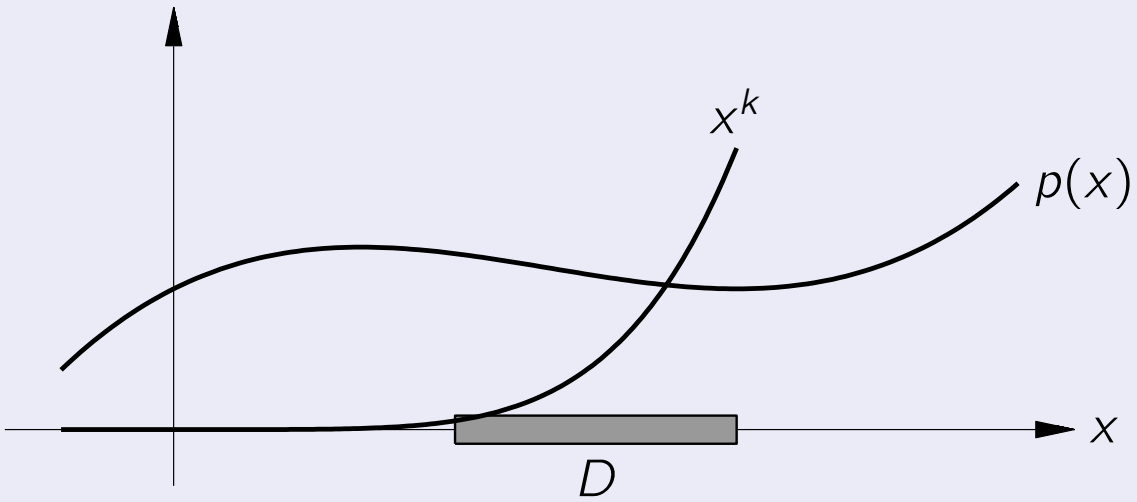
Monomial Form

A (real) polynomial p of degree n is a linear combination of the monomials $x \mapsto x^k\,,\ k = 0, \ldots, n$:

$$p(x) = c_0 + c_1 x + \cdots + c_n x^n\,,$$

with coefficients $c_k \in \mathbb{R}$ and $c_n \neq 0$. The coefficients correspond to the derivatives at $x = 0$:

$$k!\, c_k = p^{(k)}(0), \quad k = 0, \ldots, n\,.$$



The polynomials of degree $\leq n$ form a linear vector space of dimension $n+1$, denoted by $\mathbb{P}^n$. More precisely, we write $\mathbb{P}^n(D)$ if the variable x is restricted to a particular nondegenerate interval D.