

While Surfing a Handbook

Handbook of Discrete and Combinatorial Mathematics. *Kenneth H. Rosen, Editor-in-Chief.* CRC Press, Boca Raton, Florida, 2000, 1231 pages, \$99.95.

I've had a soft spot for compendia and handbooks ever since I had to buy B.O. Peirce's *A Short Table of Integrals* (third edition, 152 pages, \$1.80) for my second-year calculus course. My old copy of B.O. Peirce is full of notes, insertions, tables of my own devising, and over the years it's become dog-eared from use. Only bits and pieces of its original cover remain.

BOOK REVIEW

By Philip J. Davis

Compendia of one sort or another have been coeval with mathematics itself. Among the earliest mathematical documents we have are arithmetic tables of various sorts.

As mathematicians push forward their research, they can't afford the time to work out everything required from scratch.

And there's no need: Handbooks rush in where experience is missing or where limited memories falter. But having said this, I wonder whether research is, in fact, the principal use that these giant handbooks are getting.

I like the book under review. Its 17 chapters, 115 sections, and I don't know how many subsections were produced by 77 individuals, kept nicely in line (no mean task) by five editors who, in turn, got the go-ahead signal and the prestige of an advisory editorial board of 23 distinguished authorities.

I like the book's format. There are glossaries at the beginning of each chapter. Each subsection, e.g., 10.1.2, begins with definitions, goes on to "facts," to algorithms (boxed and in pseudo-code), lists of mathematical objects, illuminating pictures, examples, lists of applications, and, finally, bibliography and Web resources. Much of the material included was developed in the last decade.

I love the sub-sub-sub-sections called "facts." What is a mathematical fact? Well, according to the samples here, listed and numbered, a fact might be a theorem. It might be a strategy, a conjecture, a bit of history. A practice. A piece of advice. A goal. A connection. A value judgment. A range of applicability. An explanation in alternative terms.

Reading through a subsection of "facts," with back-and-forth glances at the definitions or the glossary, gives the uninitiated a bird's-eye view of a subject. Whether or not the uninitiated can understand what an individual fact is really saying, the reader gets a strong impression of the concerns of the theory and of its scope. The book could therefore serve in a tutorial capacity, and lead a person to decide whether s/he should dig deeper into a particular subject and move to a higher level of involvement.

I should think that any of the "fact" sections could be used as an outline for a graduate course on the topic. In my mind's ear, I hear an old instructor of mine saying "no, no, never," when, as a graduate student, I suggested that if all the material contained in B.O. Peirce were explained in depth, it would engender a very substantial knowledge of mathematics. "No, no, never," he said, "it's only formal stuff and no more interesting than a telephone book." I never bothered to tell him I thought he was wrong.

A quarter century ago, the thought was abroad in the land that continuous mathematics was passé and that the major educational emphasis should in the future be placed on discrete mathematics. I was invited to participate in a foundation-funded conference convened to bang this idea around. At the conference, members of the C party (partisans of the continuous) and members of the D party (partisans of the discrete) went after each other with theorematic rhetoric and with sprays of vitriol. A few mugwumps and mavericks such as myself sat back, enjoyed the donnybrook, and shook our heads thinking how silly it all was. I have the impression that the C and D controversy has slackened considerably. The writers and publishers of calculus texts are in no immediate danger of losing their revenues.

On the other hand, it is interesting that the continuous has wormed its way into this huge book on the discrete in so few places. The exponential function and logs are seen frequently. I even spotted a few derivatives and a few lonesome integral signs lurking about the pages. But the Bernoulli polynomials without the Euler–Maclaurin integral formula strike me as akin to Hamlet looking around for the Prince of Denmark. I wonder whether this was a deliberate effort by the D party to suppress the continuous any time a continuous manifestation was deemed unnecessary for an application.

Is the world continuous or discrete? Here we have a very old philosophical question. In fact, what is continuous and what is discrete? Is a limit as $n \rightarrow \infty$ part of both worlds? Is the same true of an infinite sum of algebraic expressions? How about the discretization of a differential operator? There are times when I believe that all formal mathematics is discrete because all of it is expressed in terms of finite accumulations of discrete symbols. At other times, I'm convinced that, contrary to the claims of many, mathematics cannot be totally formalized and hence goes beyond the discrete.

This handbook will appeal both to pure and to applied mathematicians. But what is applied mathematics really? Over the centuries there has been no universal agreement as to what the word "application" means. There are times when I think that all mathematics



is applied because it derives from human experience. But people have interpreted the word “application” variously.

The tender-minded say that if a theorem is used for anything whatsoever, then that is an application. One might even say that if a heavy handbook is used to maintain the crease in a pair of pants, that is an application. The tough-minded say that an application is something that yields a bottom-line profit to society outside of mathematics itself. In any case, I wish the authors of the lists of applications had told us at what level, for example, the theory of differential games (mentioned, but not developed), claimed to be used in fishery and forest management, is actually used at the down-to-earth-and-water level. I would like to know whether Arrow’s Impossibility Theorem is used in real-world economic planning and actions. I would appreciate knowing these things without having to refer to the many references cited.

A major problem confronting every contributor to a handbook is “how much is enough?” Examining a few of the sections about which I know something, I had the immediate feeling that more facts, more tables, more diagrams, more displays, more of everything would have been appropriate. But then, to avoid producing a handbook of ten or twenty thousand pages, the editorial blue pencil must be the severe and final arbiter of what’s in and what isn’t. I would suppose—in fact, I know it to be the case—that many of the chapters already have compendia of their own, each with its own individual formatting strategy. Thus, CRC Press has itself issued, in 1997, *A Handbook of Discrete and Computational Geometry*, in 1008 pages.

If I were allowed one augmentation for future editions, I would suggest, in view of several cases that have recently gone to the U.S. Supreme Court (census counting and Florida elections), that Section 11.2.4, “Application to Social Choice,” be beefed up a bit and labeled “Applications to Social and Political Choices.” Such a section might include such questions as: How do you really count large numbers of things, beyond the ideal world of arithmetic and combinatorics? What voting schemes have been proposed and discussed (e.g., proportional representation, approval voting)? What methods have been proposed for reapportioning the U.S. House of Representatives after a census? (I believe that the scheme proposed by the late E.V. Huntington is now statutory.) What about redistricting after a census? When is a gerrymander not a gerrymander? [The author, and interested readers, are encouraged to read the article on congressional apportionment (“Political Calculus,” by Barry Cipra) that appears on page 6 in this issue.]

A different problem facing the advisory board, the editors, and the authors is whether, in today’s world, to go into print or onto the Web, or into a mixture. In an article written in 1994, Andrew M. Odlyzko, the chief advisory editor of the handbook under review, pointed to the necessity for the mathematical community to switch over from paper to electronic publishing. Such a shift has come about, but principally for journal articles. In the case of the present handbook, the publisher informed me that there are no immediate plans to go electronic. The existence of this work, put together by state-of-the-art-savvy people, is evidence of the hard-to-eliminate virtues of traditional books.

Contents: 1. Foundations; 2. Counting methods; 3. Sequences; 4. Number theory; 5. Algebraic structures; 6. Linear algebra; 7. Discrete probability; 8. Graph theory; 9. Trees; 10. Networks and flows; 11. Partially ordered sets; 12. Combinatorial designs; 13. Discrete and computational geometry; 14. Coding theory and cryptology; 15. Discrete optimization; 16. Theoretical computer science; 17. Information structures.

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