

# Contents

<b>Preface</b>	<b>xi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 An Example: The Professor's Dairy . . . . .	2
1.1.1 The Setup . . . . .	2
1.1.2 Formulating the Problem and a Graphical Solution . . . . .	2
1.1.3 Changing the Problem . . . . .	4
1.1.4 Discussion . . . . .	6
1.2 Formulations . . . . .	7
1.3 Applications . . . . .	8
1.3.1 The Diet Problem . . . . .	8
1.3.2 Linear Surface Fitting . . . . .	9
1.3.3 Load Balancing Problem . . . . .	10
1.3.4 Resource Allocation . . . . .	10
1.3.5 Classification . . . . .	11
1.3.6 Minimum-Cost Network Flow . . . . .	12
1.4 Algorithms and Complexity . . . . .	14
1.4.1 The Simplex Method . . . . .	14
1.4.2 Interior-Point Methods . . . . .	15
<b>2 Linear Algebra: A Constructive Approach</b>	<b>17</b>
2.1 Jordan Exchange . . . . .	17
2.2 Linear Independence . . . . .	23
2.3 Matrix Inversion . . . . .	27
2.4 Exact Solution of $m$ Equations in $n$ Unknowns . . . . .	32
2.5 Solving Linear Equations Efficiently . . . . .	39
2.6 $LU$ Decomposition . . . . .	41
<b>3 The Simplex Method</b>	<b>45</b>
3.1 A Simple Example . . . . .	46
3.2 Vertices . . . . .	51
3.3 The Phase II Procedure . . . . .	53
3.4 The Phase I Procedure . . . . .	60
3.5 Finite Termination . . . . .	65

3.5.1	The Nondegenerate Case . . . . .	65
3.5.2	Cycling . . . . .	66
3.5.3	The General Case . . . . .	67
3.6	Linear Programs in Nonstandard Form . . . . .	72
3.6.1	Transforming Constraints and Variables . . . . .	72
3.6.2	Scheme I . . . . .	76
3.6.3	Scheme II . . . . .	80
3.6.4	Summary . . . . .	86
<b>4</b>	<b>Duality</b>	<b>89</b>
4.1	Duality and Rank in Linear Systems . . . . .	89
4.2	Duality in Linear Programming . . . . .	94
4.3	Interpretation of Linear Programming Duality . . . . .	96
4.4	Duality Theory . . . . .	97
4.5	KKT Optimality Conditions . . . . .	100
4.6	Dual Simplex Method . . . . .	102
4.7	General Linear Programs . . . . .	107
4.8	Big M Method . . . . .	110
4.9	Applications of Duality . . . . .	112
<b>5</b>	<b>Solving Large Linear Programs</b>	<b>117</b>
5.1	Foundations . . . . .	118
5.1.1	Basic Feasible Solutions and Basis Matrices . . . . .	118
5.1.2	Geometric Viewpoint . . . . .	121
5.2	The Revised Simplex Method . . . . .	123
5.2.1	Upper and Lower Bounds . . . . .	129
5.2.2	Generating Basic Feasible Solutions . . . . .	134
5.2.3	Basis Updates . . . . .	139
5.2.4	Advanced Pivot Selection Mechanisms . . . . .	142
5.3	Network Flow Problems . . . . .	143
5.3.1	Minimum-Cost Network Flow . . . . .	144
5.3.2	Shortest-Path Problem . . . . .	145
5.3.3	Max-Flow Problem . . . . .	146
5.3.4	Transportation Problem . . . . .	147
5.3.5	Assignment Problem . . . . .	149
5.3.6	Network Simplex Method . . . . .	149
<b>6</b>	<b>Sensitivity and Parametric Linear Programming</b>	<b>151</b>
6.1	Sensitivity Analysis . . . . .	151
6.2	Adding New Variables or Constraints . . . . .	155
6.3	Parametric Optimization of the Objective Function . . . . .	158
6.4	Parametric Optimization of the Right-Hand Side . . . . .	164
<b>7</b>	<b>Quadratic Programming and Complementarity Problems</b>	<b>169</b>
7.1	Nonlinear Programs: Optimality Conditions . . . . .	169
7.2	Quadratic Programming . . . . .	172

7.2.1	Basic Existence Result . . . . .	172
7.2.2	KKT Conditions . . . . .	173
7.2.3	Duality . . . . .	176
7.3	Linear Complementarity Problems . . . . .	177
7.4	Lemke's Method . . . . .	178
7.5	Bimatrix Games . . . . .	185
7.5.1	Computing Nash Equilibria . . . . .	186
7.5.2	Zero-Sum Games As Dual Linear Programs . . . . .	192
<b>8</b>	<b>Interior-Point Methods</b>	<b>195</b>
8.1	Motivation and Outline . . . . .	195
8.2	Newton's Method . . . . .	197
8.3	Primal-Dual Methods . . . . .	201
8.3.1	An Affine-Scaling Approach . . . . .	202
8.3.2	Path-Following Methods . . . . .	204
8.3.3	Solution of the Linear System at Each Interior-Point Iteration . . . . .	208
8.3.4	Practical Primal-Dual Methods . . . . .	209
8.4	Interior-Point vs. Simplex . . . . .	212
8.5	Extension to Quadratic Programming . . . . .	212
<b>9</b>	<b>Approximation and Classification</b>	<b>217</b>
9.1	Minimax Problems . . . . .	217
9.2	Approximation . . . . .	218
9.2.1	Chebyshev Approximation . . . . .	219
9.2.2	$L_1$ Approximation . . . . .	221
9.2.3	Approximate Solutions to Systems with Inequality Constraints . . . . .	223
9.2.4	Least-Squares Problems . . . . .	224
9.3	Huber Estimation . . . . .	227
9.4	Classification Problems . . . . .	230
<b>A</b>	<b>Linear Algebra, Convexity, and Nonlinear Functions</b>	<b>237</b>
A.1	Linear Algebra . . . . .	237
A.2	Convex Sets . . . . .	239
A.3	Smooth Functions . . . . .	242
A.4	Convex Functions . . . . .	242
A.5	Quadratic Functions . . . . .	244
A.6	Norms and Order Notation . . . . .	247
A.7	Taylor's Theorem . . . . .	249
<b>B</b>	<b>Summary of Available MATLAB Commands</b>	<b>251</b>
B.1	Basic MATLAB Operations . . . . .	251
B.2	MATLAB Functions Defined in This Book . . . . .	252

**Bibliography****257****Index****261**