

# Contents

<b>List of Figures</b>	<b>xi</b>
<b>Preface</b>	<b>xvii</b>
<b>1 Climate and Mathematics</b>	<b>1</b>
1.1 Earth's Climate System . . . . .	1
1.2 Modeling Earth's Climate . . . . .	2
1.3 Conceptual Models . . . . .	3
1.4 Climate and Statistics . . . . .	5
1.5 Climate Variability and Climate Change . . . . .	6
1.6 Models from Data . . . . .	8
1.7 Exercises . . . . .	10
<b>2 Earth's Energy Budget</b>	<b>13</b>
2.1 Solar Radiation . . . . .	13
2.2 Energy Balance Models . . . . .	14
2.3 Basic Model . . . . .	16
2.4 Greenhouse Effect . . . . .	17
2.5 Multiple Equilibria . . . . .	18
2.6 Budyko's Model . . . . .	19
2.7 Snowball Earth . . . . .	20
2.8 Bifurcation . . . . .	21
2.9 Exercises . . . . .	23
<b>3 Oceans and Climate</b>	<b>29</b>
3.1 Ocean Circulation . . . . .	29
3.2 Temperature . . . . .	31
3.3 Salinity . . . . .	32
3.4 Box Models . . . . .	33
3.5 One-Dimensional Model . . . . .	36
3.6 Exercises . . . . .	39
<b>4 Dynamical Systems</b>	<b>41</b>
4.1 Autonomous Differential Equations . . . . .	41
4.2 Geometrical Objects . . . . .	44
4.3 Critical Points . . . . .	48
4.4 Periodic Orbits . . . . .	49
4.5 Dynamics near Critical Points . . . . .	51
4.6 Planar Case . . . . .	52

4.7	Nonlinear Systems . . . . .	56
4.8	Exercises . . . . .	58
<b>5</b>	<b>Bifurcation Theory</b>	<b>63</b>
5.1	Bifurcation . . . . .	63
5.2	Examples . . . . .	64
5.3	From Examples to the General Case . . . . .	73
5.4	Bifurcation Points . . . . .	74
5.5	Hopf Bifurcation Theorem . . . . .	74
5.6	Exercises . . . . .	75
<b>6</b>	<b>Stommel's Box Model</b>	<b>77</b>
6.1	Stommel's Two-Box Model . . . . .	77
6.2	Dynamical System . . . . .	78
6.3	Bifurcation . . . . .	82
6.4	Comments . . . . .	82
6.5	Exercises . . . . .	83
<b>7</b>	<b>Lorenz Equations</b>	<b>87</b>
7.1	Lorenz Model . . . . .	87
7.2	Preliminary Observations . . . . .	88
7.3	Equilibrium Solutions . . . . .	89
7.4	Numerical Experiments . . . . .	91
7.5	Exercises . . . . .	92
<b>8</b>	<b>Climate and Statistics</b>	<b>95</b>
8.1	Challenges for Statistics . . . . .	95
8.2	Proxy Data . . . . .	98
8.3	Reanalysis . . . . .	101
8.4	Model Skill . . . . .	102
8.5	Exercises . . . . .	103
<b>9</b>	<b>Regression Analysis</b>	<b>105</b>
9.1	Statistical Modeling . . . . .	105
9.2	Linear Regression . . . . .	108
9.3	Simple Linear Regression . . . . .	110
9.4	Regression Diagnostics . . . . .	111
9.5	Exercises . . . . .	113
<b>10</b>	<b>Mauna Loa CO<sub>2</sub> Data</b>	<b>117</b>
10.1	Keeling's Observational Study . . . . .	117
10.2	Assembling the Data . . . . .	118
10.3	Analyzing the Data . . . . .	118
10.4	Exercises . . . . .	122
<b>11</b>	<b>Fourier Transforms</b>	<b>123</b>
11.1	Fourier Analysis . . . . .	123
11.2	Trigonometric Interpolation . . . . .	124
11.3	Discrete Fourier Transform . . . . .	125
11.4	Fast Fourier Transform . . . . .	126
11.5	Power Spectrum . . . . .	127

---

11.6	Correlation and Autocorrelation . . . . .	128
11.7	Fourier Series and Fourier Integrals . . . . .	130
11.8	Milankovitch's Theory of Glacial Cycles . . . . .	131
11.9	Exercises . . . . .	137
<b>12</b>	<b>Zonal Energy Budget</b> . . . . .	<b>141</b>
12.1	Zonal Energy Balance Model . . . . .	141
12.2	Legendre Polynomials . . . . .	146
12.3	Spectral Method . . . . .	148
12.4	Equilibrium Solutions . . . . .	150
12.5	Temperature Profile . . . . .	151
12.6	Assessment . . . . .	152
12.7	Exercises . . . . .	154
<b>13</b>	<b>Atmosphere and Climate</b> . . . . .	<b>159</b>
13.1	Earth's Atmosphere . . . . .	159
13.2	Pressure . . . . .	160
13.3	Temperature . . . . .	160
13.4	Atmospheric Circulation . . . . .	162
13.5	Exercises . . . . .	164
<b>14</b>	<b>Hydrodynamics</b> . . . . .	<b>165</b>
14.1	Coriolis Effect . . . . .	165
14.2	State Variables . . . . .	169
14.3	Continuity Equation . . . . .	170
14.4	Equation of Motion . . . . .	171
14.5	Other Prognostic Variables . . . . .	173
14.6	Equation of State . . . . .	173
14.7	Coupling Ocean and Atmosphere . . . . .	174
14.8	Need for Approximations . . . . .	174
14.9	Shallow Water Equations . . . . .	174
14.10	Further Approximations . . . . .	177
14.11	Boussinesq Equations . . . . .	178
14.12	Exercises . . . . .	179
<b>15</b>	<b>Climate Models</b> . . . . .	<b>183</b>
15.1	Climate Models as Dynamical Systems . . . . .	183
15.2	Dimension Reduction: Lorenz Model . . . . .	185
15.3	Abstract Climate Models . . . . .	189
15.4	Exercises . . . . .	191
<b>16</b>	<b>El Niño–Southern Oscillation</b> . . . . .	<b>193</b>
16.1	El Niño . . . . .	193
16.2	Recharge-Oscillator Model . . . . .	195
16.3	Delayed-Oscillator Model . . . . .	198
16.4	Delay Differential Equations . . . . .	204
16.5	Numerical Investigations . . . . .	206
16.6	Exercises . . . . .	208

<b>17</b>	<b>Cryosphere and Climate</b>	<b>213</b>
17.1	Cryosphere . . . . .	213
17.2	Glaciers, Ice Sheets, and Ice Shelves . . . . .	214
17.3	Sea Ice . . . . .	215
17.4	Exercises . . . . .	220
<b>18</b>	<b>Biogeochemistry</b>	<b>223</b>
18.1	Biosphere and Climate . . . . .	223
18.2	Carbon Cycle . . . . .	224
18.3	Carbon Transport into the Deep Ocean . . . . .	227
18.4	Ocean Plankton . . . . .	228
18.5	Algal Blooms . . . . .	230
18.6	Exercises . . . . .	235
<b>19</b>	<b>Extreme Events</b>	<b>237</b>
19.1	Climate and Weather Extremes . . . . .	237
19.2	Exceedance . . . . .	240
19.3	Tail Probabilities and Return Periods . . . . .	242
19.4	Order Statistics, Extreme Value Distribution . . . . .	244
19.5	Exercises . . . . .	248
<b>20</b>	<b>Data Assimilation</b>	<b>251</b>
20.1	Data Assimilation and Climate . . . . .	251
20.2	Example . . . . .	252
20.3	Bayesian Approach . . . . .	254
20.4	Sequential Data Assimilation . . . . .	255
20.5	Kalman Filtering . . . . .	257
20.6	Numerical Example . . . . .	259
20.7	Extensions . . . . .	261
20.8	Data Assimilation for the Lorenz System . . . . .	263
20.9	Concluding Remarks . . . . .	265
20.10	Exercises . . . . .	265
<b>A</b>	<b>Units and Symbols</b>	<b>269</b>
<b>B</b>	<b>Glossary</b>	<b>273</b>
<b>C</b>	<b>MATLAB Codes</b>	<b>279</b>
C.1	MATLAB Code for Lorenz Equations . . . . .	279
C.2	MATLAB Code for Regression Analysis . . . . .	280
C.3	MATLAB Code for Delay Differential Equations . . . . .	280
	<b>Bibliography</b>	<b>283</b>
	<b>Index</b>	<b>291</b>