

# Contents

Preface .....	xv
Chapter 1 On Some Variational Problems in Hilbert Spaces .....	1
1.1. Introduction .....	1
1.2. A particular class of linear variational problems in Hilbert spaces.....	1
1.3. On variational inequalities in Hilbert spaces. (I) Elliptic variational inequalities of the first kind .....	46
1.4. On variational inequalities in Hilbert spaces. (II) Elliptic variational inequalities of the second kind .....	56
Chapter 2 Iterative Methods in Hilbert Spaces.....	69
2.1. Introduction. Synopsis .....	69
2.2. On Newton's method.....	70
2.3. Conjugate gradient algorithms .....	81
2.4. Further comments on iterative methods .....	112
Chapter 3 Operator-Splitting and Alternating Direction Methods .....	113
3.1. Introduction. Synopsis.....	113
3.2. On alternating direction-type methods .....	114
3.3. On Lie's and Strang's operator-splitting schemes .....	136
3.4. Applications .....	144
Chapter 4 Augmented Lagrangians and Alternating Direction Methods of Multipliers.....	157
4.1. Introduction. Synopsis.....	157
4.2. A historical perspective .....	158
4.3. Decomposition-coordination methods by augmented Lagrangians ...	165
4.4. A convex finite dimensional application: The numerical solution of the Weber problem .....	170

4.5. Application to the solution of a nonconvex problem from nonlinear elasticity .....	172
4.6. Application to the solution of the Dirichlet problem for the two-dimensional elliptic Monge–Ampère equation .....	178
4.7. Application to the solution of a nonsmooth eigenvalue problem from visco-plasticity .....	188
4.8. Further comments. References .....	201
Chapter 5 Least-Squares Solution of Linear and Nonlinear Problems in Hilbert Spaces .....	203
5.1. Generalities.....	203
5.2. Least-squares conjugate gradient solution of nonlinear problems in Hilbert spaces .....	209
5.3. Least-squares conjugate gradient solution of linear problems in Hilbert spaces .....	224
5.4. Further comments and references.....	236
Chapter 6 Obstacle Problems and Bingham Flow: Application to Control.....	237
6.1. Introduction. Synopsis.....	237
6.2. A penalty/Newton/conjugate gradient method for the solution of obstacle problems for linear second order elliptic operators .....	238
6.3. A penalty/Newton/conjugate gradient method for the simulation of Bingham flows in cylinders.....	246
6.4. Application to the optimal control of distributed parameter systems modeled by parabolic variational inequalities of the obstacle type .....	268
6.5 Further comments on penalty methods .....	282
Chapter 7 $-\nabla^2 u = \lambda u^3$ and Other Nonlinear Eigenvalue Problems .....	285
7.1. Introduction. Synopsis.....	285
7.2. Numerical solution of the Lane–Emden problem (7.1).....	287
7.3. On the numerical solution of the steady von Kármán equations for nonlinear elastic thin plates .....	321
7.4. On the numerical solution of a nonsmooth eigenvalue problem from visco-plasticity.....	337
7.5. A variant of the Bratu problem: Vortex condensation in the Chern–Simons Higgs model.....	343

---

Chapter 8 Eikonal Equations .....	353
8.1. Introduction. Synopsis.....	353
8.2. A calculus-of-variations approach to the solution of (8.1).....	354
8.3. An equivalent variational formulation of problem (8.9). An associated initial value problem .....	355
8.4. Time discretization of problem (8.15) by operator splitting .....	356
8.5. Finite element implementation.....	358
8.6. On the Eikonal equation $ \nabla u  = 1$ .....	360
8.7. Numerical experiments.....	365
8.8. Further comments.....	370
Chapter 9 Fully Nonlinear Elliptic Equations .....	379
9.1. Introduction. Synopsis.....	379
9.2. On the least-squares solution of the Dirichlet problem for the elliptic Monge–Ampère equation in dimension 2 .....	380
9.3. A least-squares/operator-splitting method for solving the Dirichlet problem for a two-dimensional elliptic Pucci equation.....	404
Epilogue .....	421
Bibliography .....	425
Author Index .....	449
Subject Index .....	455