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Synthetic Aperture Radar

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Synthetic Aperture Radar image of Washington, DC. From Sandia National Laboratories.

The process of remote sensing can be thought of as a flow:

data → image → image features → information.

The features of an image include aspects like textures, edges, and shapes. The desired information can help us answer important questions about the image: Is this tumor malignant or benign? Is this a school bus or a missile launcher? Much of the imaging science conference focused on the process of extracting features from images. The emphasis here, and in my invited talk at the conference, is on the first step in the diagram, namely the transformation of raw data into an image. This step, of course, has implications for the downstream processes.

Synthetic Aperture Radar (SAR) is a very successful remote sensing technology that has been developed entirely within the engineering community. It is almost unknown in the mathematical community, and yet the key technology is mathematics! It has close connections with tomography and integral geometry, and with other applied fields that are better known to mathematical scientists, such as seismic inversion. SAR is of interest in imaging science because it produces a tremendous number of images that require processing and interpretation.

In SAR imaging, a plane or satellite carrying an antenna moves along a flight path. The antenna emits pulses of electromagnetic radiation, which scatter off the terrain; the same antenna detects the scattered waves. The received signals are then used to produce an image of the terrain (see Figure 1).

SAR data D can be written in the form

$$D(t,\vec{x}) = \iint e^{i\omega\left(t-2|\vec{x}-\vec{z}|/c\right)} A(\vec{z},\vec{x},\omega)\, d\omega\, V(\vec{z})\, d\vec{z} \qquad (1)$$

where $\vec{x}$ is a point on the flight path, $\vec{z}$ is a point on the ground, c is the speed of light, A is an amplitude that includes such factors as the antenna beam pattern, geometric spreading, and the frequency-domain waveform sent to the antenna, and V is the ground reflectivity function. From knowledge of D, we wish to find V.

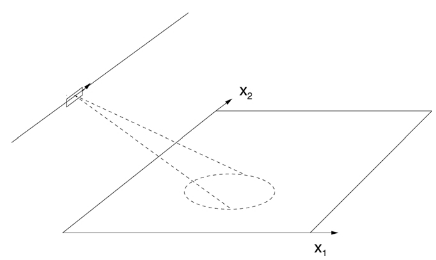

Figure 1. *SAR imaging: An antenna on a plane or satellite emits pulses of electromagnetic radiation, which scatter off the terrain; the scattered waves are detected by the antenna, and the received signals are used to produce an image of the terrain.*

A number of observations can be made about (1). The first is that the SAR problem is essentially a seismic inversion problem in which the speed of waves in the background medium is a constant, namely the speed of light. The constant background speed makes the problem easier in some respects than the seismic prospecting problem. Because SAR data is much more restricted, however, the SAR problem is in other respects more difficult. For SAR, backscattered data are known only along a single curve, whereas seismic prospecting generally offers the luxury of an entire surface for the placement of sources and receivers.

If we take $A = 1$ in (1), the SAR problem reduces to the integral geometry problem of finding a function V from its integrals over spheres. If V is assumed to be supported on a plane (i.e., $V(\vec{x}) = V(x_1, x_2)\,\delta(x_3)$), then the problem is that of finding V from its integral over circles (see Figure 2). If V is supported on a more complicated surface, the integral geometry problem becomes even more interesting.

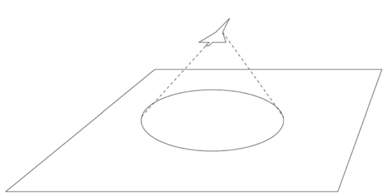

Figure 2. *When the ground reflectivity function V is assumed to be supported on a plane, the SAR problem consists of finding V from its integral over circles.*

Another connection with a large field of mathematics is evident from the form in which we have written (1): The right side of (1) has the form of an oscillatory integral, to which the techniques of microlocal analysis apply.

Further Reading

[1] M. Cheney, *A mathematical tutorial on SAR*, SIAM Rev., 43 (2001), 301–312.
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[3] C. Elachi, *Spaceborne Radar Remote Sensing: Applications and Techniques*, IEEE Press, New York, 1987.
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[5] V.P. Palamodov, *Reconstruction from limited data of arc means*, J. Fourier Anal. Applic., 6 (2000), 25–42.

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