Error of Polynomial Approximation

For a hyperrectangle $R = [a_1, b_1] \times \cdots \times [a_d, b_d]$, the error of the orthogonal projection $P^n f \in \mathbb{P}^n(R)$ defined by

$$\int_R f\, q = \int_R (P^n f)\, q \quad \forall q \in \mathbb{P}^n(R)$$

can be estimated by

$$|f(x) - (P^n f)(x)| \leq c(d, n) \sum_{\nu=1}^{d} h_\nu^{n_\nu+1} \, \|\partial_\nu^{n_\nu+1} f\|_{\infty,R} \quad \forall x \in R\,,$$

where h_ν denotes the width of R in the νth direction.