

Preface

In the past decade there has been a surge of interest in algebraic approaches to optimization problems defined in terms of multivariate polynomials. Fundamental mathematical challenges that arise in this program include understanding the structure of nonnegative polynomials, the interplay between efficiency and complexity of different representations of algebraic sets, and the development of effective algorithms. Remarkably, and perhaps unexpectedly, convexity provides a new viewpoint and a powerful framework for addressing these questions. This naturally brings us to the intersection of *algebraic geometry*, *optimization*, and *convex geometry*, with an emphasis on algorithms and computation. This emerging area has become known as *convex algebraic geometry*.

Our aim is to provide an accessible and unifying introduction to the many facets of this fast-growing interdisciplinary area. Each chapter addresses a fundamental aspect of convex algebraic geometry, ranging from the well-established core mathematical theory to the forefront of current research and open questions. Throughout we showcase the rich interactions between theory and applications.

This book is suitable as a textbook in a graduate course in mathematics and engineering. The chapters make connections to several areas of pure and applied mathematics and contain exercises at many levels, providing multiple entry points for readers with varied backgrounds.

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