Assembly of the Ritz–Galerkin System

The entries of the matrix and right side of the Ritz–Galerkin system are computed by adding the contributions from each grid cell:

$$
\begin{array}{l}
\tilde{G} = 0,\ \tilde{F} = 0 \\
\textbf{for } D_\ell \subseteq R \\
\quad \textbf{for } k \sim \ell \\
\quad\quad \tilde{f}_k = \tilde{f}_k + \lambda_{k,\ell} \\
\quad\quad \textbf{for } k' \sim \ell \\
\quad\quad\quad \tilde{g}_{k,k'-k} = \tilde{g}_{k,k'-k} + a_{k,k',\ell} \\
\quad\quad \textbf{end} \\
\quad \textbf{end} \\
\textbf{end}
\end{array}
$$

Multiplication by the Ritz–Galerkin matrix

Assume that the matrix $(g_{k,k'})_{k,k'\sim R}$ is stored in an array $(\tilde{g}_{k,s})_{k\sim R,|s_\nu|\le n}$ with the second index s corresponding to the offsets $k'-k$. Then, for a vector $(u_k)_{k\sim R}$, the product $V = GU$ can be computed with the following algorithm:

$V = 0$
for $s \in \{-n, \dots, n\}^d$
  for $k \sim R$
    $v_k = v_k + \tilde{g}_{k,s} u_{k+s}$
  end
end

where entries u_{k+s} with indices $\not\sim R$ are set to zero.