Hierarchical Splines

A hierarchical spline space $S^n_{\underline{\underline{=}}}(D)$ is spanned by B-splines $b^n_{k,\xi}$ from knot sequences ξ, which are nodes of a tree Ξ. It is assumed that each of the children η is a refinement of its parent ξ, and the enclosing hyperrectangles $[\eta]$ do not intersect. Moreover, if $D \cap \operatorname{supp} b^n_{k,\xi}$ is contained in $[\eta]$, then this B-spline must be representable as linear combination of B-splines from η on D.

A basis for $S^n_{\underline{\underline{=}}}(D)$ consists of those relevant B-splines for D,

$$b^n_{k,\xi}, \quad k \in K_\xi, \; \xi \in \Xi,$$

which are nonzero at a point in the interior of D outside of the hyperrectangles $[\eta]$ for any of the children η of ξ.