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Nonlinear Time Scale Systems in Standard and Nonstandard Forms: Analysis and Control. *By Anshu Narang-Siddarth and John Valasek*. SIAM, Philadelphia, 2014. $94.00. xvi+219 pp., hardcover. ISBN 978-1-611973-33-4.

Singular perturbation methods in control have provided a very successful application of many asymptotic techniques, involving applied mathematicians and engineers. The early work was summarized in Kokotovic, Khalil, and O'Reilly [1], now reprinted as a SIAM Classic. This new book has developed from the recent thesis research of Professor Narang-Siddarth at Texas A&M University, where Professor Valasek was her advisor. It is less specifically oriented toward aerospace applications than Ramnath [2] and indeed, it begins with a presentation of multiple time scale phenomena quite generally before considering design aspects and stabilizing controls.

The standard problem consists of the initial value problem for the coupled slow-fast vector system

$$\dot{x} = f(t, x, z, u),$$
$$\epsilon\dot{z} = g(t, x, z, u),$$

with a small positive parameter ϵ. Its limiting outer solution away from a thin initial layer results when we can solve the limiting algebraic constraint for

$$z = h(t, x, u),$$

resulting in a reduced-order control problem. These authors call the problem nonstandard when they can't solve for z in this way. They naturally seek ways to transform the given problem to a standard one. The control aspect makes the problem interesting, and computed solutions for aerospace and other examples provide a check on any intuitive design choices made.

Not surprisingly, inner and outer (or slow and fast) problems arise, and one naturally seeks the composite control as the sum of slow and fast parts. Stability hypotheses naturally involve Liapunov functions, and extensions with a hierarchy of several small parameters multiplying derivatives occur. There's a nice overview of classical results, including an emphasis on a role of the slow manifold. Most significant and novel, however, is the treatment of nonstandard examples. This is very worthy of further development, regarding both theory and practice. The authors deserve our thanks for their successful and provocative developments.

REFERENCES

[1] P. Kokotovic, H. K. Khalil, and J. O'Reilly, *Singular Perturbation Methods in Control: Analysis and Design*, Academic Press, London, 1986.

[2] R. V. Ramnath, *Multiple Scales Theory and Aerospace Applications*, AIAA Education Series, Reston, VA, 2010.

Robert E. O'Malley, Jr.
University of Washington