

Contents

Preface	xi
Acknowledgments	xvii
List of Figures	xxi
List of Tables	xxiii
List of Algorithms	xxv
I Atmospheric Dynamical Models	1
1 Introduction	3
1.1 Introduction—The Spectral Method	3
1.2 A History of Numerical Weather Prediction	9
2 Governing Atmospheric Dynamics	13
2.1 Introductory Comments	13
2.2 Global Atmospheric Energetics	14
2.3 The Complete Governing Equations	19
2.4 Energy Conservation	24
2.5 Potential Vorticity	25
2.6 Angular Momentum	31
2.7 Exercises	41
3 The Primitive Equations	45
3.1 Introductory Comments	45
3.2 The Hydrostatic Primitive Equations	46
3.3 Potential Vorticity in the HPEs	49
3.4 Angular Momentum	53
3.5 The HPEs with Vertical Coordinate σ	53
3.6 Vorticity and Divergence Equations	60
3.7 Streamfunction and Velocity Potential	66
3.8 The Thermodynamic Equation	67
3.9 Summary of PEAK Model Equations	67
3.10 Conservation of Total Energy	69
3.11 Exercises	72

4	The Shallow-Water Model	75
4.1	Introductory Comments	75
4.2	Formulation of the Shallow-Water Model	78
4.3	Energetics of the SWM	82
4.4	Angular Momentum in the SWM	83
4.5	Vorticity and Divergence Equations	84
4.6	The SWM on the Sphere	86
4.7	Nonlinear Balance of Mass and Wind	88
4.8	A Stationary Solution on the Sphere	95
4.9	Exercises	99
5	The Barotropic Vorticity Equation	103
5.1	Introductory Comments	103
5.2	The Nondivergent Barotropic Model	104
5.3	The Barotropic Model in Spherical Geometry	105
5.4	Rossby–Haurwitz Waves	108
5.5	Barotropic Instability—By Example	110
5.6	Exercises	113
6	Balanced Flow	115
6.1	Introductory Comments	115
6.2	QG Scaling in the SWM	117
6.3	Baroclinic QG Flow	126
6.4	Jet Stream Dynamics	131
6.5	Hydrodynamic Flow Instability	145
6.6	Spectral Vertical Normal-Mode Decomposition	155
6.7	Exercises	163
II	Spectral Numerical Models	167
7	The Spectral Method	169
7.1	Introductory Comments	169
7.2	Series-Expansion Methods	170
7.3	The Spectral Method in NWP	176
7.4	Galerkin and Collocation—Linear Advection	182
7.5	Galerkin and Collocation—Nonlinear Advection	185
7.6	Galerkin and Collocation—Discussion	187
7.7	The Transform Method	195
7.8	Galerkin and Collocation—Burgers Equation	197
7.9	Aliasing—A Bit of Theory	217
7.10	Aliasing—The Simplest Example	223
7.11	Spectral Expansion in NWP Models	228
7.12	Routine <code>sp2gg</code> and the Collocation Grid	232
7.13	Routine <code>gg2sp</code>	237
7.14	The Transform Method in the BVE	245
7.15	Spectral Isotropic Correlations	250
7.16	Exercises	255

8	Vertical Discretization	257
8.1	Introductory Comments	257
8.2	Vertical Staggering of Variables	258
8.3	Formulation of the Vertical Discretization	260
8.4	Energetics of the Vertical Discretization	269
8.5	Exercises	272
9	Time Integration	275
9.1	Introductory Comments	275
9.2	Gravity-Wave Separation	276
9.3	Semi-Implicit Time Differencing	283
9.4	Leapfrog Time Stepping	290
9.5	Exercises	294
10	Code Structure of PEAK	297
10.1	Introductory Comments	297
10.2	Numerical Implementation of PEAK	302
10.3	Description of Standard PEAK Routines	309
10.4	Spectral-Transform Routines	338
10.5	Numerical-Recipes-Based Routines	339
10.6	Numerical Algorithms Group (NAG) Routines	340
10.7	Exercises	340
11	Experimentation with PEAK	343
11.1	Introductory Comments	343
11.2	Setting Up PEAK	344
11.3	PEAK in NAG-Using Mode (NAGM)	347
11.4	Validation of PEAK	349
11.5	Exercises	364
12	Barotropic PEAK Configurations	371
12.1	Introductory Comments	371
12.2	The Spectral BVE Model in PEAK	372
12.3	The Spectral SWM in PEAK	381
12.4	Exercises	399
III	Appendices	401
A	Tensor Analysis	403
A.1	Introductory Comments	403
A.2	Vector Calculus	404
A.3	Orthogonal Curvilinear Coordinates	405
A.4	Standard Spherical Polar Coordinates	411
A.5	Modified Spherical Polar Coordinates	413
A.6	Gradient of a Scalar	414
A.7	The Curl	415
A.8	The Divergence	416
A.9	The Laplacian	417
A.10	The Physical Components of a Vector	418

A.11	Advection of a Scalar	420
A.12	The Helmholtz Decomposition	420
A.13	Generalized Mass Continuity	421
A.14	Generalized Vertical Velocity	423
A.15	The HPE Material Derivative	424
A.16	Solutions to Selected Exercises	425
B	Spectral Basis Functions	433
B.1	Associated Legendre Functions	433
B.2	Tabulated Legendre Polynomials	435
B.3	Computation of $\hat{P}_n^m(\mu)$ in Routine <code>plgndr2</code>	435
B.4	Spherical Harmonics	438
B.5	Gaussian Quadrature	438
C	The PEAK Model Code	441
C.1	The PEAK Code	441
C.2	Spectral Transform Routines	463
C.3	Numerical-Recipes-Based Routines	464
C.4	Spectral Transform Routines with NAG	466
C.5	Normal-Mode Computations Code	468
C.6	Burgers-Equation Code	469
C.7	The Fourier-Matrix Code	471
	Afterword	473
	Bibliography	475
	Index	493