Contents

Preface **vii**

1 Characteristics of Inverse Problems **1**
1.1 Preliminaries . . . 1
1.2 The least squares estimator . . . 6
1.3 The statistical properties of $\mathbf{x}_{\mathrm{LS}}$ and ill-posedness . . . 8
1.4 An illustrative example . . . 11
Exercises . . . 12

2 Regularization by Spectral Filtering **15**
2.1 Spectral filtering methods . . . 15
2.2 Regularization parameter selection methods . . . 19
2.3 Periodic and data-driven boundary conditions . . . 25
Exercises . . . 29

3 Two-Dimensional Test Cases **33**
3.1 Two-dimensional image deblurring . . . 33
3.2 Computed tomography . . . 42
3.3 The preconditioned conjugate gradient iteration . . . 45
Exercises . . . 47

4 Bayes' Law, Markov Random Field Priors, and MAP Estimation **53**
4.1 Bayes' law and regularization . . . 53
4.2 Choosing $p(\mathbf{x}|\delta)$: Gaussian Markov random fields . . . 54
4.3 Choosing $p(\mathbf{x}|\delta)$: Laplace Markov random fields . . . 65
4.4 The infinite-dimensional limit . . . 68
Exercises . . . 72

5 Markov Chain Monte Carlo Methods for Linear Inverse Problems **77**
5.1 Sampling from high-dimensional Gaussian random vectors . . . 77
5.2 Hierarchical modeling of λ and δ and sampling from $p(\mathbf{x}, \lambda, \delta|\mathbf{b})$ . . . 81
5.3 Alternative MCMC methods for sampling from $p(\mathbf{x}, \lambda, \delta|\mathbf{b})$ . . . 89
Exercises . . . 99

6 Markov Chain Monte Carlo Methods for Nonlinear Inverse Problems **105**
6.1 A general setup for nonlinear inverse problems . . . 105
6.2 Levenburg–Marquardt nonlinear least squares optimization . . . 106

6.3 Randomize-then-optimize as a proposal for Metropolis–Hastings . 109
6.4 Nonlinear test cases 112
6.5 Hierarchical modeling of λ and δ and sampling from $p(\mathbf{x}, \lambda, \delta|\mathbf{b})$. 118
Exercises 122

Bibliography **125**

Index **131**