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SIAM's UK/Republic of Ireland Section Meets in Bath

By Marco Marletta and Alison Ramage

The 2003 Annual Meeting of the United Kingdom and Republic of Ireland SIAM Section was held on January 10. Forty-five mathematicians from a wide variety of backgrounds assembled at the University of Bath for a one-day conference organised by David Parker (president), Ivan Graham (vice-president), and Alison Ramage (secretary/treasurer).

Helen Byrne of the University of Nottingham opened the meeting with a talk on multiphase models of tumour growth. The cell types in a tumour, and their growth, depend on the presence of nutrients, on chemicals expressed by other cells, and on mechanical constraints, such as the stiffness of surrounding tissues or drag between cells of different types. In the first part of her talk, Byrne developed a model for a two-phase tumour in which the cell types are classified as tumour and nontumour. In the second part, she examined the phenomenon of tumour encapsulation; this requires a three-phase model, the third cell type being collagen.

The mechanisms by which tumours become encapsulated are not fully understood; the two main competing hypotheses are (a) compression of surrounding tissue (mechanical action only), and (b) excess collagen production in surrounding tissue resulting from the presence of tumour cells. Modelling these processes with a coupled system of nonlinear parabolic PDEs, Byrne was able to show that the first hypothesis could provide a possible explanation of what has been observed experimentally, but that the second does not lead to the formation of a capsule (although tumour growth can be suppressed). She concluded that, in practice, a balance between the two effects is likely.

Doug Arnold of the University of Minnesota then spoke on differential complexes in numerical analysis. Arnold showed that the use of differential complexes, such as the well-known (to differential geometers) de Rham complex, gives a systematic method for constructing mixed finite element formulations satisfying the LBB stability conditions.

As his simplest example, Arnold considered the discretisation of the elliptic partial differential equation $-\nabla \cdot (C\nabla u) = f$, treating the quantities u, ∇u, $C\nabla u$, and $-\nabla \cdot (C\nabla u)$ as variables. Within this framework, he represented each variable using finite element spaces that relate to each other through the actions of precisely the same differential operators that express the relations between the original variables themselves. In this way, he obtained a discretization satisfying the LBB stability conditions.

The application of this principle to more complicated PDEs is highly nontrivial, and the construction of a stable mixed finite element formulation for the equations of elasticity was published only very recently (2001). In the future, Arnold hopes to be able to tackle even more demanding problems arising in numerical general relativity.

John Toland (University of Bath) opened the afternoon session with a talk on analytic theory of global bifurcation. He began with a question: Given real Banach spaces X and Y, an open set $U \subseteq \mathbb{R} \times X$ and a real analytic map $F : U \mapsto Y$, what can be said about the structure of the set E of solutions of the equation $F(\lambda,x) = 0$? Toland showed that under fairly mild hypotheses, in a deleted neighbourhood of a singular point the set E consists either of no points (isolated singularity) or of a finite *even* number of branches that can be matched in *unambiguous pairs*. In particular, it is not possible to have an odd number of solution branches in a neighbourhood of a singular point. Although there was little time to discuss the ramifications for Stokes wave theory, many members of the audience were clearly excited about the possibilities.

In the second "mathematical modelling" talk of the day, John Billingham (University of Birmingham) described joint work with Andy King on solid oxide fuel cells, which convert energy from combustion of fuel directly into electricity. A typical planar fuel cell consists of a layer of zirconia sandwiched between two electrodes, operating at around 700°C. Fuel is fed to one electrode and oxygen to the other; during the combustion, O_2^- ions pass through the zirconia, causing a current to flow between the electrodes. The problem was modelled in a lubrication approximation (the zirconia is very thin), leading to a system of three coupled nonlinear ODEs in the radial variable. Asymptotic solutions were developed, leading to a nonlinear eigenvalue problem that was solved by shooting. The model enabled Billingham to show that the steam-reforming reaction is the most important in determining the performance of the cell.

He also considered tubular fuel cells, which are much easier to manufacture. In them, a burnt region travels along the tube, and heat transfer occurs primarily by convection, although the governing equation is parabolic. Matched asymptotic solutions were obtained; a fully numerical treatment is planned for the future.

Philippe Toint of the University of Namur concluded the academic proceedings with a talk titled "The Filter Idea and its Applications to Nonlinear Equations and Nonlinear Least Squares." The purpose of filter methods is to overcome some of the disadvantages of penalty function methods for constrained nonlinear optimisation. A filter (or "waffle") is constructed by rejecting any point that is dominated by its predecessors in the sense that (a) it gives greater constraint violation, and (b) it also gives a worse value of the objective function.

Filter methods can also be adapted to the problem of finding feasible regions, by using more dimensions for the filter space. Toint showed that when implemented with a Gauss–Newton method, the filter idea substantially outperforms trust region methods. He also made the point that these results illustrate the power of non-monotone methods, which allow more freedom for the fast convergence of Gauss–Newton, even with constrained problems. The new algorithms are implemented in the FILTRANE software, to be included in the GALAHAD package.

The meeting ended with a brief business session for SIAM members. The first item on the agenda was the election of a president and

vice-president, as the terms of David Parker and Ivan Graham end on March 31. The members expressed their thanks to Parker and Graham, and nominations were solicited for their replacements, with elections scheduled for February. The main topic of discussion was the possibility of a joint meeting in the UK co-sponsored by SIAM.

Further details about the UKIE SIAM Section can be found at http://www.maths.strath.ac.uk/ukiesiam.

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