ODE Text *Mathematica* Examples

Example 1.1.10

```
A = {{1, 2, 3}, {4, 5, 6}, {7, 8, 2}};
b = {-1, 4, -7};
newa = Transpose[Insert[Transpose[A], b, 4]];
MatrixForm[RowReduce[newa]]
```

$$\begin{pmatrix} 1 & 0 & 0 & \frac{139}{21} \\ 0 & 1 & 0 & -\frac{152}{21} \\ 0 & 0 & 1 & \frac{16}{7} \end{pmatrix}$$

Matrix Algebra

```
A = {{1, 2, 3}, {4, -5, 6}};
B = {{-2, 5, -8}, {-7, 9, 3}};
Cee = {{-3, 5}, {2, 1}, {9, -7}};
a = {4, -7, 9};
b = {2, -6, 5};
AB = 2 * A - 5 * B;
ab = 4 * a + 7 * b;
AC = A.Cee;
CA = Cee.A;
ACinv = Inverse[AC];
MatrixForm[AB]
MatrixForm[ab]
MatrixForm[AC]
MatrixForm[CA]
MatrixForm[ACinv]
MatrixForm[AC.ACinv]
```

$$\begin{pmatrix} 12 & -21 & 46 \\ 43 & -55 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 30 \\ -70 \\ 71 \end{pmatrix}$$

$$\begin{pmatrix} 28 & -14 \\ 32 & -27 \end{pmatrix}$$

$$\begin{pmatrix} 17 & -31 & 21 \\ 6 & -1 & 12 \\ -19 & 53 & -15 \end{pmatrix}$$

$$\begin{pmatrix} \frac{27}{308} & -\frac{1}{22} \\ \frac{8}{77} & -\frac{1}{11} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Example 1.4.5

```
A = {{1, 2, 3}, {-1, 2, -3}, {5, 6, 7}};
Ainv = Inverse[A];
MatrixForm[Ainv]
```

$$\begin{pmatrix} -1 & -\frac{1}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{2} & -\frac{1}{8} & -\frac{1}{8} \end{pmatrix}$$

Example 1.5.6

```
RowReduce[{{3, 4, 7, -1}, {2, 6, 8, -4}, {-5, 3, -2, -8}, {7, -2, 5, 9}}]

MatrixForm[%]
```

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
RowReduce[{{1, 2, 3, -3}, {2, 1, 3, 0}, {1, -1, 0, 3}, {-3, 2, -1, -7}}];
MatrixForm[%]
```

Example 1.7.9

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
RowReduce[{{1, 2, 3, 4}, {-1, 2, 1, 0}, {5, 6, 11, 16}, {2, 4, 6, 8}}];
MatrixForm[%]
```

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
A = {{-3, 4, 8, -2}, {2, 6, 8, -4}, {-5, -9, -2, -8}, {7, -2, 5, 9}};
MatrixForm[%]
Det[A]
```

Compute the determininant

$$\begin{pmatrix} -3 & 4 & 8 & -2 \\ 2 & 6 & 8 & -4 \\ -5 & -9 & -2 & -8 \\ 7 & -2 & 5 & 9 \end{pmatrix}$$

-6624

Example 1.12.7

```
A = {{1, 2, 3}, {-1, 2, -3}, {5, 6, 7}};
MatrixForm[A]
N[Eigenvectors[A], 5]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & -3 \\ 5 & 6 & 7 \end{pmatrix}$$

{{0.39 - 0.03 i, -0.59 + 0.49 i, 1.0},
{0.39 + 0.03 i, -0.59 - 0.49 i, 1.0}, {-2.0, 0.35, 1.0}}

Case Study 1.13.2

Eigenvalues and associated eigenvectors for **M**

```
M = {{9/10, 3/100, 1/10}, {2/100, 85/100, 2/10}, {8/100, 12/100, 7/10}};
Eigenvectors[M]
```

$$\left\{\left\{\frac{35}{24}, \frac{55}{36}, 1\right\}, \{-7, 6, 1\}, \left\{-\frac{1}{4}, -\frac{3}{4}, 1\right\}\right\}$$

Rescaled eigenvector **v1**

```
v1 = {35/24, 55/36, 1};
MatrixForm[v1/(35/24 + 55/36 + 1)]
```

$$\begin{pmatrix} \frac{15}{41} \\ \frac{110}{287} \\ \frac{72}{287} \end{pmatrix}$$

Case Study 1.13.3

Eigenvalues and associated vectors for **A**

```
A = {{2/10, 0, 1/100}, {8/10, 5/10, 0}, {0, 5/10, 99/100}};
Eigenvectors[A]
Eigenvalues[A]
```

$$\left\{\left\{\frac{1}{80}, \frac{1}{50}, 1\right\}, \left\{\frac{1}{100}\left(29 - \sqrt{641}\right), \frac{1}{100}\left(-129 + \sqrt{641}\right), 1\right\}, \left\{\frac{1}{100}\left(29 + \sqrt{641}\right), \frac{1}{100}\left(-129 - \sqrt{641}\right), 1\right\}\right\}$$

$$\left\{1, \frac{1}{200}\left(69 + \sqrt{641}\right), \frac{1}{200}\left(69 - \sqrt{641}\right)\right\}$$

Rescale eigenvector **v1**

```
v1 = {1/80, 1/50, 1};
MatrixForm[v1/(1/80 + 1/50 + 1)]
```

$$\begin{pmatrix} \frac{5}{413} \\ \frac{8}{413} \\ \frac{400}{413} \end{pmatrix}$$

Case Study 1.13.4

Eigenvalues and associated eigenvectors for **A**

```
A = {{0, 0, 33/100}, {18/100, 0, 0}, {0, 71/100, 94/100}};
N[Eigenvectors[A], 5]
N[Eigenvalues[A], 5]
```

```
{{0.33550, 0.061398, 1.0000}, {-0.1678 - 1.5848 i, -1.35464 + 0.29003 i, 1.0000},
 {-0.1678 + 1.5848 i, -1.35464 - 0.29003 i, 1.0000}}
```

```
{0.98359, -0.02180 + 0.20592 i, -0.02180 - 0.20592 i}
```

Rescaled eigenvector **v1**

```
v1 = {0.3355, 0.0614, 1};
MatrixForm[v1/(0.3355 + 0.0614 + 1)]
```

$$\begin{pmatrix} 0.240175 \\ 0.0439545 \\ 0.715871 \end{pmatrix}$$

Solve **Pc=x0** by finding inverse of **P**

```
v1 = {0.240177541311969, 0.0439531075092037, 0.715869351178827};
v2 = {1, -0.009150128540143671-
    0.8644469584876894 * I, -0.06604960570424193 + 0.6239953953635560 * I};
v3 = {1, -0.09150128540143671 + 0.8644469584876894 * I,
    -0.06604960570424193-0.6239953953635560 * I};
Ptr = {v1, v2, v3};
P = Transpose[Ptr];
Pinv = Inverse[P];
N[MatrixForm[P], 5]
```

$$\begin{pmatrix} 0.240178 & 1.0000 & 1.0000 \\ 0.0439531 & (0. - 0.0079098\,i) - & -0.0915013 + 0.864447\,i \\ 0.715869 & -0.0660496 + 0.623995\,i & (0. - 0.0412146\,i) - \end{pmatrix}$$

Example 2.4.4

```
a = -Exp[-3 * t] * Sin[2 * t];
A = Integrate[a, t]
```

$$\frac{1}{13} e^{-3 t} (2 \text{Cos}[2 t] + 3 \text{Sin}[2 t])$$

Example 2.5.4

```
f = t^2 / (4 + t);
F = Integrate[f, t];
xp = (4 + t) * F
```

$$\frac{1}{2} (4 + t) \left(-48 - 8 t + t^2 + 32 \text{Log}[4 + t]\right)$$

Example 2.5.11

```
f = t * Sin[2 * t] * Exp[-5 * t];
F = Integrate[f, t];
xp = Exp[5 * t] * F
```

$$\frac{1}{841} (-2 (10 + 29 t) \text{Cos}[2 t] - (21 + 145 t) \text{Sin}[2 t])$$

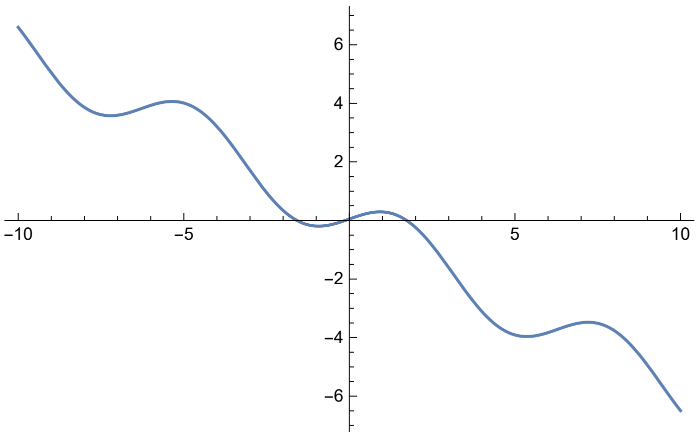
Case Study 2.6.1

```
Clear[a, c, t, w, v]
f = Exp[a * t] * (1 - Cos[w * t]);
F = Integrate[f, t];
xp = a * c * v * Exp[-a * t] * F
```

$$-\frac{c v \left(-a^2 - w^2 + a^2 \text{Cos}[t w] + a w \text{Sin}[t w]\right)}{a^2 + w^2}$$

Exercise 2.6.9 - finding zeros of a function

```
f = Sin[x] - 0.6 * x + 0.05;
Plot[f, {x, -10, 10}]
```




```
FindRoot[f, {x, -2}]
FindRoot[f, {x, -1}]
FindRoot[f, {x, 1}]
```

{x → -1.58321}

{x → -1.58321}

{x → 1.72915}

Case Study 3.7.1

```
Clear[a, c, w, V]
Cee = {{-2 * a, a, -w, 0}, {a, -2 * a, 0, -w}, {w, 0, -2 * a, a}, {0, w, a, -2 * a}};
cf = {a * V * c, 0, 0, 0};
cans = LinearSolve[Cee, cf]
```

$$\left\{-\frac{2 a^2 c V (3 a^2 + w^2)}{9 a^4 + 10 a^2 w^2 + w^4}, -\frac{a^2 c V (3 a^2 - w^2)}{9 a^4 + 10 a^2 w^2 + w^4}, -\frac{a c V w (5 a^2 + w^2)}{9 a^4 + 10 a^2 w^2 + w^4}, -\frac{4 a^3 c V w}{9 a^4 + 10 a^2 w^2 + w^4}\right\}$$

Simplify each component of the solution vector

```
Factor[cans[[1]]]
Factor[cans[[2]]]
Factor[cans[[3]]]
Factor[Det[Cee]]
```

$$-\frac{2\, a^2\, c\, V\, (3\, a^2 + w^2)}{(a^2 + w^2)\, (9\, a^2 + w^2)}$$

$$-\frac{a^2\, c\, V\, (3\, a^2 - w^2)}{(a^2 + w^2)\, (9\, a^2 + w^2)}$$

$$-\frac{a\, c\, V\, w\, (5\, a^2 + w^2)}{(a^2 + w^2)\, (9\, a^2 + w^2)}$$

$$(a^2 + w^2)\, (9\, a^2 + w^2)$$

Case Study 3.7.2

```
A = {{-13/360, 272/21875, 7/200000}, {1/90, -1/35, 0}, {7/1800, 0, -7/200000}};
N[Eigenvectors[A], 5]
N[Eigenvalues[A], 5]
```

```
{{-11.482, 7.9164, 1.0000},
 {-5.1338, -6.6548, 1.0000}, {0.0011204, 0.00043617, 1.0000}}
```

```
{-0.044687, -0.020000, -0.000030643}
```

```
f = {49 + 3/10, 0, 0};
a0 = LinearSolve[-A, f]
```

$$\left\{\frac{1848750}{1027}, \frac{2156875}{3081}, \frac{616250000}{3081}\right\}$$

Case Study 3.7.3

```
A = {{-1/2, 1/80, 73/80}, {1/4, -961/80, 0}, {1/4, 12, -73/80}};
N[Eigenvectors[A], 5]
N[Eigenvalues[A], 5]
```

```
{{-0.078406, -0.92159, 1.0000},
 {-0.97691, -0.023087, 1.0000}, {1.8260, 0.038001, 1.0000}}
```

```
{-11.991, -1.4338, 0}
```

```
{1, 20/961, 38420/70153}/(1 + 20/961 + 38420/70153)
```

$$\left\{\frac{70153}{110033}, \frac{1460}{110033}, \frac{38420}{110033}\right\}$$

```
A = {{-1, 1/80, 1/16}, {1/2, -961/80, 0}, {1/2, 12, -1/16}};
N[Eigenvectors[A], 5]
N[Eigenvalues[A], 5]
```

```
{{-0.0045464, -0.99545, 1.0000},
 {-0.95632, -0.043677, 1.0000}, {0.062533, 0.0026028, 1.0000}}
```

```
{-12.010, -1.0648, 0}
```

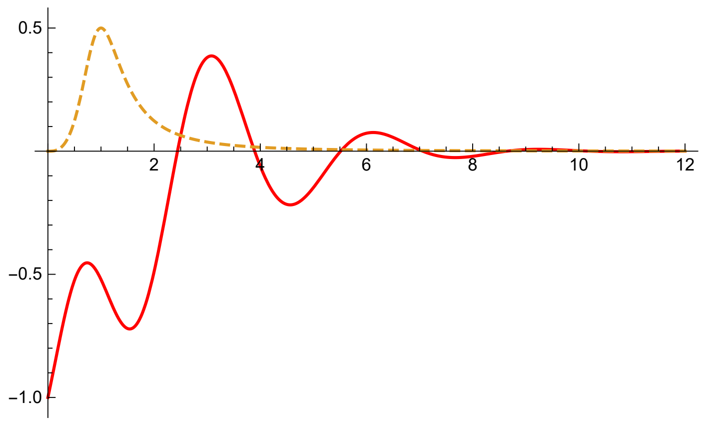
```
{1, 40/961, 15368/961}/(1 + 40/961 + 15368/961)
```

$\left\{\frac{961}{16369}, \frac{40}{16369}, \frac{15368}{16369}\right\}$

Plot multiple functions on one graph

```
f = Exp[-x] * (x^2 * Cos[2 * x] - 1);
g = x^3/(1 + x^6);
Plot[f, g]
```

```
Plot[{f, g}, {x, 0, 12}, PlotRange → All, PlotStyle → {Red, Dashed, Thick}]
```



Example 4.1.3 - roots of a polynomial will be found by finding the eigenvalues of the companion matrix for the ODE

```
A = {{0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}, {-9, -6, 3, 0}};
MatrixForm[A]
N[Eigenvalues[A], 5]
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & -6 & 3 & 0 \end{pmatrix}$$

```
{1.5259 + 1.3459 i, 1.5259 - 1.3459 i, -1.9190, -1.1328}
```

Example 5.2.10

```
f1 = Exp[-s * t] * t;
f2 = Exp[-s * t] * (t^2 - 2);
F1 = Integrate[f1, {t, 0, 2}];
F2 = Integrate[f2, {t, 2, Infinity}];
F = Factor[F1 + F2]
```

ConditionalExpression$\left[\frac{e^{-2 s} \left(2 + 3 s + e^{2 s} s\right)}{s^3}, \mathrm{Re}[s] > 0\right]$

Example 5.2.13

```
F = (s^4 + 6 * s^3 - 15 * s^2 - 10 * s + 50) / (s^3 * (s^2 - 2 * s + 10));
Apart[F]
```

$$\frac{5}{s^3} - \frac{2}{s} + \frac{2 + 3 s}{10 - 2 s + s^2}$$