

Contents

Preface	vii
1 Schwarz methods	1
1.1 Three continuous Schwarz algorithms	1
1.2 Connection with the block Jacobi algorithm	5
1.3 Algebraic algorithms: Discrete partition of unity	8
1.4 Iterative Schwarz methods: RAS and ASM	13
1.5 Convergence analysis	14
1.6 More sophisticated Schwarz methods: P. L. Lions' algorithm	17
1.7 Schwarz methods using FreeFem++	18
2 Optimized Schwarz methods	35
2.1 P. L. Lions' algorithm	35
2.2 Helmholtz problems	41
2.3 Implementation issues	46
2.4 Optimal interface conditions	54
2.5 Optimized interface conditions	59
2.6 FreeFem++ implementation of ORAS	72
3 Krylov methods	77
3.1 Fixed-point iterations	77
3.2 Krylov spaces	79
3.3 The conjugate gradient method	82
3.4 The GMRES method for nonsymmetric problems	88
3.5 Krylov methods for ill-posed problems	94
3.6 Schwarz preconditioners using FreeFem++	97
4 Coarse spaces	103
4.1 Need for a two-level method	103
4.2 Nicolaides coarse space	107
5 Theory of two-level additive Schwarz methods	113
5.1 Introduction of a spectral coarse space	113
5.2 Variational setting	116
5.3 Additive Schwarz setting	117
5.4 Abstract theory for the two-level ASM	120
5.5 Definition and properties of Nicolaides and spectral coarse spaces	122
5.6 Convergence theory for the ASM with Nicolaides and spectral coarse spaces	125

5.7	Functional analysis results	127
5.8	Theory of spectral coarse spaces for scalar heterogeneous problems . .	129
6	Neumann–Neumann and FETI algorithms	131
6.1	Direct and hybrid substructured solvers	131
6.2	Two-subdomain case at the continuous level	133
6.3	Two-subdomain case at the algebraic level	139
6.4	Many-subdomain case	143
6.5	Neumann–Neumann method in FreeFem++	147
6.6	Nonstandard Neumann–Neumann-type methods	154
7	Robust coarse spaces via generalized eigenproblems: The GenEO method	161
7.1	Reformulation of the additive Schwarz method	162
7.2	Mathematical foundation	164
7.3	Finite element setting	172
7.4	GenEO coarse space for additive Schwarz	173
7.5	Hybrid Schwarz with GenEO	178
7.6	FreeFem++ implementation	182
7.7	SORAS-GenEO-2 coarse space for the optimized Schwarz method . .	186
7.8	Balancing Neumann–Neumann	190
8	Parallel implementation of Schwarz methods	203
8.1	A parallel FreeFem++ script	203
8.2	Numerical experiments	213
8.3	FreeFem++ algebraic formulation	218
	Bibliography	223
	Index	237