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A Nonlinear Incompressible Model of the Human Tongue

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The human tongue plays important roles in the production of intelligible speech, in the maintenance of an airway during breathing and swallowing, and in the processing of food in the mouth. An understanding of the tongue's dynamics and control would be very helpful to those who study speech production, to those who treat macroglossia surgically [5], and to those who use certain approaches in treating sleep apnea [6].

The biomechanics of the tongue is unusual. There is no rigid structure, such as bone, in the tongue. This makes it one of a small group of biomechanical systems—other examples are the squid's tentacles and the elephant's trunk—known as "muscular hydrostats" [3]. The rich collection of movements of which these systems are capable results from the interaction of actively contracting muscles with the incompressibility of the overall structure. In the case of speech, it is the shape of the tongue that is important.

There have been many efforts to develop a computer model of the tongue's dynamics, beginning with the work of Perkell [2]. All models begin with a discretization of the tongue, usually in the form of finite elements. Mathematical questions have been largely ignored. Our work is novel in part because we have created a genuine mathematical model of the tongue. As a result, we can at least ask whether the finite element approximation might converge in the limit as the elements are made smaller and smaller.

The first steps in developing a mathematical description of the dynamics of the tongue are very straightforward. The tongue is well approximated by an incompressible elastic structure that undergoes large deformations. This means that the material form for the dynamics is preferable. The model thus begins as follows: Let $\underline{z}$ denote a material point of the tongue. The tongue consists of a set of points $\underline{z} \in B \subset R^3$. Let $\underline{p}(\underline{z},t)$ denote the position of material point $\underline{z}$ at time t. Then,

$$\underline{F}\,(\underline{z},t) = \underline{p}_{\underline{z}}\,(\underline{z},t), \tag{1}$$

$$\underline{C}\,(\underline{z},t) = \underline{F}^{\,T}\,(\underline{z},t)\underline{F}\,(\underline{z},t), \tag{2}$$

where the subscript denotes partial differentiation with respect to $\underline{z}$, $\underline{F}(\underline{z},t)$ is the deformation gradient, and $\underline{C}(\underline{z},t)$ is the right Cauchy–Green deformation tensor. The fundamental equation describing the dynamics is then

$$\nabla \cdot \underline{T}^{T} + \underline{f} = \underline{p}_{tt}\,(\underline{z},t). \tag{3}$$

The double subscript denotes the second partial derivative with respect to t. $\underline{T}$ is the first Piola–Kirchoff stress tensor, and $\underline{f}$ is the body force due to gravity; we assume the mass density of the tongue to be a constant, which we have normalized to one. The incompressibility of the tongue imposes the constraint

$$\det \underline{C}(\underline{z},t) = 1. \tag{4}$$

The boundary conditions are straightforward. The normal stress is zero at any point on the boundary of the tongue that is not attached to a bone. The points that are attached to bone, either the hyoid or the styloid process, cannot move.

The real issue in describing the tongue mathematically is the description of the muscles that make up and actuate the tongue. Each tongue muscle consists of a set of material points $M_i \subset B$. In this way, all of the points in extrinsic muscles (those that connect the tongue to surfaces outside it, such as the hyoid bone) are treated as points within the tongue. Each point of a muscle lies on a muscle fiber. Mathematically, a muscle fiber is just a curve extending from one point of the muscle to another. These curves can be taken to be as smooth as necessary. Physically, only one muscle fiber can pass through any single point of the tongue. Fibers from different muscles can be closely interleaved, even when the fibers are nearly perpendicular. Mathematically, we approximate this by allowing a point to have one fiber from each of several muscles pass through it.

Lastly, the stress–strain characteristics of a muscle are described by a distributed version of the Zajac lumped muscle model [7] aligned along the muscle fiber. In Zajac's model, the muscle force has the form

$$F(l, l, a(t)) = f(l(t), \dot{l}(t), a(t)) = f_l(l(t)) f_v(\dot{l}(t))\, a(t), \tag{5}$$

where F is the muscle force, $l(t)$ is the length of the muscle, $\dot{l}(t)$ is the shortening velocity (+ when the muscle is shortening) of the muscle, and $a(t)$ is an activation, $0 \le a(t) \le 1$. The input in Zajac's model is not $a(t)$; instead, $a(t)$ is determined by a nonlinear first-order ordinary differential equation with input $u(t)$, where $u(t)$ is related to the observed EMG of the muscle. Because we have no way to determine the EMG for muscles in the tongue, we take $a(t)$ for each muscle as the inputs to our model. In addition to simplifying our model, this feature reflects our commitment not to exceed the resolution of the experimental data; the precise details can be found in [1].

Real muscles consist primarily of contractile fibers, as described in the previous paragraph, and a lattice of collagen fibers. There are also blood cells, nerve cells, and fat cells, which we believe can be ignored at the level of accuracy of our model. The lattice of collagen fibers in a muscle is aligned so that one part of the lattice is parallel to the contractile fibers. Collagen fibers offer no resistance to either compression or shear. They have a smooth, monotonic increasing force–length curve in extension. We model the collagen fibers as a homogeneous isotropic nonlinear elasticity throughout the tongue. Their stress is added to that from the contractile fibers.

The model is a partial differential equation that we approximate by finite elements and use in two ways. In the forward direction, we start with a given set of piecewise-continuous (in time) muscle activations $a_i(t)$ (i being an index for each independently controllable section of muscle). We use the model to compute the resulting tongue shape, again as a function of time.

In the inverse, or backward, direction, the given information is the position versus time of each point in a set of material points of the tongue. We then invert the finite element model to compute the muscle activations that could produce the observed movements. Because equations far outnumber unknowns, it is generally impossible to find a solution. However, the best solution in the sense of minimizing the mean-squared error is both reasonable and useful. In fact, the number of independently controllable activations is unknown. Anatomical data indicates that the muscles in the tongue, unlike those in the limbs, can have different activations along their lengths [4]. As we increase the number of independent activations, we improve the match between the inverted model and the data.

Figure 1. *The first frame from an untagged cine MRI sequence of images of the mid-sagittal plane of a human head during normal speech. The image is black where there is bone or air., and the tongue is the large blob in the middle of the frame. The speaker is saying "aaaaoooo."*

Our main emphasis is on the inverse problem, because we do not have a good way to measure activation. Thus, the inputs to the forward problem can come only from conjecture or from solution of the inverse problem. In contrast, tagged cine MRI images of the tongue can give us the position versus time for any point within the tongue. Thus, as we refine our finite elements, we can get the position versus time for every vertex. The velocity and acceleration of each point are also needed as inputs to the full inverse problem. We are working on a way to estimate the velocities directly from the MRI. Otherwise, we need to differentiate and filter the positions, with considerable attendant inaccuracy.

One way around the difficulty with velocity and acceleration is to find a quasi-static solution, i.e., to set both velocity and acceleration to zero and solve the resulting static inverse problem at each instant for which data is available. We have done this by creating a simplified description of part of the tongue, a mathematical model of the mid-sagittal plane of the tongue (see Figure 1). Because material points can flow out of the plane, the incompressibility constraint can be relaxed. The deformation of the tongue for the motion we study is relatively small, and a linear model is thus reasonably accurate. The quasi-static approximation then reduces the inverse problem to the form $\underline{A}\,\underline{x} = \underline{b}$, where $\underline{x}$ represents the unknown activations. The least-squares solution, constrained by $\underline{x} \geq \underline{0}$, because muscle activations must be between 0 and 1, is then easily obtained.

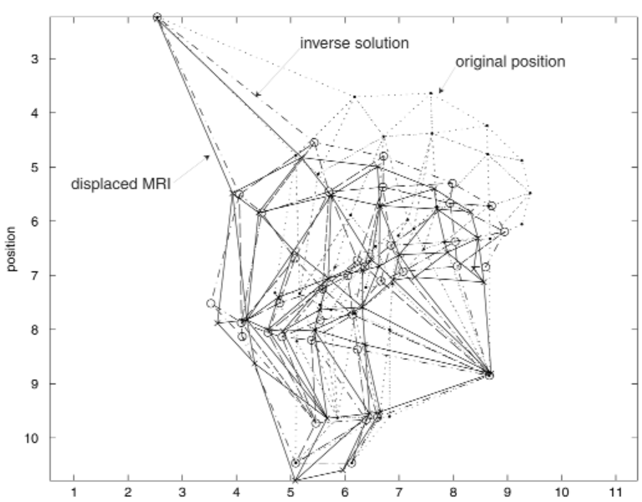

Figure 2. *Results for the simplified inverse problem. Both the original position and the displaced MRI are from the MRI data. The inverse solution corresponds to the displaced MRI. The speaker is saying "eeeeoooo." The original position corresponds to eeee, the final position to oooh.*

One result is illustrated in Figure 2. The finite elements of the original position are shown with dots at the vertices. The corresponding elements after displacement, as determined from the MRI, are shown with x's at the corners. The elements that correspond to the solution of the inverse problem are indicated by small circles. The triangular region in the upper left corner corresponds to the styloglossus muscle, which is not seen in Figure 1 because it is located at the sides of the tongue. The styloglossus, and several other muscles that are not found in the mid-sagittal plane, are included in the model because they do influence the motion. The styloglossus is shown somewhat out of its true position, which is more horizontal; a correction is in progress.

We are currently finishing the construction of both our two- and three-dimensional mathematical models of the tongue. We expect to have some results on the three-di-mensional inverse problem within a few months. We are also working on, and have some preliminary results for, some greatly simplified "models" of the tongue for which we can obtain analytical results. Although they do not correspond to a real tongue, they do provide a great deal of insight into its behavior.

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