Preface

This book is a written, slightly expanded version of selective portions of my 10 *Conference Board of the Mathematical Sciences* (CBMS) lectures presented over the week of August 13–17, 2012, at West Chester University in West Chester, PA. The intent of CBMS lectures is to introduce new material in an expository manner to students and faculty who wish to learn about an area. And so, in an introductory and expository manner, the material in this book describes mathematical ideas reflecting concerns from certain social and behavioral science disciplines. (Most technical details are left for references.)

My doctoral thesis and a continuing research interest (mathematics of the Newtonian N-body problem) are far removed from the social and behavioral sciences. So when I stumbled on this area, it was a delight to discover the challenging, interesting associated mathematics where there is so much to explore. There are, of course, many who have made deep, excellent contributions to these areas, but they can describe what they have done much better than I. As such, what follows is my personal take on how mathematics can and should help address issues from these areas; it reflects the content of my CBMS lectures. As the book's title suggests, my goal is to promote an inquiry about how the differing nature of these concerns can require developing more appropriate forms of mathematics.

The mathematics needed to advance the social and behavioral sciences most surely differs from what has proved to be successful for the physical sciences. Remember, a strong portion of contemporary mathematics reflects a fruitful symbiotic relationship enjoyed by mathematics and the physical sciences over a couple of millennia: Advances in one area motivated advances in the other. As it must be expected, this intellectual relationship shaped some of our mathematics and influenced the way in which certain physical sciences are viewed. Centuries of experimentation in the physical sciences, for instance, led to precise measurements and predictions, which motivated the creation of mathematical approaches, such as differential equations, that allow precision predictions.

To illustrate with Newton's equations, bounded two-body motion lies on an ellipse, so had the universe consisted solely of the Sun and the planet Mercury, Mercury would forever circle the Sun on a fixed elliptic orbit. But Mercury is not isolated; our Solar System is populated with other planets, including its near neighbors of Venus, Earth, and Mars, that tug on Mercury and alter the orientation of its elliptic motion. Features of the orbit, such as the perihelion location (Mercury's closest approach to the Sun), slowly change.

In 1859, the French mathematician Urbain Le Verrier tested Newton's equations by comparing the theoretical change in Mercury's perihelion position with data. His stunning conclusion created a crisis concerning the legitimacy of Newton's laws: It uncovered a *discrepancy of* 43 *seconds of arc per century*. (This Earth-year century translates into about 415 Mercury years.) To put Le Verrier's finding in perspective, suppose an object is moving on a circle with a 2 foot diameter; the goal is to predict the object's position a century from now. A small difference between the predicted and actual position of a mere $\frac{1}{400}$ of an inch over a *century*, which is about the thickness of a thin strand of hair, might seem to be acceptable. Not for Le Verrier; this $\frac{1}{400}$ of an inch is his 43 seconds of arc! History proved that the doubt Le Verrier's results cast on Newton's equations was on the mark; it required Einstein's theory of general relativity to explain the gap.

Contrast the precision of Le Verrier's analysis, based on mathematics available over a century and half ago, with current events from the social sciences. A particularly worrisome incident was the economic crisis of 2008. Forget predictions of the "43 seconds of arc per century" accuracy. Instead, had anyone in June 2008 been able to predict what would happen within a 50% accuracy, not within a century, but within the next couple of months, this success would have been widely acclaimed.

As fully recognized, it is unrealistic to expect "precise" predictions for many issues in the social and behavioral sciences. But, researchers have access only to a limited number of mathematical approaches, where favorite choices for theoretical models tend to involve methods designed for precision predictions—not much else is available. This comment underscores the need to develop appropriate mathematical tools that, rather than designed for exactness, reflect the current status for much of the social and behavioral sciences, which requires qualitative predictors.

Compare this comment with what happens in less precise physical science topics, such as earthquake analysis. Those of us luxuriating in sunny Southern California would embrace an accurate prediction of when and where will be the next "big one." As this is not possible, sharper qualitative information would be appreciated. The same holds for the social and behavioral sciences; whenever precise predictions are not realistic, tools allowing sharper qualitative information would be welcomed; but doing so requires different mathematical approaches. Ideally, such challenges would create a symbiotic tie between mathematics and the social and behavioral areas. We are nowhere near there yet, but I can hope.

This leads to Chapter 1. The social and behavioral sciences are dominated by change. Everything changes; opinions change, economics change, politics change, preferences change. As it is not clear how to model this, until recently "change" rarely was examined. Resembling the story of a drunk searching around a street light for keys lost elsewhere because "The light is better here!" there is a tendency to emphasize what can be analyzed with currently available techniques: Results are sought where there is sufficient "light" such as finding equilibria without any knowledge or exploration of the associated dynamics.

Many factors hinder our understanding of how to model change, including a lack of reliable information. Often, what is best known reflects behavior in specialized, local settings. And so the qualitative approach developed in Chapter 1 emphasizes how to connect local information into a global dynamic. With so little known about the dynamics, the emphasis must be on qualitative modeling where refinements must come from the host area data.

For another topic, even high school students know about Adam Smith's "invisible hand" story, which is a supply-and-demand aggregation process that combines the economic agents' preferences and resources. The key word of "aggregation" is central across the social and behavioral sciences. Statistical methods, probabilistic predictions, understanding migration, social movements, political processes, and on and on involve aggregations where even reasonably correct assertions require sound methods. But, to the best of my knowledge, no complete, general mathematical analysis exists describing what can go right, what can go wrong, and what are potential pitfalls of aggregation rules.

An obvious obstacle is the overwhelming number of dissimilar aggregation approaches which clouds how to address this issue. One way to handle this problem is to embrace Occam's razor, which in contemporary terms is the KISS philosophy.[1] Thus my initial emphasis (Chapter 2) has been to examine a particular aggregation class—voting methods. These approaches often are linear aggregations, which makes it reasonable to expect that lessons learned will transfer to more complicated settings. This is because a standard mathematical way to identify characteristics of a system is to use linear approximations of derivatives, tangent spaces, and so forth.

Examining voting systems might seem to be mathematically trivial. After all, commonly used voting rules just sum ballots; what can go wrong? This reflects my seriously mistaken initial attitude! A clue should have come from actual events with pundits wondering "how did that so-and-so win the election!" In fact, a telling measure of the intricacies of paradoxical outcomes is that the characteristics, number, and kinds of these mysteries can be identified with the complexity of chaotic dynamics! (Sect. 2.3, [87].) The numbers of difficulties are mind boggling: Using even a thousand of the fastest available computers, it would be impossible to *count* (not even list) just the plurality vote paradoxical outcomes that arise with only *eight* candidates—even had the counting started at the Big Bang.

Of importance, these troubling, unanticipated behaviors help to identify unexpected properties of other aggregation methods. As illustrations, understanding paradoxical behavior in voting provides guidelines to discover similar actions in aggregation methods as diverse as bizarre features of the aggregate excess demand function from Adam Smith's supply-and-demand story to puzzling behavior in nonparametric statistics. A selection of these topics is discussed in Chapters 3 and 4.

Moving to something else, when introducing vectors to an undergraduate class, or eigenvectors in a linear algebra course, my approach is to confess that there are far too many vectors—even in just two-dimensions—to intimately know all of them. A convenient approach is to become acquainted with carefully selected choices, such as $\mathbf{i}, \mathbf{j}$, or the eigenvectors, and then describe all other vectors in terms of how they relate to our newly acquired friends.

This commentary reflects the common mathematical methodology of dividing a construct into component parts to clarify the analysis. Aspects of this useful approach are used in areas such as psychology to differentiate features of observations, but it has not been generally adopted to address mathematical concerns in the social and behavioral sciences. Indicating the strong advantages of doing so are themes of Chapters 4 and 5. To ensure consistency in the described

[1] Keep It Simple, Stupid!

topics, the illustrating choices come from the first two chapters: I first show how to decompose voting rules and then games. (Many of the game theory results involve joint work with Dan Jessie.) As shown, both decompositions significantly simplify the discovery of new conclusions.

The approaches depend upon inherent symmetries, which accurately suggests that mathematical ways to extract consequences of symmetries were used to discover the decompositions. Well, this is how results were discovered, but not how they are described. To explain, while many researchers in the social and behavioral sciences have strong mathematical backgrounds, most do not. Thus, for results to be understood and adopted by the intended audiences—the social and behavioral sciences—outcomes must be described in a mathematically more assessable manner. And so, after discovering new properties, it can take another six months to a year to learn how to make the conclusions more readable.

While my "assessable" descriptions may remain mathematically obscure for many in the intended audiences, they reach a much larger number than if I did not try. But this objective conceals the mathematics responsible for discovery. Others, such as Mike Orrison and his coauthors (e.g., [23, 25]), have published descriptions and nice extensions of some of these mathematical symmetries. I recommend their papers.

The final chapter addresses the commonly used reductionist approach; probably all readers are familiar with this whole-parts system analysis. This is the realistic approach of handling a complex problem by dividing it into tractable parts, solving the questions posed by each part, and then assembling the answers into a solution for the whole.

Although widely used, it is shown why this approach can suffer serious, unexpected problems. Indeed, as outlined at the end of this concluding chapter, many of the complexities described in this book reflect unanticipated consequences of this approach. The positive side is that understanding the source of the difficulties helps to identify what causes many of the complexities suffered by the social and behavioral sciences. Actually, as indicated, this description of "what can go wrong" extends to shed light even on problems from engineering and the physical sciences, such as the compelling dark matter mystery of astronomy—a topic briefly outlined. Again, by understanding what causes problems focuses attention in a search for resolutions.

These chapters describe some of the topics described in my CBMS lectures. Other themes included the currently hot topic of gerrymandering (where Joe Gerver supplied examples illustrating its mathematical complexity) that often reaches and baffles the US Supreme Court, power indices (briefly mentioned in Sect. 3.4), concerns from psychology where certain resolutions involve simple algebraic topology, or something called "meaningfulness" (which is full of possible mathematical concerns; e.g., see Narens' book [65]), spatial voting, and dynamics in economics. When writing up these notes, it became clear that trying to include everything covered in my CBMS lectures would result in an overly bulky book that might never be completed. Maybe elsewhere.

There are organizations and many people to thank; my apologies to those I missed. Let me start with Ron Rosier, the former director of the CBMS, for his persistent, friendly reminders to complete this book; reminders that were appropriate because I finally finished this project about four years past the original deadline. Ron served as the CBMS Director for over a quarter of a century where, thanks in large part to his efforts, the CBMS has grown in influence and

prestige. Indeed, my thanks to the CBMS for inviting me (a second time[2]) to be a CBMS lecturer: What a wonderful experience! These CBMS lectures provide an excellent opportunity to describe results and approaches to an attentive audience. The math department of West Chester University proved to be an excellent host; this praise holds particularly for the conference chair and organizer, Mike Fisher, who ensured a smoothly functioning, excellent time (while doing much of the work). *Thanks Mike!*

Thanks to the many who influenced my thinking on these topics; a full listing is not realistic as it would take pages, but it includes friends from Europe (in particular, Finland and France) as well as many friends, colleagues, and coauthors from the Midwest. More recent are the members of the *Institute for Mathematical Behavioral Sciences* here at the University of California, Irvine; there is much to learn from the weekly colloquia, subsequent discussions, and IMBS workshops. This list includes Bernie Grofman (a political scientist making excellent use of mathematics) and, definitely, Louis Narens (a mathematician who became a cognitive scientist) where the number of my conversations on these topics with Narens may define infinity. Included are the participants of our weekly "Social Dynamics" discussion group headed by Brian Skyrms (a philosopher who is radically changing his field by using dynamics to explain long-standing puzzles) and Narens. Similarly, the lively, explorative conversations characterizing my weekly meeting with graduate students (which the students call "The Don Squad") always are informative: With respect to this book, let me mention former squad members Dan Jessie, Ryan Kendall, and Tomas McIntee.

Of particular importance is Duncan Luce, who in 2000 recruited me from Northwestern University to direct the *Institute for Mathematical Behavioral Sciences* that Luce created. Trained as a mathematician (his mathematics PhD is from MIT), Luce was a pioneer in using mathematics to advance the behavioral sciences; he was, for instance, one of the founders of the contemporary area of mathematical psychology. His seminal contributions are reflected by his many honors, including the *President's Medal of Science*. My intent was to dedicate this book *to* Luce, not in his memory. Sadly, Duncan died the day before my CBMS lectures started; he will be remembered; his influence continues!

Anyone who has even attempted to write a book recognizes that most of the time-consuming effort is done at home after completing day-job professorial responsibilities: This draws from family attention and responsibilities. And so, my very strong thanks, with deep love, to my wife Lillian for her understanding and constant support for our 51 wonderful years of marriage!

Irvine, California
September 2017

[2]My first CBMS lecture series, in 2002, discussed the Newtonian N-body problem [95].