adds. "There's a lot of interesting geometry. And there are so many open questions."

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