Properties of Bernstein Polynomials

The Bernstein polynomials of degree $n$ are nonnegative on the standard parameter interval $[0, 1]$ and sum to one:

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\sum_{k=0}^{n} b^n_k(x) = 1.
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Moreover, $b^n_k$ has a unique maximum at $x = \frac{k}{n}$ on $[0, 1]$. 

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At the interval endpoints 0 and 1, only the first and the last Bernstein polynomials are nonzero, respectively:

\[
\begin{align*}
  b^n_0(0) &= 1, & b^n_1(0) &= \cdots = b^n_n(0) &= 0, \\
  b^n_0(1) &= \cdots = b^n_{n-1}(1) &= 0, & b^n_n(1) &= 1.
\end{align*}
\]

As a consequence, a polynomial in Bernstein form, \( p = \sum_{k=0}^{n} c_k b^n_k \), is equal to \( c_0 \) at \( x = 0 \) and equal to \( c_n \) at \( x = 1 \). This property is referred to as endpoint interpolation.
Ids for Bernstein Polynomials

The Bernstein polynomials $b_k^n$, $k = 0, \ldots, n$, satisfy the following identities.

Symmetry:

$$b_k^n(1 - x) = b_{n-k}^n(x).$$
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$$b^n_k(x) = x b^{n-1}_{k-1}(x) + (1 - x) b^{n-1}_k(x).$$
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We note that $b^{n-1}_{n-1} = b^{n-1}_n = 0$ in the second and third identities, according to the standard convention.