
Index

- $A \times B$ Cartesian product, 550
 C^∞ , 66
 $C_c^k(\mathbb{R})$, 301
 $C_c^\infty(\mathbb{R})$, 243
 $E = mc^2$, 39
 $J_0(a)$ Bessel function, 575
 $J_1(x)$ Bessel function, 576
 $J_n(x)$, Bessel function, 92
 $L^1(\mathbb{R})$, 230
 $L^2([0, 1])$, 51
 $L^2(\mathbb{R})$, 309
 $L^p(\mathbb{R})$, 239
 $P_n(x)$, Legendre polynomial, 83
 $\mathcal{F}f$, 109
 $\mathcal{F}^{-1}f$, 110
 Λ_p , 77, 127
 Π_p , 126
 S , 239
dinc, 448
 δ , 250
 $\delta(ax)$, 283
 δ_a , 257
 $\hat{f}(n)$, 12
jinc, 576
 \mathbb{C} , 48, 666
 \mathbb{C}^n , 48
 \mathbb{R} , 44
 \mathbb{R}^n , 44
 \mathbb{Z}_N , 429
 \mathcal{C} , 243
sgn, 288
 III_p , 334
 σ_a , 129
sinc, 103
 τ_b , 129
 $\underline{\mathcal{F}}f$, 415
 $\underline{\text{II}}$, 444
 $\underline{\delta}$, 421
 $\underline{\omega}$, 417
 $\underline{\text{III}}$, 446
 $e^{\pm 2\pi int}$, 8
 $e^{i\theta}$, 669
 $f : A \rightarrow B$, 661
 i , not j , 9, 665
absolute convergence, 391
AC component, 8
accumulator, 536
adjoint, 425, 427
Ahlfors, Lars, xv, 247, 312
airtight, 287
alias, 373
aliasing, 371, 378
Amis, Kingsley, 367
analytic signal, 153, 516
antenna radiation patterns, 325
Archimedes, 161, 164
arithmetic-geometric mean inequality,
73
Arnold, V. I., xiv
artifacts, 625
autocorrelation, 215
 Fourier series, 88
average value, 12
averaging and least-squares, 520
averaging operator, 94, 319, 509, 537

- Bach, J. S., 7
 band-pass filter, 171
 discrete, 529
 bandlimited, 244, 362
 discrete, 446
 on \mathbb{R}^2 , 605
 bandwidth, 15, 362, 366
 endpoint problem, 376
 infimum, 366
 basis
 dual, 599
 natural, 47
 orthonormal, 47
 basis of \mathbb{C}^N
 complex exponentials, 424
 bed of nails, 593
 bell-shaped curve, 115
 Benford's Law, 223
 Bessel function, 92
 Bessel function of the first kind, 352
 first-order, 576
 zero-order, 575
 Bessel, F., 55
 Bessel's inequality, 55
 bit reversal, 463
 via permutation matrices, 463
 Black-Scholes equation, 227
 bounded-input bounded-output
 (BIBO), 545
 Boyd, Stephen, xv, 60, 531
 Bra-ket notation, 257
 Bracewell, R., xv, 1, 29, 110, 118, 325,
 334, 509, 511, 515, 549, 593
 Bravman, J., xv
 Briggs, W., and V. Hensen, 412
 Brownian motion, 585
 bump function, 300
 butterfly diagram, 461
 Butterworth filter, 171
 buzz, 42

 Cantor, G., 2
 Cartesian product, 550
 Cauchy distribution, 219, 221
 Cauchy principal value, 266
 Cauchy-Schwarz inequality, 70
 and matched filters, 508
 causal function or signal, 510
 causality, 509
 and linear systems, 510
 and linear time-invariant systems, 510
 and physical realizability, 511

 Central Limit Theorem, 185, 187, 198
 central slice theorem, *see also*
 projection-slice theorem
 change of variables formula
 for multiple integrals, 570
 characteristic function (probability),
 218
 chirp signal, 154
 chirp transform, 537
 chopper amplifier, 408
 Chowning, John, 352
 Chuaqui, M., xv
 circ function, 575
 circle impulse, 637
 circulant matrices, 504
 circularly (radially) symmetric function,
 574
 Clark, Latimer, 37
 compact, 243
 compact set, 240
 compact support, 243, 362
 completeness of real numbers, 366
 complex exponentials
 n -dim, 553
 and Fourier series, 8
 discrete, 417
 eigenfunctions of LTI systems, 505
 operator decomposition, 506
 complex number
 argument or phase, 668
 conjugate, 666
 magnitude, 667
 polar form, 668
 real and imaginary parts, 666
 computed tomography (CT), 611, 621
 concentration at a point, 251
 conjugate symmetry, 49
 conservation of energy, 155
 continuity of Fourier transform, 230
 convergence
 Fourier series, 60
 of power integral, 590
 partial sums
 Dirichlet kernel, 61
 pointwise, 27, 60
 pointwise vs. uniform, 68
 uniform, 60
 convolution, 33, 159
 and probability density functions, 194
 and Schwartz functions, 305

- as a smoothing operation, 185, 298, 300
- as smoothing and averaging, 165
- definition, 161
- derivative theorem, 298
- derivatives, 169
- different normalizations, 164
- discrete, 441, 442
- for periodic functions, 33
- Fourier series, 88
- identities, 168
- in frequency domain, 163
- interpretation, 164
- Mellin, 542
- properties, 167
- theorem, 161
- visualization, 165
- Cool Hand Luke, 507
- Cooley, J., 411
- cosine transform, 151
- Cover, T., xv
- cross correlation, 214
- crystals, 333, 603
 - and dual lattice, 603
 - and electron density distribution, 333
- cumulative density function (cdf), 190
- cumulative probability, 190
- cycle, 4
- Davis, P., 504
- DC component, 8
 - in discrete case, 432
- deconvolution, 168
- delta, 250
 - n -dim, 578
 - along a function, 344
 - along a line, 617
 - approximated by heat kernel, 253
 - approximated by rectangle functions, 251
 - as a limit, 250
 - convolution, 252
 - convolution property, 293
 - derivative, 289
 - discrete, *see also* discrete delta
 - Fourier transform, 271
 - Kronecker, 47
 - product with a function, 279
 - sampling property, 279
 - scaling, 283
 - shifted, 252
- derivative of
 - δ -function, 289
 - $\ln |x|$, 289
 - distribution, 285
 - signum (sign) function, 288
 - unit ramp function, 288
 - unit step function, 286
- DFT, *see also* discrete Fourier transform
- DFT properties, 472
- Diaconis, P., 531
- difference operators, 480
- differential calculus, 285
- differential equations
 - Bessel, 92
 - Legendre, 83
 - solution via Fourier transform, 174
- differential operator, 174
- diffraction, 323
 - by a single slit, 330
 - by two point sources, 332
 - X-ray, 333
- diffraction gratings, 333
- diffusion, 28, 39
 - submarine telegraphy, 37, 178
- digital filters, 519
 - analysis in the frequency domain, 523
 - computation in the frequency domain, 530
- digital watermarking, 558
- dinc, 448, 528
- Dirac comb, 334
- Dirac, Paul, 233, 235
- Dirichlet kernel, 40, 61
- Dirichlet problem
 - disk, 90
 - half-plane, 357
- Dirichlet, Lejeune, 27
- discrete III, 446
- discrete complex exponentials, 417
 - as eigenvectors, 506
 - orthogonality, 423
 - periodic version, 429
 - periodicity, 418
- discrete convolution, 442
 - and random variable probability, 195
- discrete correlation, 475
- discrete delta, 421
 - periodic version, 429
 - properties, 443
- discrete filters
 - band-pass, 529

- low-pass, 527
- discrete Fourier transform, 411, 414, 415
 - alternative definition, 431
 - convolution, 441
 - downsampling, 472
 - duality relations, 438
 - general properties, 421, 439
 - higher-dimensional, 609
 - inverse, 426
 - matrix form, 419
 - modulation theorem, 440
 - negative frequencies, 431
 - notation, 416
 - of reversed signals, 436
 - Parseval's identity, 440
 - periodicity, 428
 - reciprocity relationship, 420
 - replication, 472
 - shift theorem, 440
 - upsampling, 472
 - vector form, 418
- discrete Fourier transform of
 - discrete complex exponential, 424
 - discrete delta, 421
 - discrete rect, 444
 - shifted deltas, 423
- discrete impulse response, 496
- discrete linear systems examples
 - matrix multiplication, 488
- discrete linear time-invariant systems, 503
- discrete rectangle function, 444
- discrete signals
 - periodic assumption, 429
- discrete time Fourier transform (DTFT), 479
- dispersion, 154
- distribution (probability)
 - Cauchy, 219, 221
 - infinitely divisible, 219
 - mean or expected value, 190
 - normal (or Gaussian), 187, 193, 219
 - uniform, 192
 - variance and standard deviation, 190
- distributions, 229, 234, 249
 - approximation, 265
 - as linear functionals, 256
 - continuity, 258
 - convolution, 293, 296
 - defined by function, 260
 - derivative, 286
 - dual space of test functions, 259
 - evenness and oddness, 277
 - limits, 264
 - linearity, 258
 - pairing, 257
 - principal value, 266
 - product with a function, 278
 - pullback, 637
 - regularizing, 308
 - reversal, 275
 - shifts, 281
 - stretches (scaling), 283
 - tempered, 234, 260
- distributions and Fourier transform
 - convolution theorem, 292
 - definition, 268
 - derivative theorem, 291
 - duality relations, 276
 - shift theorem, 282
 - stretch theorem, 284
- divided time signal, 209
- Donaho, D., and P. Stark, 205
- Doppler effect, 547
- downconversion, 398
- downloading problem files, 75
- downsampling, 80, 472, 473
- Doyle, P., and J. Snell, 585
- driven harmonic oscillator, 319
- dual (reciprocal) basis, 599
- dual lattice, 598
- dual space, 259
- duality, 275
- Duren, Peter, xv, 63
- Dutton, R., xv
- duty cycle, 100, 147
- Dym, H., and H. McKean, 35, 66, 386
- edge detection, 573
- El Gamal, A., xv
- electron density distribution, 333
- electrostatic capacity per unit length, 37
- energy of a signal, 50
- energy spectrum, 15, 108
- Epstein, C., 612
- Euler's formula, 669
- Euler, L., 116
- Ewald sphere, 342
- existence of Fourier transform, 229
- expected value, 190
- exponential decay
 - one-sided, 113

- two-sided, 128
- far-field diffraction, 329
- Faraday, Michael, 37
- Farmelo, G., 235
- Fast Fourier Transform, 449
 - algorithm, 449
 - butterfly diagram, 461
 - calculation, 453
 - computational complexity, 460
 - description, 455
 - divide and conquer approach, 458
 - sorting indices, 461
 - via matrix factorization, 458
- Ferry, G., 605
- Feynman, R., 324, 637
- FFT, *see also* Fast Fourier Transform
- filtering, 169
- filters, 169
 - band-pass, 171
 - Butterworth, 171
 - discrete, 527
 - effect on magnitude and phase, 170
 - high-pass, 172, 573
 - low-pass, 171, 573
 - notch, 173
 - pass-band, 171
 - stop-band, 171
 - two-dimensional, 573
- finite sampling theorem, 382
- FM, 352
 - and music, 352
 - carrier frequency, 352
- Folland, G., xv, 28
- Fourier, xii
 - analysis, 1
 - coefficients, 12
 - finite series, 12
 - infinite series, 15
 - study of heat flow, 28
- Fourier coefficients
 - $\hat{f}(n)$ definition, 12
 - and Fourier transform, 144
 - as Fourier transform, 106
 - higher dimensions, 584
 - multiply under convolution, 88
 - rapidly decreasing, 66
 - size, 63
 - symmetry properties, 13
- Fourier inversion theorem, 107
- Fourier optics, 323
- Fourier pair, 110
- Fourier reconstruction
 - model brain, 625
- Fourier series
 - autocorrelation, 88
 - basic convergence theorem, 26
 - convolution, 88
 - downsampling, 80
 - summary of convergence results, 60
 - upsampling, 80
- Fourier transform, 99
 - and L^2 , 309
 - and Fourier coefficients, 144
 - continuity, 230
 - definition, 105
 - different definitions, 135, 551
 - duality, 118
 - existence, 229
 - magnitude and L^1 -norm, 231, 237
 - modulation property, 143
 - motivation, 103
 - of a Fourier series, 343
 - of a line impulse, 634
 - of Gaussian, 562
 - of reversed signals, 120
 - of sampled signal, 413
 - polar coordinates, 574
 - short-time, 152
 - symmetries, 153
 - unitary linear system, 535
- Fourier transform of
 - $1/x$, 291
 - complex exponential, 272
 - delta, 271
 - Gaussian, 116
 - shah function, 337
 - shifted delta, 272
 - signum, 291
 - sine and cosine, 274
 - unit step, 291
- Fourier transform properties
 - derivative formula, 174, 238
 - derivative theorems, 133, 572
 - duality, 118, 552
 - even and odd symmetry, 122, 552
 - general shift and stretch theorem, 130, 571
 - linearity, 123, 564
 - modulation theorem, 124, 566
 - Parseval's identity, 132, 572
 - shift theorem, 123, 564
 - stretch (similarity) theorem, 125, 567

- fractal, 542
- Fraunhofer approximation (diffraction), 325
- Fraunhofer diffraction, 326
- Frayn, Michael, 39
- frequency, 2
 - and pitch, 7
 - and spectrum, 15
 - and wavelength, 2
 - high frequencies and sharp corners, 25
 - negative, 674
 - positive and negative
 - in discrete case, 433
 - spatial, 4
- frequency domain, 107
 - polar coordinate grid, 624
- frequency response, 505
- Fresnel diffraction, 329
- Friedlander, F. G., 311, 498
- Friedrich and Kniping, 333
- Fubini's theorem, 248
- function
 - = signal, 661
 - absolutely integrable, 74
 - compact support, 243
 - even, odd, 670
 - even, odd parts, 671
 - global properties, 66
 - local properties, 66
 - notation, 661
 - orthogonal projection, 57
 - rapidly decreasing, 235
 - scaling, 115
 - smooth window, 299
 - smoothness and decay, 235
 - square integrable, 74
 - step, 244
- functional analysis, 484
- fundamental solution
 - heat flow on a circle, 33
- Gårding, L., 311
- Gauss, 115
 - and DFT, 411
- Gaussian function, 115, 186
 - n -dim, 562
 - Fourier transform, 116
 - general form, 129
 - heat kernel for infinite rod, 176
 - periodization, 185
- Gaussian integral evaluation, 117
- Gaussian vectors, 653
- Gehring, F. W., xv
- general topology, 243
- generalized function, 234, 250
- generating function, 588
- geometric sums, 678
- Gibbs, J. W.
 - phenomenon, 27
- Gill, J., xv, 643
- Gleason, A., xv
- Goldstein, J., xv
- Golub, G., and C. van Loan, 504
- Goodman, J. W., xvi, 323, 327
- Gram-Schmidt process, 84
- Gray, R., xv, 504
- Green's function
 - heat flow on a circle, 33
 - heat flow on infinite rod, 177
- gridding, 624, 658
- Haar functions, 83
- half-wave rectifier, 539
- half-wave symmetry, 79
- Hallelujah, 393
- Halmos, Paul, 661
- Hamilton, E., xv
- Hamming, R., 519
- Hankel transform, 575
 - electrostatics, 652
- Hann window, 317
- harmonic oscillator, 6
- harmonics, 6, 108
 - amplitude and phase, 8
 - energy, 15
- Hartley transform, 155, 217
 - and Fourier transform, 155
 - duality, 155
- heat equation, 30
 - infinite rod, 175, 254
- heat flow, 28, 29
 - on a circle, 30, 182
 - on a semi-infinite rod, 178
 - on an infinite rod, 175
 - spot on earth, 35
- heat kernel, 253
 - for infinite rod heat flow, 177
- Heaviside function, 286
 - in two dimensions, 631
- Heaviside, Oliver, 38, 233, 235
- Heideman, Johnson, and Burrus, 411
- Heisenberg
 - inequality, 202
 - uncertainty principle, 202

- Heisenberg, Werner, 39
 Helmholtz equation, 326
 Hermite polynomials, 157
 Hermite, Charles, 49
 Hermitian symmetry, 49
 Higgins, J. R., 391
 high-pass filter, 172, 322
 higher-dimensional DFT, 609
 inverse, 611
 Hilbert transform, 266, 289, 511
 and analytic signals, 515
 as an LTI system, 514
 as an operator, 513
 Cauchy principal value integral, 513
 inverse, 513
 of Π , 515
 of sinc, 515, 518
 of sine and cosine, 515
 histogram, 188
 Hodgkins, Dorothy, 605
 Hölder's inequality, 310
 Hubbard-Stratonovich formula, 157
 Hughes Hallett, D., xv
 Huygens's principle, 327
 Huygens, Christiaan, 324
 hypersurface, 637

 IDFT, *see also* inverse discrete Fourier transform
 impulse response, 170, 322, 496
 and convolution, 500, 504
 and Schwartz kernel, 498
 discrete, 496
 for LTI system, 500
 heat flow on a circle, 35
 impulse train, 42
 independent periods, 581
 independent random variables, 193
 indicator function
 parallelogram, 649
 triangle, 649
 inequality
 arithmetic-geometric mean, 73
 Cauchy-Schwarz, 70
 infinite sum of Gaussians, 183
 infinitely divisible probability distribution, 219
 inner product
 algebraic properties for complex vectors, 49
 algebraic properties for real vectors, 47
 geometric formula, 72
 in n -dim, 583
 of functions in $L^2([0, 1])$, 52
 vectors in \mathbb{C}^n , 48
 vectors in \mathbb{R}^n , 46
 instantaneous frequency, 153, 518
 integer lattice, 583, 594
 self-dual, 599
 integral
 convergence, 249
 improper, 266
 tail, 249
 integration, 244
 contour and the residue theorem, 247
 positive functions, 244
 integration by parts
 formula, 23
 integrator, 208
 interpolation
 Lagrange, 390
 orthogonality, 369
 sinc, 364
 inverse n -dim Fourier transform
 definition, 550
 inverse discrete Fourier transform, 426
 inverse Fourier transform
 definition, 107
 motivation, 106
 inverse Hilbert transform, 513
 isoperimetric inequality, 87

 Jacobi, 183
 identity, 183
 theta function, 183
 James, J. F., 327
 jinc function, 576
 joint probability density, 196
 Joy of Convolution, 166
 Judson, H., 605

 Kac, Mark, 187
 Kalfayan, S., xv
 Kammler, D., 75, 141, 312, 403
 kernel
 Dirichlet, 40, 61
 of an integral operator, 489
 Poisson
 disk, 90
 Kevles, B., 611
 Knuth, D., 461
 Körner, T. W., 75, 135, 247
 Kronecker delta, 47

- Lagrange, J., 390
 Lagrange interpolation polynomial, 390
- Laplacian, 326
- lattice, 341, 594
 and shah, 597
 area, 596
 dual, 598
 fundamental parallelogram, 595
 general, 595
 reciprocal, 598
 unit cell, 595
- lattice sampling formula, 608
- law of squares, 38
- Lebesgue dominated convergence theorem, 247
- Lebesgue integral, 247
- Lebesgue, H., 51
- Legendre polynomials, 83
- Legendre, A.-M., 83
- Levitt, T., 329
- light
 linearly polarized, 325
 monochromatic, 325
 point source, 332
 waves, 325
- limits of distributions, 264
- line
 (ρ, ϕ) parametrization, 616
 (m, b) parametrization, 615
 line coordinates, 627
 normal vector, 626
 parametric description, 615
- line impulse, 617, 626, 629, 637
 Fourier transform, 634
 in terms of 1-form, 630
 in terms of 1D δ , 630
 scaling, 633
 symmetry, 633
- linear attenuation coefficient, 613
- linear change of variables, 567
- linear combination, 486
- linear functional, 259
- linear interpolation, 360, 472
 convolution, 388
- linear modulation invariant (LMI) systems, 541
- linear operator, 485
- linear scale-invariant (LSI), 541
- linear systems, 484
 additivity, 485
 and causality, 510
 and convolution, 491
 and translation, 492
 composition or cascade, 492
 homogeneity, 485
 superposition theorem, 497
 symmetric et al., 490
 via integration against a kernel, 489
- linear systems examples
 direct proportion, 487
 integration, 489
 multiplication, 487
 periodization, 492
 sampling, 488
 switching, 487
- linear time-invariant systems
 and causality, 510
 and Fourier transform, 504
 definition, 499
 discrete, 503
 superposition theorem, 501
 transfer function, 505
- Lippman, G., 186
- Littlewood, J. E., 362
- Lord Kelvin, 37
- Lorentz profile curve, 115
- Los Alamos, 39
- low-pass filter, 171
 discrete, 527
- LTIS, *see also* linear time-invariant systems
- Macovski, A., 612
- magnetic resonance imaging (MRI), 611, 657
 sensitivity encoding, 655
- Mandelbrot, B., 542
- manifold, 638
- Markov process, 585
- matched filter, 507
 matched filter theorem, 508
- matrices and dot products, 570
- matrix
 adjoint, 425, 427
 circulant, 504
 Hermitian, 427
 inverse transpose, 567
 orthogonal, 427, 569
 rotation, 569
 symmetric, 427
 Toeplitz, 504
 transpose, 427

- unitary, 427
- McCallum, W., xv
- mean, 190
- measure, 247
- measure zero, 110
- medical imaging, 611
 - numerical computations, 623
 - reconstruction, 622
- Melin transform, 541
- Michelson and Stratton's device, 27
- Miller, D., xv, 327
- modulation property, 143
- modulation theorem, 124, 566
- module, 595
- multi-index, 578
- musical
 - pitch, 7
 - tone, 39
 - tuning, 7
- Mutapcic, A., xvi

- n -dim Fourier series, 581, 584
 - coefficients, 584
- n -dim Fourier transform, 549
 - convolution theorem, 572
 - definition, 550
 - derivative theorems, 572
 - different conventions, 551
 - duality, 552
 - general stretch theorem, 567
 - inverse, 550
 - linearity, 564
 - modulation theorem, 566
 - notation, 549
 - Parseval's identity, 572
 - polar coordinates, 574
 - properties, 559
 - rotation theorem, 569
 - shift and stretch theorem, 571
 - shift theorem, 564
 - spectrum, 556
 - stretch theorem, 566
 - symmetry, 552
- n -dim Fourier transform of
 - δ -function, 579
 - separable functions, 559
- n -dim Schwarz functions, 578
- n -dim complex exponential, 553, 582
- n -dim convolution, 572
- n -dim functions
 - circ, 575
 - delta, 578
 - Gaussian, 562
 - jinc, 576
 - rect, 560
- n -dim periodic functions, 581
- n -dim stretch theorem derivation, 569
- n -dim tempered distributions, 578
- two-dimensional Fourier transform of
 - radial function, 573
- Nahim, Paul, 77, 235
- narrowband signal, 517
- near-field diffraction, 329
- negative frequency, 674
- Nehari, Z., xv
- Newton's law of cooling, 29
- Newton, I., 324
- Nietzsche, F., 584
- Nishimura, D., xv, 612
- Noll, A., 612
- Noll, W., xv
- nonrecursive filter, 520
- norm
 - L^1 , 230
 - L^2 (square), 52
- normal approximation, 187
- notch filter, 173, 323
- null space, 487
- Nyquist, Harry, 364

- one-sided exponential decay, 113
- Oppenheim, A., and A. Willsky, 75, 400, 408
- ordinary differential equations
 - solution, 174
- orthogonal
 - functions in $L^2([0, 1])$, 52
- orthogonality, 42, 43
 - of discrete complex exponentials, 423
 - vectors, 45
- orthonormal basis, 47
 - complex exponentials, 67

- Papoulis sampling theorem, 406
- parallelogram law, 95
- Parseval's theorem
 - for Fourier transforms, 109
- partial Fourier transform, 563
- partial sums, 26
- pass-band, 171
- Pauly, J., xv
- penmanship, 243
- period
 - fundamental, 4

- periodic distributions, 342
 and Fourier series, 342
 periodic extension, 336
 periodic functions, 2, 4
 summation of, 5
 periodicity
 and integer lattice, 594
 independent, 581
 spatial, 2
 temporal, 2
 periodization, 78
 with shah function, 336
 periodizing sinc functions, 383
 permutation matrix, 455
 perpendicularity, 43
 Perutz, Max, 605
 phase retrieval, 604
 phasor, 673
 Pierce, J., 612
 Plancherel's theorem, 109, 311
 plane wave field, 326
 Plummer, J., xv
 pointwise convergence, 27, 60
 Poisson distribution, 225
 Poisson kernel
 disk, 90
 Poisson summation formula, 338
 n -dim case, 601
 Poisson transform, 225
 Poisson's equation, 652
 polar coordinates Fourier transform,
 574
 polar form, 668
 polarization identity, 95
 Pólya, G., 587
 power integral convergence, 590
 power spectrum, 15, 108
 principal value distribution, 266
 principle of superposition, 485, 486
 principles, declaration of, 665
 probability, 188, 189
 generating function, 588
 probability density function (pdf), 187
 probability distribution, 187, 188
 problem files
 downloading, 75
 projection
 and Radon transform, 618
 vector, 46
 projection-slice theorem, 621
 pullback, 345, 630
 pullback of a distribution, 637
 Pythagorean theorem, 43
 quadrature function, 514
 quantum mechanics
 inequalities, 202
 momentum, 205
 observable, 204
 particle moving in one dimension, 204
 position of particle, 205
 radial function, 574
 Radon transform, 614, 615
 evenness, 620
 linearity, 620
 projection, 618
 properties, 620
 shift, 620
 Radon transform of
 circ, 618
 Gaussian, 619
 Radon, Johann, 615
 random variable, 187
 continuous, 188
 discrete, 188
 expected value, 190
 independent and identically
 distributed (iid), 197
 mean (or average), 190
 standard deviation, 190
 variance, 190
 random vector, 586
 random walk, 584
 recurrent, 585, 587
 theorem, 587
 transient, 585, 587
 rapidly decreasing Fourier coefficients,
 66
 rapidly decreasing functions, 235, 238
 Fourier inversion, 241
 Fourier transform, 240
 Parseval identity, 243
 Rayleigh's identity, 15, 55
 higher dimensions, 584
 reciprocal lattice, 598
 reciprocal or dual lattice, 341
 reciprocal relationship
 and inverse transpose, 568
 and stretch theorem, 126
 changing period, 18
 frequency and wavelength, 3
 spatial and frequency domain, 554

- time and frequency domain, 20
- rectangle function, 99
 - discrete, 444
 - for a strip, 628
 - stretched, 126
- recursive filter, 520
- Reed, M., and B. Simon, 485, 497
- reflection across a line, 648
- replication, 472
- reproducing function, 212
- reproducing kernel, 95
- resistance per unit length, 37
- reversed signal, 120
 - discrete case, 436
 - Fourier transform, 120
- ridiculous statement, 116
- Riemann-Lebesgue lemma, 233, 304
- Riesz-Fischer theorem, 68
- ringing, 528
- Rodrigues's formula, 84
- Roentgen, William, 333
- root of unity, 417
- Rosel, P., 463
- rotation matrix, 569
- running average, 520
 - five point, 522
- sample-and-hold, 360
- sample-rate conversion, 474
- sampling, 359
 - demodulation, 399
 - discrete, 446
 - downsampling, 473
 - endpoint problem, 376
 - for bandlimited periodic signal, 380
 - for bandlimited signals, 362
 - in frequency domain, 413
 - sampling theorem, 364
 - sines and bandlimited signals, 359
 - stable in energy, 403
 - upsampling, 473
 - with shah function, 336
- sampling on a lattice, 605
- sampling oscilloscope, 400
- sampling theorem
 - discrete, 446
- Savage, Sam, 187, 585
- savings account, 542
- scale invariance, 151
- scaled triangle function, 136
- scaling operator, 95, 129, 162, 283
 - as pullback, 346
- Schäffer, J., xv
- Schiffer, M., 592
- Schwartz functions, 239
- Schwartz kernel, 498
- Schwartz, Laurent, 234
- Schwartz, M., 327
- self-dual lattice, 599
- separable functions, 559
- separation of variables method
 - in partial differential equations, 559
- Serber, Robert, 39
- shah
 - III, 334
 - distribution, 335
 - Fourier transform, 337
 - scaling identity, 337
- shah function
 - discrete, 446
- Shannon, C., 235, 364
- Shepp, L., and J. Kruskal, 625
- Sheppard, J., 10
- Sherwood, D., and J. Cooper, 341, 605
- shift operator, 95, 129
 - as pullback, 346
- shift theorem, 123, 564
 - as pullback, 346
- shifted delta
 - δ_a , 257
 - Fourier transform, 272
- short-time Fourier transform, 152, 351
- signal
 - bandlimited, 15, 105, 111, 362
 - bandwidth, 15, 362
 - reversal, 120
- signal = function, interchangeable terms, xii
- signal conversion
 - continuous to discrete, 412
- signal processing, 159
- signal-to-noise ratio, 508
- signum (sign) function, 288
- similarity, 125
- sinc function, 103
 - as a convolution identity, 368
 - discrete version, 448
 - in dreams, 103
 - orthonormal basis, 369
- sine transform, 152
- sinusoid, 6
 - amplitude, 6
 - frequency, 6

- phase, 6
- Siripuram, A., xv, 446
- Smith, R., 17
- SNR, *see also* signal-to-noise ratio
- Sobolev, Sergei, 234
- Solymar, L., 38
- Sommerfeld, A., 35, 323
- sorting algorithm
 - merge and sort, 450
- spatial frequency, 553
- spatial variable, 550
- spectral leakage, 475
- spectral power density, 108
- spectral theorem for matrices, 496
- spectrum, 15, 107, 556
 - continuous, 105
 - for a discrete signal, 418
 - musical instruments, 16
 - trombone, 16
 - trumpet, 17
 - unbounded, 362
- spherical harmonics, 577
- square integrable function, 42, 50
- square sampling formula, 607
- square wave, 20
- stable LTI systems, 539
- standard deviation, 190
- stationary phase, 154
- Steiglitz, K., 42, 450
- step functions, 244
- step response, 536
- Stokes, G., 37
- stop-band, 171
- stretch theorem, 125, 566
 - as pullback, 346
- stretched rectangle function Π_p , 126
- stretched triangle function Λ_p , 127
- Strichartz, R., 311
- Sturm-Liouville theory, 43
- Sullivan, E., xv
- superposition
 - principle, 485, 486
- superposition integral, 497
- superposition theorem
 - for discrete linear time-invariant systems, 503
 - for linear systems, 497
 - for linear time-invariant systems, 501
- support, 243
- system, 483
- Talmud, 288
- Tekalp, A. M., 341
- telegraph equation, 39
- temperature, 29
- tempered distribution, 260
 - Fourier inversion, 270
 - Fourier transform, 268
 - inverse Fourier transform, 270
- tensor products, 562
- test functions, 257
 - class, 234
- thermal capacity, 29
- thermal resistance, 29
- Thomson, William (Lord Kelvin), 37, 116, 178
- time domain, 107
- time domain multiplexing, 474
- time-dependent Schrödinger equation, 214
- timelimited vs. bandlimited, 367, 386
- Titchmarsh, E. C., xiv
- Toeplitz matrices, 504
- Toeplitz, Otto, 504
- tomography, 611, 613, 621
- top-hat function, 99
- trade secret, 45
- transfer function, 170, 322
 - of linear time-invariant system, 505
- transpose, 427
- trapezoidal rule approximation, 623
- triangle function, 112
 - stretched, 77, 127
- triangle inequality, 72
- triangle wave, 22
- tritone, 7
- Tuesday Night Live, xv
- Tukey, J., 68, 411
- tuning
 - equal tempered scale, 7
 - natural, 7
- two slits experiments, 324
- two-dimensional shah, 593
- two-sided exponential decay, 128
- uncertainty principle, 202
 - discrete, 476
- uniform convergence, 60
- unit cliff, 631
- unit impulse, 250
- unit ramp function, 288
- unit step function, 286
 - in two dimensions, 631
- unit vector, 44

- unitary matrix, 427
- upsampling, 80, 472, 473

- vacuum tubes, 234
- van Loan, C., 459
- Vandenberghe, Lieven, 531
- variance, 190
- vector, 44
 - inner (dot) product for complex vectors, 48
 - inner (dot) product for real vectors, 46
 - norm, 44
 - projection, 46
 - underline notation, 44
 - unit, 44
- Venus, image of, 557, 573
- voice scrambling, 146
- von Laue, Max, 333
- von Neumann, John, 71

- Walsh functions, 82
- watermarking, 656
- wave equation, 326
- wave motion, 2
- wave number, 326
- wavefront, 325
- Weierstrass
 - M -test, 70
- white noise, 508
- Whitehouse, Edward, 38
- Whittaker, E., 364
- Wilf, H., 588
- windowing, 242, 299
- windowing functions, 317
- Wirtinger's inequality, 86
 - discrete form, 481
- Wu, W., xv, 446

- X-ray diffraction, 333, 603
- X-rays, 333

- Young's experiment, 331
- Young, Thomas, 324

- Zemanian, A., 311, 498
- zero padding, 463, 464
- zero phase, 553, 559
- zero-order hold, 534
- Zygmund, A., 311