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FOREWORD TO THE CLASSICS EDITION

Cornelius Lanczos described his philosophy of technical writing in a fifty-five minute video, *About Mathematics*, which is now freely available on YouTube. To the three University of Manchester faculty members who collectively interviewed him in 1972, five years after publication of *Fourier Series*, he said, “You can make, to my mind, any subject interesting without the technical language. Most of the books are written in an intolerable fashion. Most of the textbooks are terribly dull because the writer automatically puts himself on a pedestal, handing out Wisdom.

“I say, why not talking to your audience as if you were face to face to him? Why could you not express it in a more chatty way? Express it in a way as if you were talking to a friend?”

Lanczos was very careful. He wrote out his lectures beforehand with great care, lavishing five or six hours of preparation on each hour of lecture. His words in the classroom, as in this book, are chatty and informal, but he knew exactly what he wanted to say and followed always the shortest path to his goals.

*Fourier Series* [4] is based on a lecture series for upper-level undergraduates and beginning graduate students at the University of Washington, given when he was already middle-aged. Although he had spent many years as sometimes a physicist, sometimes a mathematician, at various universities, he had also guided engineers during five years at the National Bureau of Standards (now NIST) and in aviation. Mathematics was not merely a theoretical’s tool useful during his years as Einstein’s assistant in the late 1920s, and in the work in general relativity that continued throughout his long life, with a major article published at eighty; he was also a man who had worked for Boeing Aircraft and had spent a lot of time applying Fourier analysis to experimental data and airfoils.

He had no students or disciples. He was forced from academic positions by anti-Semitism in his native Hungary in the 1920s, by more of the same, amplified, in Germany in 1933, and lastly in the United States in the 1950s when he could no longer work for the government because he was a foreigner from a country that had turned communist. His first wife died of tuberculosis; he escaped with his seven-year-old son to the U.S. just before the outbreak of World War 2, and lost almost all his remaining family to the Holocaust. When Purdue University rescued him, he spoke almost no English; a year of research was sacrificed to learning and simultaneously lecturing in a language he did not know. His position was always visiting professor, half-time appointment, until a new physics head forced him out. He was a kind of a Flying Dutchmen of mathematical physics, always on the move, until he ended his days at the Institute of Advanced Studies in Dublin.

A wasted, bitter life one might suppose. The rigid career path of a profes-
sor at a modern university is that One Must Build the Big Research Group, recruit doctoral students more vigorously than the head football coach, bombard the federal agencies with grant applications more numerous than the pollen falling from the heavens in spring, and leave the paper writing and the research to the postdocs, research associates, and students who do all the bench work and all the computer programming. A professor is chained to his previous topics by his Big Group, his network of contacts built up laboriously over decades, and the impossibility of large funding except in areas where the grantee has grown the group from a corner of the building to an entire floor. The senior tenure-track faculty at a research university—the "silverbacks" in anthropological jargon—are bound by invisible chains stronger than the strongest steel to a narrow range of what the Prevailing Consensus agrees are Very Important Problems. The aspiring scientist is confronted with the reality that his mentors are all business managers.

Lanczos managed nothing, grew nothing in the sense of groups and funding. Nevertheless, he wrote eight books and a hundred plus research papers as cataloged in his *Collected Works* [2] and Barbara Gellai's short biography [1]. Fourteen editors were required to generate the six volumes of *Collected Works*, totaling 3200 pages, with translations of his dissertation and forty seven articles first published in a language other than English, plus seventeen introductions to parts and sections and fifty-two commentaries on specific papers and his research areas. General relativity, differential geometry, numerical approximation, computational algorithms for matrices and differential equations, and essays on the relationships of science, mathematics, Judaism, and art. His publications span 56 years and enormous swathes of physics and mathematics.

Mathematical fields, like sports, usually evolve through the collective incremental innovations of many hands. Among sports, basketball is the anomaly; although the jump ball after made baskets has long since disappeared, the teams shrunk from nine to five, the stationary ball handling replaced by the dribble, and the peach baskets, ten feet above the gym floor, have been replaced by metal rings, supporting a net and hung from a glass rectangular backboard, the modern game is recognizably the grownup version of the child fathered entirely by the brain of Dr. James Naismith at Springfield College in 1891. Similarly, despite the rise of supercomputers, the Fast Fourier Transform (independently invented by Lanczos in 1942), Smolyak sparse grids, and the open software system Chebfun, the huge field of Chebyshev polynomial and Fourier spectral and pseudospectral methods traces its lineage to a Lanczos paper of nearly 100 pages that appeared in 1938 [3]. It is all there: Chebyshev polynomials, collocation methods, the modified Galerkin method (known as the tau method), spectral methods triumphant for a function that lacks even a power series about one singular
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endpoint. He wrote it while an ocean away from his dying wife and his young son, the reflections from teaching a course on practical numerical methods that somehow coalesced into a stupendous act of genius in an area completely orthogonal to his previous interests in general relativity and other physics. It has been dubbed “The Lanczos Thunderbolt,” but it was only one of many, for he was a man who lived with lightning.

Only a handful of exact solutions to the Einstein equations have been found in the century since Einstein published his theory. One was found by Lanczos in 1924. Rediscovered years later by another physicist, Lanczos’s solution is now known as the “van Stockum Dust.” Lanczos was not upset; the scientific literature is densely populated by items bearing his name, including the Lanczos algorithm for calculating the eigenvalues of large matrices, the Lanczos sampling method in signal processing and Fourier analysis, the Lanczos potential in general relativity, and the Lanczos approximation for the gamma function.

His books are extremely useful to engineers and scientists because he was not interested in building a house of abstract theorems, but rather in deducing how things work. Richard Feynman, the Nobel laureate in physics, wrote in Lanczos’s spirit. He invented Feynman diagrams, a graphical language of wiggly, dashed, and solid lines and curves in which each segment is one species of particle interacting with another and simultaneously a factor in the integrals that are the terms of perturbation theory: quantum field theory condensed into a kindergartner’s doodles.

Feynman shared his Nobel prize with his Bronx Science High School classmate, Julian Schwinger, but it was said in physics that “Schwinger wrote papers that only Schwinger could understand, whereas Feynman wrote papers that everyone could understand.”

A Discourse on Fourier Series contains no mention of Hilbert spaces or indeed of almost any of the machinery of functional analysis. Instead, in one chapter, he discusses some thorny technical questions through a Platonic dialogue:

**QUESTION.** Is the sectional existence of \( f'(x) \) really necessary for the validity of Fourier’s integral theorem?

**ANSWER.** No, it is not. The Fourier theorem holds even without differentiability. In fact, the considerations of section 26 have demonstrated that for any function which vanishes outside the interval \([-L, L]\), and which satisfies the Dirichlet conditions, the Fourier integral is automatically valid, since for such functions the convergence of the Fourier series was demonstrated before, and the Fourier series changed over into the Fourier integral merely by extending the base line of analysis to infinity. . . .
QUESTION. I see a contradiction in the demand that $f(x)$ has to be absolutely integrable, and the Dirichlet factor, which seems to have a Fourier integral, although it is not absolutely integrable.

ANSWER. It is true that the Dirichlet factor... is not integrable in an infinite interval. But in this case, although the Fourier synthesis is still perfectly regular, the Fourier analysis fails to converge... The proper method here is to introduce the damping factor $e^{-\varepsilon |x|}$ which makes the function integrable, and then go to the limit $\varepsilon \to 0$.

This is a radically different approach from modern mathematics texts, which tend to hide behind vast arrays of symbols and formalism. Lanczos, like Feynman, was so brilliant that even very complicated mathematics physics seemed simple to him. His goal was to help the reader see how simple it all was, too.

He was so successful that he was honored with the Chauvenet Prize, the highest award for mathematical expository writing, by the Mathematical Association of America in 1960, six years before the publication of A Discourse on Fourier Series. He says in the video, “I always considered that one of my strengths is that I could project myself... into another... and try to understand the world with his eyes.”

One hundred and sixty-one problems are integrated into the text, not gathered at the end of each chapter. The book is illustrated with thirty line drawings.

The University of Manchester also filmed Lanczos’s autobiography. There is no interviewer, no props, no blackboard. There is only a serene old man talking in the same style as in his books, as if talking to a friend.

Lanczos believed in covering a modest amount of material thoroughly, so his Fourier Analysis is not long and is unlikely to be adopted as the primary text for a class. Rather, his book is a lifeline, the book that a student turns to in despair when his 600-page textbook fails him, hiding the foundations under a bruising hail of formalism. Lanczos hides nothing, aims for the foundations, develops the most important topics slowly and carefully, explaining that it is all quite simple if you walk with him on the simplest path.

His book is not dry; a person inhabits it. “And why is it not allowed,” he says in the video, “to put in a certain emotional emphasis? If you’re enthused about something, why not say so?”

The joyful old man of the Manchester videos, almost eighty, less than 18 months from death, had endured a very hard life, but he was not bitter. Rather, he felt his life had been greatly blessed despite its many tragedies. He refused to be a victim. He felt the same uplift of spirit in great science and mathematics that is also found in great art as he expressed clearly both in the videos and in a series of essays included in his Collected Papers.
It will be a blessing to future generations of students that they will have the opportunity through SIAM Classics to gain a new perspective on Fourier series and integrals. It is not a blizzard of formalism. It is not a lifeless catalogue of theorems either. It is rather a joyful come-with-me into ideas that are both great art and engineering, guided by an unpretentious docent who was both a great artist and a great mathematical engineer.

John Boyd
University of Michigan
June 2016

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PREFACE

This book on the mathematical theory of the Fourier series grew out of a lecture course on "The Fourier Series and its Applications," given by the author in the fall of 1947 as Walker-Ames Lecturer at the University of Washington, Seattle, Wash., at the invitation of Professor F. B. Farquharson, Director of the Engineering Experimental Station. The audience was composed of students of mathematics, physics and engineering, on the half-graduate and graduate level, and the problem was to present the material in a manner which would be stimulating to everybody. The gratifying response of the students demonstrated that such an endeavour is in fact possible.

By the nature of things it was necessary to develop the subject from its early beginnings and this explains the fact that even so-called "elementary" concepts, such as the idea of a function, the meaning of a limit, uniform convergence and similar "well-known" subjects of analysis were included in the discussion. Far from being bored, the students found this procedure highly appropriate, because very often exactly the apparently "elementary" ideas of mathematics—which are in fact "elementary" only because they are relegated to the undergraduate level of instruction, although their true significance cannot be properly grasped on that level—cause great difficulties in proceeding to the more advanced subjects. An added advantage of this method was that it encouraged the students to ask questions of a fundamental nature, without fear of displaying an ignorance of things which they should know.

The subject was particularly well suited to this procedure since it was exactly the remarkable and apparently paradoxical nature of the Fourier series which led to a constant revision and deepening of the fundamental concepts and methods of higher analysis. Hence a close tie with the historical development seemed appropriate, although the author is well aware that this exposes him to the charge of datedness. We have to be "modern" and there are those who believe that before the advent of our own blessed era the pursuers of mathematics lived in a kind of no-man's-land, bumping against each other in the gloomy haze that pervaded everything ("Euclid must go!"). But there are others (and the author belongs to the latter group), who believe that the great masters of the eighteenth and nineteenth centuries, Lagrange, Euler, Gauss, Cauchy, Riemann, Fourier, Dirichlet, and many others, were not necessarily lacking mathematical intelligence and some of them might even be comparable to the geniuses of today.

To this has to be added that it was not the author's intention to
present the theory of the Fourier series in its pure aspects. To sever
the theory from its relation to the physical universe was not desirable,
since the manifold applicabilities of the Fourier series to problems of
physics and engineering—which will be pursued in a second volume—
warranted a treatment which is not interested in pathological cases
but in the properties of the series in relation to a fairly well behaved
class of functions. This does not mean, however, that anything less
than a rigorous treatment could be tolerated. Within the class of
Riemann-integrable functions the mathematical deductions are entirely
rigorous and those who are acquainted with the principles of Lebesgue-
integrability, will have no difficulty in extending the proofs to all
Lebesgue-integrable functions.

The primary aim of the author was to convey something of the
excitement and enthusiasm which imbued the hundreds of mathe-
maticians who have contributed to this remarkable chapter of analysis.
To display formal fireworks, which are so much in the centre of many
mathematical treatises—perhaps as a status-symbol by which one
gains admission to the august guild of mathematicians—was not the
primary aim of the book. The insight in and comprehension of the
basic problems and the tools developed for their mastery was more
in the focus of discussion than the details of the formal manipulations.
Hence even a student whose primary aim is a broad understanding of
the subject, can peruse the book to his advantage, while those who
go to the trouble of working through the numerous problems, will
develop their technical skill to no small degree. In this manner the
author hopes to have served the needs of a large group of students
and scientific workers, who either tangentially or centrally come in
touch with this eternally fascinating subject.

The author is much indebted to Professor D. E. Rutherford for
accepting this book within his renowned series of monographs. This
gave him an opportunity to elaborate on a subject which is near to
his heart through years of research in this field. The author's own
contributions concerning differentiation of the Fourier series, smoothing
of the Gibbs oscillations, application to interpolation and noise
problems, together with some unusual exercise material, could naturally
be interwoven with the classical treatment.†

The author wishes to express his thanks to Mr Frederick O'Connor
of Trinity College, Dublin, for his help in proof reading, and particu-
larly to Professor Rutherford for his invaluable criticism, which contrib-
uted so much to an improved clarity of both text and formalism.

† The abbreviations A.A. (for "Applied Analysis") and L.D.O. for "Linear
Differential Operators") refer to two previous books of the author; see Bibliography.
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