

Contents

List of Figures	ix
List of Algorithms	xiii
Preface	xv
I Introduction to Linear Systems with Time Delays	1
1 Introduction	3
1.1 Why Time Delays?	3
1.2 Effects of Delays on Dynamic Response	4
1.2.1 Destabilizing Effects	4
1.2.2 Stabilizing Effects	5
1.2.3 Performance Degradation	5
1.3 Delays in Linear Time-Invariant Dynamical Systems	5
1.3.1 Discrete versus Distributed Delays	6
1.3.2 Single- versus Multiple-Delay Systems	6
1.3.3 Retarded- versus Neutral-Type Systems	7
1.4 Studies on Stability Analysis	8
1.4.1 Early Studies	8
1.4.2 Energy-Based Methods versus Eigenvalue Approaches	8
1.4.3 Frequency Domain Techniques	9
1.5 Concluding Remarks	12
2 Stability in Delay and System Parameter Spaces	13
2.1 Characteristic Functions	13
2.2 Stability Condition and Definitions	14
2.3 Stability Condition and Challenges	15
2.4 Critical Crossings over the Imaginary Axis	16
2.5 Sensitivity of Critical Roots	17
2.6 Decomposition of Parameter Space	19
2.7 Concluding Remarks	21
3 Analyzing Stability: Two Fundamental Examples	23
3.1 Destabilizing Effect of Delay	23
3.2 Stabilizing Effect of Delay	27
3.3 Performance Analysis	31
3.4 Concluding Remarks	32

4	Discussion	35
II	Problems with Single Delay	37
5	Preliminaries	39
5.1	Motivations and Mathematical Formulations	39
5.2	Examples 1–3	40
6	Stability of SISO Systems	43
6.1	Computation of Critical Roots	43
6.1.1	Elimination of the Exponential Function	43
6.1.2	A Geometric Interpretation	44
6.2	Sensitivity of Critical Roots	46
6.3	Concluding Remarks	48
7	Stabilizing PI/PID Controllers via Frequency Sweeping	49
7.1	Stabilizing PI Controllers	49
7.1.1	Identification of Critical Gains	49
7.1.2	Identification of Stable versus Unstable Regions	50
7.1.3	Performance Analysis	51
7.2	Stabilizing PID Controllers	54
7.2.1	Identification of Critical Gains	54
7.2.2	Identification of Stable versus Unstable Regions	54
7.2.3	Performance Analysis	56
7.3	Concluding Remarks	57
8	Degenerate Cases	59
8.1	System Poles Tangent to Imaginary Axis	59
8.2	Zero Crossing at the Origin	62
8.3	Crossing of Double Imaginary Poles	64
8.4	Concluding Remarks	67
9	Discussion	69
III	Problems with Commensurate Delays	71
10	Preliminaries	73
10.1	Motivations and Mathematical Formulations	73
10.2	Example 4	73
11	Kronecker Operations	77
11.1	Approach 1: Solve $ z = 1$ first, $s = j\omega$ next	77
11.1.1	Solving $z := e^{-\tau s}$ from a Finite Dimensional Eigenvalue Problem	78
11.1.2	Solving $s = j\omega$ from a Finite Dimensional Eigenvalue Problem	79
11.1.3	Study of Example 1	80
11.1.4	Study of Example 2	80
11.1.5	Study of Example 3	81
11.1.6	Study of Example 4	82

11.2	Approach 2: Solve $s = j\omega$ first, $ z = 1$ next	84
11.2.1	Solving $s = j\omega$ from a Finite Dimensional Eigenvalue Problem	84
11.2.2	Solving $z := e^{-\tau s}$ from a Finite Dimensional Eigenvalue Problem	85
11.2.3	Study of Example 1	86
11.2.4	Study of Example 2	86
11.2.5	Study of Example 3	87
11.2.6	Study of Example 4	87
11.3	Concluding Remarks	88
12	Bilinear Transformation and Its Variations	91
12.1	Properties of Bilinear Transformation	92
12.1.1	Geometric Interpretations	92
12.1.2	Root Crossings and Sensitivity	93
12.1.3	Special Points	94
12.2	Study of Example 1	95
12.3	Study of Example 2	96
12.4	Study of Example 3	97
12.5	Study of Example 4	97
12.6	Concluding Remarks	98
13	CTCR and Root Clustering	99
13.1	CTCR Propositions	99
13.1.1	Proposition 1: Finite Number of Crossings	99
13.1.2	Proposition 2: Root Tendency Invariance	100
13.2	Root Clusters, Kernel, and Offspring	100
13.3	Example 5	103
13.4	Example 6	104
13.5	Concluding Remarks	106
14	Other Approaches to Handle Commensurate Delays	107
14.1	Frequency Sweeping and Geometric Interpretations	107
14.1.1	Study of Example 4	108
14.2	Elimination of Exponential Functions	109
14.2.1	Study of Example 4	111
14.3	Schur–Cohn Criterion	111
14.3.1	Example 7	114
14.4	Concluding Remarks	115
15	Discussion	117
IV	Problems with Two Independent Delays	119
16	Preliminaries	121
17	CTCR and Root Clustering	123
17.1	Crossings and Stability Switches	123
17.2	CTCR Propositions	124

17.2.1	Proposition 1: Kernel and Offspring Curves	124
17.2.2	Proposition 2: Root Tendency Invariance	125
17.3	Concluding Remarks	126
18	CTCR Concepts and Bilinear Transformation	129
18.1	Example 8	130
18.1.1	Computation of CFS Ω and PSSC φ	131
18.1.2	Stable versus Unstable Regions	133
18.1.3	Demonstration of CTCR Propositions	134
18.2	Concluding Remarks	137
19	Kronecker Operations	139
19.1	Study of Example 8	140
19.2	Example 9	141
19.3	Concluding Remarks	143
20	Frequency Sweeping and Geometric Interpretations	145
20.1	Computation of CFS Ω and PSSC φ	146
20.2	Stable versus Unstable Regions	147
20.3	Study of Example 9	149
20.4	Example 10	153
20.5	Concluding Remarks	156
21	Discussion	157
	Bibliography	159
	Index	171

List of Figures

2.1	An example closed-loop system with linear controller $C(s)$, plant $G(s)$, sensor $H(s)$, and delay τ represented with $e^{-\tau s}$ in the Laplace domain with s being the Laplace variable.	14
2.2	Representative rightmost root locations (marked with bold crosses) and the spectral abscissa μ in (2.4) for a given delay τ	16
2.3	Representative locus of characteristic roots with respect to the delay parameter.	17
2.4	Representative locus of characteristic roots with respect to the delay parameter.	18
2.5	Representative locus of characteristic roots.	19
2.6	Examples of stability-instability composition of the delay parameter space.	20
3.1	Critical delays (3.6) form boundaries with respect to the controller gain K	26
3.2	(Left) Critical delays form boundaries with respect to the controller gain K . (Right) Zoom-in around a stability region and to demonstrate the location of critical delays on the boundaries.	30
3.3	MATLAB/SIMULINK diagram of the closed-loop dynamics in Subsection 3.1.	31
3.4	(Left) Simulation of the closed-loop system studied in Subsection 3.1. (Right) Simulation of the closed-loop system studied in Subsection 3.2.	32
6.1	Locus of $L(\omega)$ and its interaction with the unit circle on the complex plane for Example 2.	45
7.1	(Left) Critical controller gains k_p, k_i obtained by sweeping $\omega \in [0, 10]$ in (7.2)–(7.3) with a step size of 0.01 and $\alpha = 10$ (light curves). Moreover, the dynamics has a zero pole at $k_i = 0$ for any k_p , which is indicated with a bold curve. (Right) Zoom-in of the left panel. The system is stable inside the boundaries.	50
7.2	The real part of the rightmost (dominant) pole with respect to PI controller gains (grayscale) superimposed with the stability boundaries obtained in Figure 7.1.	52
7.3	Unit step response of the closed-loop dynamics in Figure 2.1 with $C(s) = k_p + k_i/s$, $G(s) = 1/(s + \alpha)$, $H(s) = 1$, and $\tau = 1$ [s].	53

7.4	(Left) PSSC (solid) on the plane of $k_p - k_i$ for fixed $k_d = 0.5$ and loop delay $\tau = 1$ for frequency sweep parameter $\omega > 0$. Whenever $k_i = 0$, the system always has a zero pole (marked with a bold line). (Right) PSSC (dashed) when the delay is reduced to $\tau = 0.75$ compared with those (solid) in the left panel with $\tau = 1$	55
7.5	The real part of the rightmost (dominant) pole with respect to PI controller gains (grayscale) with $k_d = 0.5$ and loop delay $\tau = 1$, superimposed with the stability boundaries obtained in Figure 7.4.	56
7.6	Unit step response of the closed-loop dynamics with the characteristic function (7.4).	57
8.1	(Left) Location of two system poles (dark circles) in the vicinity of the imaginary axis for $\tau \in [0.01, 0.5]$ and $K = \tilde{K} \approx 4.9749$. The locations of the delay-free system ($\tau = 0$) poles at $s = -0.5 \mp 5.4521j$ are marked with light circles. (Right) Zoom-in of system poles in the vicinity of $s = 4.9497j$ and $\tau = 0.3377$ where the locus for $K = \tilde{K}$ makes a tangent to the imaginary axis. Two other loci are also presented, one for $K = 4.8$ and the other for $K = 5.1$, to demonstrate the progression of locus. Both plots are obtained by the TRACE-DDE toolbox [23, 24], specifically with the SPIGCHART.M MATLAB file. Arrows are manually inserted to demonstrate the movement of the poles in the increasing direction of the delay parameter.	61
8.2	Real part of the two dominant poles of the dynamics represented by the characteristic function (8.8) with respect to delay τ , plotted by TRACE-DDE [23, 24].	64
8.3	The system described by the characteristic function (8.12) has double complex conjugate poles $s = \mp 3j$ for delay $\tau = 0$ (only the positive part of the imaginary axis is shown).	66
8.4	The rightmost roots of the system described by the characteristic function (8.12) for a delay range of $\tau \in [0.01, 2.5]$ to demonstrate how the poles create a locus tangent to the imaginary axis from the right, at $s = 3j$. Arrows are placed at the exact locations as in Figure 8.3 as a point of reference.	67
10.1	Two-mass vibration problem with $m_1 = 0.5$ kg, $m_2 = 2$ kg, $c = 20$ N/(m/s), $k_1 = 100$ N/m, and $k_2 = 175$ N/m. Inputs to the system are u_1 and u_2 , and system states are $x = (x_1(t), \dot{x}_1(t), x_2(t), \dot{x}_2(t))^T$	74
11.1	Time simulation of the dynamics in Example 4 with ODE4 and fixed time step of one millisecond and initial conditions $(1, 0, 0, 0)^T$	88
12.1	Given $\omega = 1$, the unit circle \mathcal{C}_1 (left) is plotted with respect to τ , while the unit circle \mathcal{C}_2 (right) is obtained with respect to T	93
13.1	A representative example with three clusters, whereby all critical characteristic roots $s = j\omega_k$ in each cluster are created by infinitely many delays τ_{k,ℓ_k} marked with symbols and presented separately along a particular axis.	102
13.2	Delay axis with all delay values from the three clusters carried over from Figure 13.1.	102

14.1	Example 4.	109
17.1	Schematic demonstration of grid size change with respect to $\omega \in \Omega$ and the position of critical delay values (markers) with respect to each other.	124
18.1	The points on the PSSC \wp obtained with Rekasius transformation, $k_p = 2, k_d = 0.5$	133
18.2	The points on the PSSC \wp from Figure 18.1.	133
18.3	Example 8. Kernel and offspring curves construct the PSSC.	135
18.4	Example 8. Demonstration of root tendency invariance at some points on the PSSC.	136
18.5	Stability regions (shaded) of the dynamics represented by the characteristic function (18.7).	136
19.1	The points on the PSSC \wp obtained with Kronecker operations, $k_p = 2, k_d = 0.5$	142
19.2	Example 9. The points on the PSSC \wp obtained with Kronecker operations.	143
20.1	Geometric interpretation of the system characteristic function with phasors on the complex plane.	146
20.2	Demonstration of tangent and normal vectors at a point on the PSSC.	149
20.3	Example 9. Frequency sweeping to check the conditions for triangle formation.	150
20.4	Example 9. PSSC obtained by frequency sweeping.	152
20.5	Demonstration of the tangent vector at a point on the PSSC in the direction of increasing ω	152
20.6	Stability regions (shaded) of Example 9.	153
20.7	Example 10. Frequency sweeping to check the conditions for triangle formation.	154
20.8	Example 10. (Left) PSSC obtained by frequency sweeping. (Right) Stable regions (shaded) obtained by TRACE-DDE [23, 24].	155

List of Algorithms

Algorithm 3.1	Detection of critical delay values of (3.2)	26
Algorithm 3.2	Detection of critical delay values of (3.10)	30
Algorithm 7.1	TRACE-DDE and spigchart.m/PI controller	51
Algorithm 8.1	TRACE-DDE and spigchart.m/poles tangent to imaginary axis	60
Algorithm 11.1	Algorithm based on Approach 1/Kronecker sum	82
Algorithm 11.2	Algorithm based on Approach 2/Kronecker sum	87
Algorithm 13.1	MAPLE code to solve the critical crossings using elimination theory	104
Algorithm 14.1	Algorithm to compute the critical roots z based on frequency sweeping	108
Algorithm 18.1	MAPLE code to obtain $\mathcal{R}_{11}, \mathcal{R}_{21}, \mathcal{R}_{22}$	131
Algorithm 18.2	MATLAB code to obtain the points on PSSC	131
Algorithm 19.1	Two-delay stability algorithm based on Kronecker operations	140
Algorithm 20.1	Example 9. Algorithm to check the conditions for triangle formation	149
Algorithm 20.2	Example 9. Algorithm to plot the points on the PSSC	151
Algorithm 20.3	Example 10. Computing the real part of rightmost roots using TRACE-DDE	154

Preface

In many dynamical systems surrounding us, events often unfold with aftereffects. This is because it takes time for an event to take place upon being triggered by a stimulus. The duration of time between when a stimulus is applied and the onset of the event is the *time delay*. The effects of time delays on the behavior of dynamical and control systems form the main focus of this book.

The presence of delays as has been widely observed and studied draws attention to a number of interesting characteristics in dynamical and control systems in the way such systems evolve and interact with each other, and how decisions are formulated. In many cases such characteristics can be intriguing but difficult to assess, and can challenge one's intuition; this very likely explains the large body of literature on the topic spread across mathematics, engineering, physics, and economics. The presence of delays also brings about interesting philosophical questions regarding how events take place, how we perceive and react to them, how our past actions propagate through time and space, and how such actions may influence our current states.

In the context of dynamical and control systems, delays arise due to a number of reasons, including the time needed to transmit information between two physical points, to formulate complex decisions based on the information available, and to implement decisions/policies. Delays can arise in sensors and information infrastructure, in controllers, e.g., due to long computation times, and in actuators that do not instantaneously respond to input commands. In supply chain management one finds analogous observations, where supplies take time to displace between physical points, supply chain managers need time to evaluate certain factors before implementing key decisions, and such decisions when implemented can take time to take effect.

Numerous manuscripts, books, and conference proceedings have been published on the topic of *time delay systems*, with one primary topic of interest being *stability*. In the context of linear time-invariant (LTI) systems, stability analysis can be performed mainly under two well-recognized directions, namely, in the time domain and in the frequency domain. This book focuses on mastering frequency domain techniques for two main reasons, one being that the topic falls within the expertise of the author, and second, these techniques for time delay systems can be easily explained using the tools of classical control for systems without delays, e.g., in terms of eigenvalues, pole placement, complex analysis, root locus, and Nyquist stability criterion. Therefore this book is a convenient starting point, especially for readers new to the area and readers who would like to rapidly develop skills based on graduate level understanding of classical and modern control.

What inspired me the most to develop this book were the many questions I have received from colleagues regarding how to analyze the stability of various types of LTI systems with time delays. One other motivation was my desire to spread knowledge as clearly as possible not only to experts but also to readers willing to learn about the field

but require some guidance at the beginning. In that regard, I hope this book achieves the purpose of disseminating key knowledge and can stimulate interest on the topic, expand the utilization of existing tools to new fields, become a valuable reference source, and help train graduate students.

Many years have gone into studying the literature, reading, and hands-on practice of the topic. This would not have been possible if I had not been introduced to the topic during my graduate studies at the University of Connecticut (2000–2005), where I worked with Professor Nejat Olgac on the stability analysis of LTI systems with single and multiple delays, and if I had not been guided in my postdoctoral position at HeuDiaSyC (CNRS) at Université de Technologie de Compiègne, France (2005–2006), where under a Chateaubriand postdoctoral fellowship of the French government I worked with Professor Silviu-Iulian Niculescu (currently at Ecole Centrale-Supelec (CNRS)) on time delay systems from supply chain management, car following dynamics and human reaction delays, and from the complex behavior arising due to the network infrastructure of coupled dynamical systems.

During my training, I was also fortunate to be exposed to studies in the areas of frequency domain techniques by, in no particular order, Gabor Stepan, Silviu-Iulian Niculescu, Jie Chen, Keqin Gu, Arild Thowsen, Zenonas V. Rekasius, Kenneth L. Cooke, Pauline van den Driessche, Richard F. Datko, Jack K. Hale, David Hertz, Ezra Zeheb, Eliahu I. Jury, John Chiasson, Edward W. Kamen, Chieh-Su Hsu, S. J. Bhatt, Fatihcan M. Atay, Zaihua Wang, Stephen Barnett, Wim Michiels, Dirk Roose, Nejat Olgac, Dimitri Breda, Stefano Masset, Rossana Vermiglio, Sabine Mondie, Elias Jarlebring, Tomas Vyhlidal, Yusuf Altintas, Hitay Ozbay, Dirk Helbing, Frank Allgower, and A. Galip Ulsoy. The list, as one may guess, is much much longer, but I have kept it brief to guide the reader to excellent research being conducted throughout the world on this topic.

With regard to what this book is about, I would note that it is slightly different in spirit and organization from traditional books. It is mainly focused on explaining in great detail a number of fundamental stability analysis techniques and how these techniques are implemented to solve problems. To serve this main purpose, these techniques are introduced and discussed in relation to other techniques to interweave them and establish connections, which are then followed by step-by-step solutions of numerical problems. The main reason for this was to help the reader to view the literature within a broader perspective to understand how key building blocks link with one another, and was to enable effective problem solving, which is rarely published in sufficient detail in archival journals inhibiting scientific progress and dissemination of knowledge. While preparing the book with this perspective, I also did my best in the introduction (Chapter 1) to provide sufficient background and a thorough list of key references. The main reason for this concise presentation was to avoid duplicating the work in excellent books already available in the literature, but instead to set the base for the subsequent discussions pertaining to the techniques and treatment of case studies.

One critical point about this book is that various techniques are not necessarily covered in isolation from one another. First, this helps the reader to see how different techniques are interrelated, if any are, and enriches the discussions. Needless to say, all techniques are properly cited and approaches are credited. Second, this approach helps educate the reader on the philosophy of the techniques and enables the reader to see them from various perspectives. In this regard, this book is not a compilation of stand-alone techniques, but instead the presentation is optimized to make the most of the material when teaching it. In that sense, the presentation does not always flow

linearly in chronological order but, when appropriate, pulls other relevant information together to make it more complete.

It is important to note that this book is provided as a reference resource to study and solve various problems, to be used as a refresher, and to train graduate students, but it is by no means complete in terms of references. One would appreciate the depth and richness of the literature, and I have done my best to present some key references to shed light on the problems studied. The book rather provides certain techniques and references therein as anchor points; it is therefore recommended that the reader utilize and connect these anchor points within their own literature reviews and research. In relation to this, it is important to note that the material presented should not be perceived as the first and/or the sole results on the topic, but rather a nucleation point from which to begin studying on the topic. Throughout the book, I have done my best not to insert any subjective opinions on the techniques since the goal here is to solve relevant problems, rather than champion a particular trend of approaches. In this sense, it is left to the reader to formulate a comprehensive understanding of the techniques and relevant literature when making comparisons between certain research trends. Nevertheless, I have still provided some objective ways to compare and contrast the approaches; whenever I did so, this was intended to provide the reader with a way to critically think, but not to diminish one approach over another.

To be able to comfortably follow this book, the reader should have graduate-level understanding of LTI systems, as well as classical and modern control. Specifically, knowledge of stability, root locus, the dominant pole concept, complex analysis, and stability-instability characterization in parameter space is needed as a foundation to easily extend to the understanding of how to analyze the stability of LTI systems with delays. While each chapter is self-contained and clearly organized into sections with headings, I would definitely recommend starting with the preliminaries in Chapters 2–4, and next examining Chapters 5–9, which are related to the fundamentals and treatment of single-delay cases. Once this is established, the reader can comfortably move on to the study of commensurate delay problems (Chapters 10–15) and followed by two-delay problems (Chapters 16–21). Moreover, the chapters are presented with interlinks to enable the reader to see how various techniques may have certain relationships; these cross references can guide the reader through various topics of interest.

The best way to learn is to practice. I would therefore strongly recommend that the reader reproduce the examples presented, comparing them side by side with the solutions provided, and next expand on more complex problems. Some solutions are also supported by MATLAB and MAPLE routines within the chapters and are marked as Algorithms to assist the reader with coding. While I am not a proficient coder, I hope that what I provide is sufficient to guide the exercises. I feel that once the reader solves the presented examples step by step, he/she will have gained a decent understanding in handling a variety of problems with time delays. To further assist the reader, some relevant MATLAB codes will be made available through the link

www.siam.org/books/cs20

All such current and future content is provided “as is”; neither SIAM nor the author are responsible for unfortunate errors in the online content.

Last but not least, I must mention that it was not possible to cover all possible problems; the focus is on those that have little ambiguity in their solution, including some degenerate cases. The interested reader can follow the most recent trends at conferences organized by IEEE, IFAC, and ASME. In particular, the IFAC Workshop on Time Delay Systems attracts a large group of researchers working on various aspects of time delay

systems. Moreover, we see an increase in the number of edited volumes being published on the topic; a large collection can be found in the Springer Lecture Notes in Control and Information Sciences series and more recently in Springer Advances in Delays and Dynamics.

Before I close, I would like to express my sincere regards to SIAM, Executive Editor Elizabeth Greenspan, and the anonymous reviewers for their support and trust in me in providing this book for the community. I also would like to acknowledge the many opportunities I have had that enabled me to work on time delay systems, specifically through the support of the National Science Foundation and working with wonderful graduate students. None of this work would have been possible without a collegial and productive working environment. I am therefore indebted to the support of Northeastern University, College of Engineering, and the Department of Mechanical and Industrial Engineering. I also express my special thanks to Mr. Yanchao Wang, who provided valuable feedback on Parts I and II of this book.

And of course, none of this would be possible without support in one's life for the good days and the tough days; I cannot express in words how indebted I am to my wife Alix for her unlimited support and trust in what I do, and in what I love to do. As a tiny token of thanks, I dedicate this book to her and our eternal love.

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Index

- algebraic geometry, 11
- analytic geometry, 148, 158

- backward continuation, 14
- bifurcation, 12
- bilinear transformation, 11, 91, 122, 129
- business cycles, 8, 9

- characteristic equation, 13
- characteristic function, 13
- characteristic root, 8, 9, 15
- characteristic time, 51
- cluster treatment of
 - characteristic roots (CTCR), 11, 99, 123, 157
- commensurate degree, 7
- compensator, 9
- consensus, 103, 104
- controller
 - proportional derivative (PD), 40, 130
 - proportional integral (PI), 10, 41, 49, 81
 - proportional integral derivative (PID), 10, 54, 58
 - proportional-integral-retarded, 58
 - proportional-retarded, 58
- core curve, 130
- critical crossing, 16
- critical delay, 17
- critical eigenvalues, 9
- crossing frequency set, 40, 123
- CTCR (cluster treatment of characteristic roots), 11, 99, 123, 158

- DDE-BIFTOOL, 12
- decomposition, 8, 19
- decomposition theorems, 11

- delay
 - commensurate, 73, 158
 - destabilizing effect, 4, 23, 32
 - discrete, 6
 - distributed, 6
 - finite, 14
 - infinite, 14
 - loop, 13
 - multiple, 6, 11, 121, 157
 - performance degradation, 5
 - pseudo-, 11, 91
 - single, 6, 7, 73
 - stabilizing effect, 5, 27, 33
- delay algebraic equation, 7
- delay-free system, 15
- delay-independent stable, 25, 158
- delay-independent unstable, 26
- delay margin, 25, 115
- differential and difference equations, 7
- dominant eigenvalue, 12
- dominant pole, 31
- dominant root, 31, 51

- eigenvalues, 9
- elimination of exponential functions, 43, 109
- elimination theory, 11, 98, 106
- equilibrium, 14
- existence of solutions, 14
- exponential decay, 14

- final value theorem, 32
- first-in first-out delay, 6
- forward continuation, 14
- frequency domain technique, 9
- frequency sweeping, 10, 44, 107, 122, 147
- full-state feedback, 73

- generalized eigenvalue problem, 10, 115
- geometric interpretation, 10, 44
- Grashof four-bar linkage theory, 156

- high-speed networks, 158

- imaginary root, 17
- implicit function theorem, 147, 156
- input delay, 103
- instability, 15
- invariance properties, 11, 100, 125
- inventory regulation, 8

- kernel
 - CTCR, 100, 124
- Kronecker operation, 10
- Kronecker product, 77, 85, 88
- Kronecker sum, 77, 79, 84, 88, 122, 139, 158

- Lambert W functions, 12
- law of cosines, 147
- linear matrix inequality, 9
- Lyapunov functionals, 15
- Lyapunov–Krasovkii framework, 9

- Möbius transformation, 92
- matrix pencils, 10

- neutral system, 7
- nonlinear optimization, 57
- nonminimum phase, 57
- normal vector to PSSC, 148
- \mathcal{NP} -hard, 121, 157
- Nyquist stability criterion, 9

- offspring
 - CTCR, 100, 124

- ordinary differential equations, 14
- Orlando's formula, 112
- overshoot, 33, 53
- Padé approximation, 92
- performance, 51
- performance analysis, 31
- perturbation, 14
- phasor, 145
- pole
 - double imaginary, 64, 68
 - double zero, 62
 - invariant pole, 62
 - placement, 73
 - standing, 62
 - tangent to imaginary axis, 60
 - triple, 68
- Pontryagin's energy principle, 9
- posicast control, 8
- potential stability switching
 - curves (PSSC), 26, 123, 132, 141, 157
- QPmR, 12
- quadrilateral formation
 - conditions, 156
- quasi-polynomial, 8, 9, 13
- Rekasius transformation, 11, 91, 106, 122, 129
- responsible eigenvalue, 104
- resultant, 104
- retarded system, 7
- rightmost eigenvalue, 12
- rightmost root, 12, 16, 51
- root clustering, 67, 99, 123
- root clusters, 100, 124
- root continuity, 9, 11
- root counting, 9
- root tendency invariance, 100, 125
- Routh's array, 130
- Schur–Cohn criterion, 111
- Schur–Cohn matrix, 112
- Schur–Cohn–Fujiwara matrix, 111
- sensitivity
 - root, 11, 17, 100, 125
- settling time, 33, 53, 57
- SIMULINK diagram, 31
- single-input single-output (SISO) system, 57, 104
- Smith predictor, 8
- smoothness, 11, 148
- spectral abscissa, 51
 - function, 16
- spectral radius, 8
- spectrum, 15
- spectrum separation, 51
- stability, 14
 - asymptotic, 15
 - condition, 15
 - exponential, 14
 - marginal, 15, 33, 103
- stability analysis
 - nonconservative, 33
- stability-instability transition, 23
- stability map, 11, 158
 - cross-sectional, 158
- stability reversals, 19, 20, 33
- stability robustness, 158
- stability surface, 158
- state-space dynamics, 6, 60
- step response, 31, 33, 53, 57
- Sylvester's matrix, 98, 104
- tangent vector to PSSC, 148
- Taylor's series, 63
- time-domain simulation, 31
- TRACE-DDE, 12, 51, 60, 153, 154
- transfer function
 - closed-loop system, 13
 - proper, 14
 - strictly proper, 14
- triangle formation conditions, 145, 146
- two delays, 121
- two-variable polynomials, 10
- undershoot, 57
- unit circle, 10, 92, 139, 158
- unstable system, 15
- vibration control, 73
- YALTA, 12