

# Contents

<b>Preface</b>	<b>xi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 General considerations . . . . .	1
1.1.1 The role of mathematics . . . . .	1
1.1.2 Modelling . . . . .	2
1.2 Aims of the lectures . . . . .	3
<b>2 Preliminaries</b>	<b>5</b>
2.1 Simplifying assumptions . . . . .	5
2.2 The governing equations . . . . .	11
2.2.1 The equation of mass conservation . . . . .	11
2.2.2 The equation of motion: Euler's equation . . . . .	12
2.2.3 The boundary conditions . . . . .	13
2.2.4 Well-posedness . . . . .	15
2.3 Vorticity . . . . .	18
2.3.1 Irrotational flows . . . . .	26
2.3.2 Currents . . . . .	27
2.4 Appendix for Chapter 2 . . . . .	29
2.4.1 Surface tension . . . . .	29
2.4.2 Dissipation of energy . . . . .	31
2.4.3 Flow-invariant sets . . . . .	33
2.4.4 The compression rate of a fluid . . . . .	37
2.4.5 Sobolev spaces . . . . .	38
2.5 Notes for Chapter 2 . . . . .	40
<b>3 Wave-current interactions</b>	<b>41</b>
3.1 Steady periodic waves of small amplitude . . . . .	46
3.1.1 The linearization . . . . .	47
3.1.2 Existence of nonlinear waves of small amplitude . . . . .	53
3.1.3 The dispersion relation . . . . .	63
3.2 Rotational steady waves of large amplitude . . . . .	72
3.2.1 The global continuum . . . . .	72
3.2.2 Nodal pattern and bounds . . . . .	79
3.2.3 Numerical simulation of waves of large amplitude . . . . .	87

3.3	Symmetry of rotational steady water waves . . . . .	90
3.4	Regularity of the streamlines . . . . .	94
3.5	Appendix for Chapter 3 . . . . .	97
3.5.1	Fredholm operators . . . . .	97
3.5.2	Local bifurcation . . . . .	99
3.5.3	Global bifurcation and degree theory . . . . .	100
3.5.4	Elliptic boundary value problems . . . . .	114
3.5.5	Maximum principles . . . . .	125
3.6	Notes for Chapter 3 . . . . .	126
<b>4</b>	<b>Fluid kinematics for wave trains</b>	<b>131</b>
4.1	Particle paths beneath a Stokes wave . . . . .	132
4.2	Pressure beneath a Stokes wave . . . . .	152
4.3	Appendix for Chapter 4: Gerstner's wave . . . . .	155
4.4	Notes for Chapter 4 . . . . .	160
<b>5</b>	<b>Solitary water waves</b>	<b>163</b>
5.1	Particle paths beneath an irrotational solitary water wave . . . . .	166
5.2	Pressure beneath an irrotational solitary water wave . . . . .	170
5.3	Solitons . . . . .	174
5.4	Appendix for Chapter 5 . . . . .	180
5.4.1	Integrable systems . . . . .	180
5.4.2	Alternative model equations within the same regime . . . . .	235
5.5	Notes for Chapter 5 . . . . .	238
<b>6</b>	<b>Breaking waves</b>	<b>241</b>
6.1	Long waves of small amplitude . . . . .	243
6.2	Wave breaking for long waves of moderate amplitude . . . . .	248
6.3	Appendix for Chapter 6 . . . . .	257
6.3.1	Semigroup theory approach toward well-posedness . . . . .	257
6.3.2	The evolution of extrema . . . . .	265
6.4	Notes for Chapter 6 . . . . .	266
<b>7</b>	<b>Modelling tsunamis</b>	<b>275</b>
7.1	General considerations . . . . .	276
7.2	The December 2004 tsunami . . . . .	285
7.3	The 1960 Chile tsunami . . . . .	287
7.4	Appendix for Chapter 7 . . . . .	289
7.4.1	Nondimensionalization and multiple scales approach . . . . .	289
7.4.2	The highest wave ever measured . . . . .	295
7.4.3	The March 2011 Japan tsunami . . . . .	296
7.5	Notes for Chapter 7 . . . . .	297
	<b>Bibliography</b>	<b>299</b>
	<b>Index</b>	<b>319</b>