Preface

Many numerical approximations rely on a simple iteration, $X_{j+1} = f(X_j)$, to generate a sequence $\{X_j\}$ of approximations to a certain target. A doubling algorithm is an idea to accelerate the process by skipping all X_j but those for $j = 2^i$. This idea traces back to the 1970s for solving continuous-time algebraic Riccati equations and discrete-time algebraic Riccati equations, and matrix Riccati differential equations from the optimal control theory. In the last 20 years or so in large part due to an extremely elegant new formulation that preserves structures and analysis, research in doubling algorithms has expanded and continues to be very active, leading to beautiful theories, numerically effective and robust algorithms for not only the aforementioned Riccati equations but also new types of nonlinear matrix equations that have found diverse applications including, beyond the optimal control theory, applied probability and transportation theory, Markov-modulated fluid queue theory, quantum transport in nano research, and vibration analysis of fast trains, to name just a few. In this book, we seek to develop a unified framework and theory of two structure-preserving doubling algorithms for solving nonlinear matrix equations in association with the eigenspaces of certain regular matrix pencils. It is broad enough to include, but is not limited to, the aforementioned nonlinear matrix equations.

This book will be of use to researchers as well as computational scientists and engineers who are seeking efficient methods for certain nonlinear matrix equations. Much of the book requires graduate-level linear algebra knowledge for full comprehension.

The first chapter introduces precisely the types of nonlinear matrix equations that will be included in our general framework, discusses the motivation and brief history of the structure-preserving doubling algorithms, and outlines the main contributions of this book, as well as most of the common notations that we will be adopting throughout. Chapter 2 gives some of the relevant basics of matrix theory, including the Weierstrass canonical form of a regular matrix pencil, and Hamiltonian and symplectic matrices/matrix pencils and their special eigenstructures. Our general theory of doubling algorithms for nonlinear matrix equations in association with the eigenspaces of certain regular matrix pencils is fully developed in Chapter 3. While it is mostly inspired by existing research already published, there are new material and results. For example, the second standard form differs from the one in [33], the primal-dual view is much deeply developed, and incorporating initial guesses has never been discussed before. In particular, the framework is much broader than any of the existing literature, which is usually focused on one particular type of nonlinear matrix equation at a time.

In Chapters 4 and 5, the focus is on applications to the algebraic Riccati equations from the control theory that places special structural properties on the coefficient matrices. These properties allow us to say more about the convergence of the first doubling algorithm such as monotonicity, robustness, and global convergence. The use of a doubling algorithm to solve the M-matrix algebraic Riccati equations (MARE) is explained in detail in Chapter 6. In Chapter 7, we treat H^*-matrix algebraic Riccati equations which relate to and have the same sources as one of those for MARE and Bethe–Salpeter algebraic Riccati equations arising from the Bethe–Salpeter equation in the quantum chemistry and material science. Finally, Chapter 8 is devoted to the nonlinear matrix equation of the form $X + BX^{-1}A = Q$, which has been seen in the solutions of quadratic eigenvalue problems, the Green's function approach for quantum transport in nano research, and others. In particular, we will highlight its very successful story for the palindromic quadratic eigenvalue problem from vibration analysis of fast trains. We have included illustrative numerical examples for each of the presented applications.

Acknowledgments

Huang's research has been supported by the Ministry of Science and Technology and the National Center of Theoretical Science, Taiwan. Li's research has been supported by grants from the National Science Foundation and a gift from the Intel Corporation from 2013 to 2015. Lin's research has been supported by the Ministry of Science and Technology, the National Center of Theoretical Science, and the ST Yau Center at Chiao Tung University, Taiwan.

We are indebted to our many collaborators for the fruitful collaborations on doubling algorithm research over the years. Much of the convergence analysis throughout either is a direct consequence of these collaborative efforts or follows similar lines of argument. In particular, we would like to thank Chun-Hua Guo (University of Regina), Linzhang Lu (Xiamen University), Wei-guo Wang (Ocean University of China), Shu-Fang Xu (Peking University), and Jungong Xue (Fudan University). Collaborations with them have made this book possible. Li is also a frequent academic visitor to Shanghai University of Finance and Economics and Xiamen University during the summer months, where part of his research on doubling algorithms was conducted. He is grateful for the opportunity provided by the institutions and wishes to thank Leihong Zhang (Shanghai University of Finance and Economics) for his hospitality. Mr. Cairong Chen (Fujian Normal University), Dr. Chun-hua Guo, and Ms. Jie Meng (Pusan National University) painstakingly read the whole draft and found numerous typos/mistakes. We are very grateful for their help.

We thank the three referees for their well-considered suggestions, comments, and advice, which improved the presentation.

It has been a pleasure working with the SIAM staff on the publication of this book. We thank, in particular, Ann Manning Allen, Elizabeth Greenspan, Gina Rinelli Harris, and Cheryl Hufnagle.