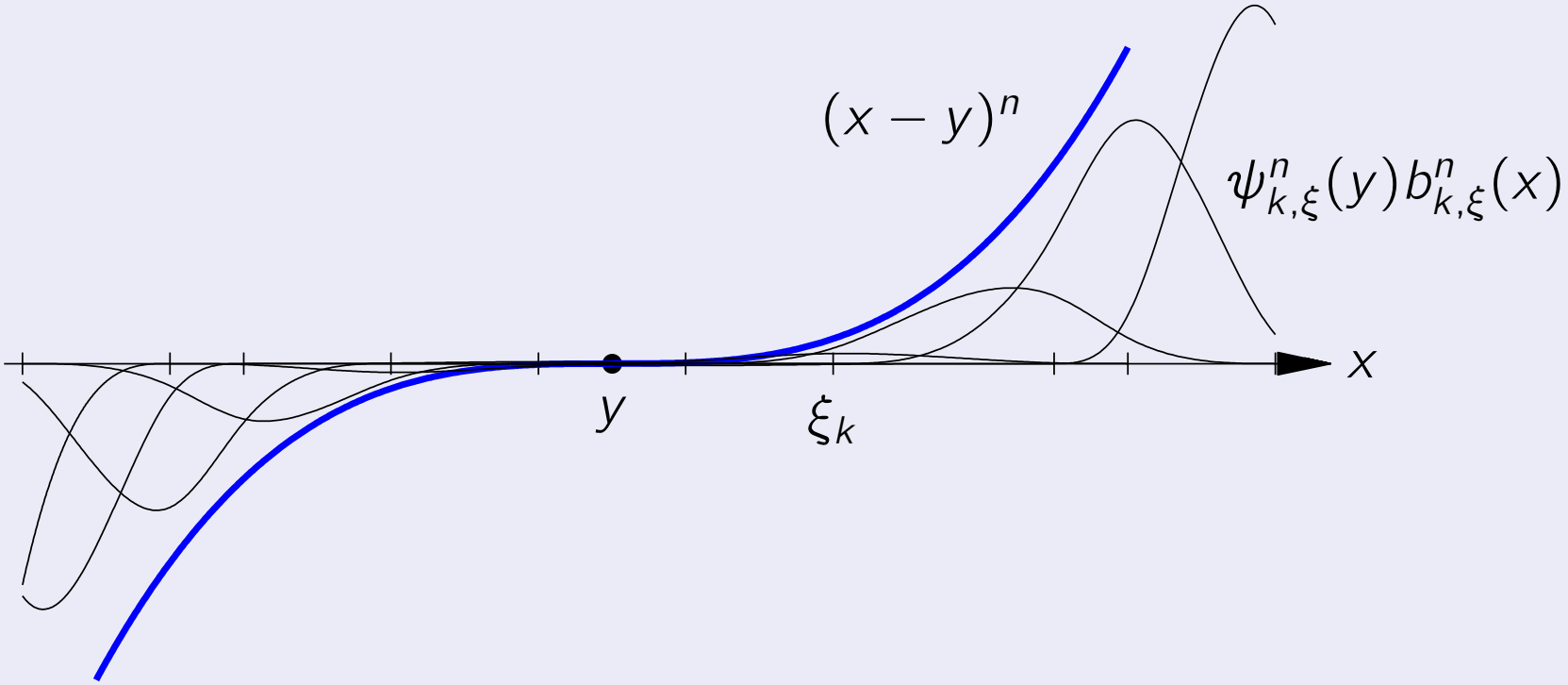
Marsden's Identity

For a bi-infinite knot sequence ξ, any polynomial of degree $\leq n$ can be represented as a linear combination of B-splines. In particular, for any $y \in \mathbb{R}$,

$$(x-y)^n = \sum_{k\in\mathbb{Z}} \psi^n_{k,\xi}(y)\, b^n_{k,\xi}(x)\,,\ x \in \mathbb{R}\,,$$

with $\psi^n_{k,\xi}(y) = (\xi_{k+1} - y)\cdots(\xi_{k+n} - y)$.



Comparing coefficients of y^{n-m} on both sides of the identity yields explicit representations for the monomials x^m. In particular, we have

$$1 = \sum_k b^n_{k,\xi}(x), \quad x = \sum_k \xi^n_k \; b^n_{k,\xi}(x)$$

with $\xi^n_k = (\xi_{k+1} + \cdots + \xi_{k+n})/n$ the so-called knot averages or Greville abscissae.