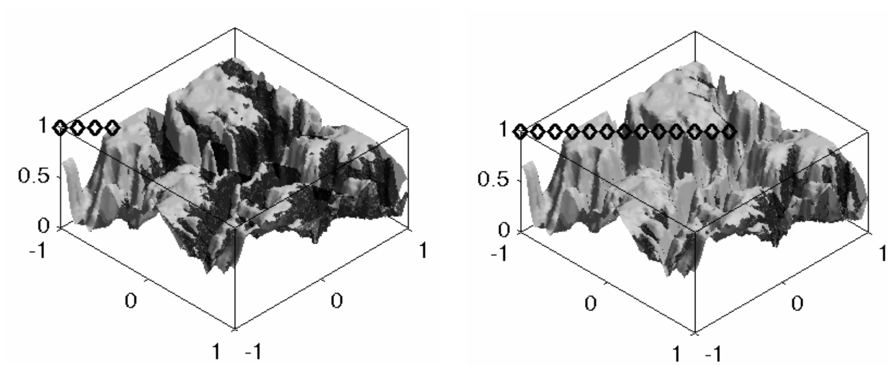
from *SIAM News*, Volume 35, Number 4

Dynamic Visibility and the Level Set Method

By Stanley Osher

The level set method (LSM), invented in 1987 by J.A. Sethian and me, has proved remarkably successful as a numerical (and theoretical) device in a host of applications, many of them in imaging science. The original reference [9] has been cited almost 600 times (according to the ISI Web of Science), and a recent query to Google's search engine gave nearly 2700 responses for "level set methods." The LSM is still an active research area (for recent results, see [7], a book published this year). In a plenary talk at the First SIAM Conference on Imaging Science, I gave a brief overview of the LSM and the associated level set technology and touched on a few imaging applications. Among them was dynamic visibility [11], which is briefly described here.



Simulation, using real data, of an airplane flying through the Grand Canyon. The regions on the occluder that have not yet become visible are dark; the diamond-shaped dots show the path of the airplane.

The problem is easily stated: Given a collection of closed surfaces representing objects in space, determine quickly the regions (in space or on the surfaces) that are visible to an observer. This question is crucial to applications in fields as diverse as rendering, visualization, etching, and the solution of inverse problems. In a 3D virtual-reality environment, knowing the visible region speeds up the rendering by enabling us to skip costly computations on occluded regions.

Representing a family of surfaces implicitly as the zero level set of a *single* function $\varphi(\vec{x})$, $\vec{x}=(x,y,z)$, has several advantages, particularly when things are moving. Topological changes are easily handled (without "emotional involvement"), and geometric quantities, such as normals and curvatures, are also easily computed dynamically. Most published work in computer graphics and computer vision uses explicit surfaces, usually constructed with triangles, but this is changing. The upcoming SIGGRAPH conference (in San Antonio, this July) will have many LSM-related papers and a full-day course on LSM and PDE-based methods in graphics. For a recent detailed report on the visibility problem with explicit surfaces, see [4].

In our approach to the dynamic visibility problem, as described in [10], we begin by laying down a simple Cartesian grid—this is step one in almost all level set methods. Next, we compute the signed distance function $\varphi(\vec{x})$ to the occluding region Ω (which generally has many disjoint pieces). Here we can use the optimally fast algorithm of Tsitsiklis [12]. The function φ approximately satisfies the eikonal equation:

$$\sqrt{\varphi_x^2+\varphi_y^2}=|\nabla\varphi|=1 \tag{1a}$$

with

$$\begin{aligned}\varphi(\vec{x})&<0 \text{ in } \Omega,\\ \varphi(\vec{x})&>0 \text{ in } \Omega^c,\\ \text{and } \varphi(\vec{x})&=0 \text{ on } \partial\Omega.\end{aligned} \tag{1b}$$

Then, for a given vantage point $\vec{x}_0$, we use a simple, easily parallelizable, multiresolution algorithm of optimal complexity to compute the visibility function $\psi_{\vec{x}_0}(\vec{x})$:

$$\psi_{\vec{x}_0}(\vec{x})\geq 0 \Leftrightarrow \vec{x} \text{ is visible to } \vec{x}_0. \tag{2}$$

Next, we allow $\vec{x}_0$ to move with velocity $d\vec{x}_0/dt$. Along the way, we obtain fairly elegant geometric-based formulae for the motion of the horizon. This is defined to be the set of visible points $\vec{x}$ lying in $\partial\Omega$ for which $\vec{x}-\vec{x}_0$ is orthogonal to $\partial\Omega$, i.e., for which

$$\psi_{\vec{x}_0}(\vec{x})=0=\varphi(\vec{x})=(\vec{x}-\vec{x}_0)\cdot\nabla\varphi(\vec{x}). \qquad 3)$$

We do the same for the motion of points on the cast horizon—that is, points that lie both on the extension of the ray connecting a horizon point $\vec{x}_0$ to $\vec{x}$ and on an occluder.

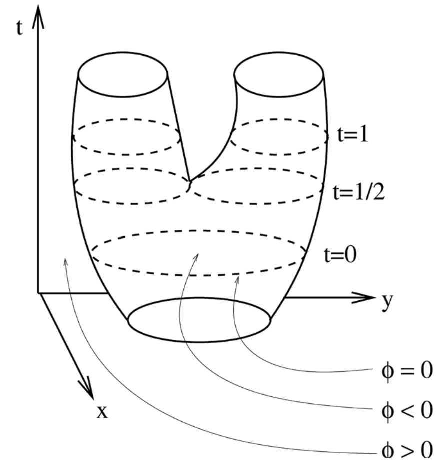
$\psi_{\vec{x}_0(t)}(\vec{x},t)$ can be found quickly at each discrete time step. As $\vec{x}_0(t)$ flies through a region, we can compute the invisible region at time *t* by computing the intersection of the sets

$$S_\tau = \left\{\vec{x} \middle| \psi_{x_0(\tau)}(\vec{x},\tau) < 0\right\} \quad (4)$$

for all $0 \le \tau \le t$. This is done by simple discrete Boolean operations.

In the plenary talk, I also gave a brief overview of the LSM and the associated level set technology, and touched on a few other imaging applications: (1) interpolation of unorganized points, curves, and surface patches [14,15]; (2) solution of PDEs on embedded manifolds with a fixed 3D local Cartesian grid [1, 2]; (3) the LSM-induced link [13] between (a) total variation-based image denoising [10], (b) active contours (snakes) [3], and (c) Mumford–Shah image segmentation [5] . All work mentioned was supported by the Office of Naval Research.

Looking to the future, we plan to modify two assumptions made in this work—that rays travel in straight lines and that the occluders are motionless. Another future direction emerged during my talk in Boston, when Arje Nachman of the Air Force Office of Scientific Research asked about visibility for radar signals, which travel around occluded regions and may return to the observer. We intend to use our new phase space-based level set approach to ray tracing [5] (developed with support from AFOSR) to analyze this and related problems.



Topological changes in (x,y) space are computed without "emotional involvement" by using the zero level set of the evolving level set function $\phi(x,y,t)$.

References

[1] M. Bertalmio, L.-T. Cheng, S. Osher, and G. Sapiro, *Variational problems and PDE's on implicit surfaces. The framework and examples in image processing and pattern formation*, J. Comput. Phys., 174 (2001), 759–780.

[2] M. Bertalmio, G. Sapiro, L.-T. Cheng, and S. Osher, *Variational problems and PDE's on implicit surfaces*, Proceedings of th 1st IEEE Workshop on Variational and LSM in Computer Vision, 186–193.

[3] T.F. Chan and L.A. Vese, *Active contours without edges*, IEEE Trans. Image Process., 10 (2001), 266–277.

[4] F. Durand, *3D visibility: Analysis, study and applications*, PhD thesis, MIT (1999).

[5] D. Mumford and J. Shah, *Optimal approximation by piecewise smooth functions and associated variational problems*, Comm. Pure. Appl. Math., 42 (1989), 577–685.

[6] S. Osher, L.-T. Cheng, M. Kang, H. Shim, and R. Tsai, *Geometric optics in a phase space based level set and Eulerian framework*, LSS report 01–01, www.levelset.com.

[7] S. Osher and R.P. Fedkiw, *The Level Set Method and Dynamic Implicit Surfaces*, Springer-Verlag, New York, 2002.

[8] S. Osher and L.I. Rudin, *Feature-oriented image enhancement using shock filters*, SIAM J. Num. Anal., 27 (1990), 919–940.

[9] S. Osher and J.A. Sethian, *Fronts propagating with curvature dependent speed: algorithms based on Hamilton–Jacobi formulations*, J. Comput. Phys., 79 (1988), 12–49.

[10] L.I. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal*, Physica D, 60 (1992), 259–268.

[11] R. Tsai, P. Burchard, L.-T. Cheng, S. Osher, and G. Sapiro, *Dynamic visibility in an implicit framework*, UCLA CAM Report 02–06, 2002.

[12] J. Tsitsiklis, *Efficient algorithms for globally optimal trajectories*, IEEE Trans. Automat. Control, 40 (1995), 1528–1538.

[13] L. Vese and S. Osher, *The level set method links active contours, Mumford–Shah segmentation and total variation restoration*, UCLA CAM report 02–05, 2002.

[14] H.K. Zhao, S. Osher, and R. Fedkiw, *Fast surface reconstruction using the level set method*, in Proceedings of the 1st IEEE Workshop on Variational and Level Set Methods in Computer Vision, 2001, Vancouver, 194–201.

[15] H.K. Zhao, S. Osher, B. Merriman, and M. Kang, Implicit and nonparametric shape reconstruction from unorganized points using variational level set method, Comput. Vision and Image Understanding, 80 (2000), 295–319.

Stanley Osher is a professor of mathematics at UCLA.