

Contents

List of Notation	ix
Preface	xiii
I Linear Analysis I	1
1 Abstract Vector Spaces	3
1.1 Vector Algebra	3
1.2 Spans and Linear Independence	10
1.3 Products, Sums, and Complements	14
1.4 Dimension, Replacement, and Extension	17
1.5 Quotient Spaces	21
Exercises	27
2 Linear Transformations and Matrices	31
2.1 Basics of Linear Transformations I	32
2.2 Basics of Linear Transformations II	36
2.3 Rank, Nullity, and the First Isomorphism Theorem	40
2.4 Matrix Representations	46
2.5 Composition, Change of Basis, and Similarity	51
2.6 Important Example: Bernstein Polynomials	54
2.7 Linear Systems	58
2.8 Determinants I	65
2.9 Determinants II	70
Exercises	78
3 Inner Product Spaces	87
3.1 Introduction to Inner Products	88
3.2 Orthonormal Sets and Orthogonal Projections	94
3.3 Gram–Schmidt Orthonormalization	99
3.4 QR with Householder Transformations	105
3.5 Normed Linear Spaces	110
3.6 Important Norm Inequalities	117
3.7 Adjoints	120
3.8 Fundamental Subspaces of a Linear Transformation	123
3.9 Least Squares	127
Exercises	131

4	Spectral Theory	139
4.1	Eigenvalues and Eigenvectors	140
4.2	Invariant Subspaces	147
4.3	Diagonalization	150
4.4	Schur's Lemma	155
4.5	The Singular Value Decomposition	159
4.6	Consequences of the SVD	165
	Exercises	171
II	Nonlinear Analysis I	177
5	Metric Space Topology	179
5.1	Metric Spaces and Continuous Functions	180
5.2	Continuous Functions and Limits	185
5.3	Closed Sets, Sequences, and Convergence	190
5.4	Completeness and Uniform Continuity	195
5.5	Compactness	203
5.6	Uniform Convergence and Banach Spaces	210
5.7	The Continuous Linear Extension Theorem	213
5.8	Topologically Equivalent Metrics	219
5.9	Topological Properties	222
5.10	Banach-Valued Integration	227
	Exercises	233
6	Differentiation	241
6.1	The Directional Derivative	241
6.2	The Fréchet Derivative in \mathbb{R}^n	246
6.3	The General Fréchet Derivative	252
6.4	Properties of Derivatives	256
6.5	Mean Value Theorem and Fundamental Theorem of Calculus	260
6.6	Taylor's Theorem	265
	Exercises	272
7	Contraction Mappings and Applications	277
7.1	Contraction Mapping Principle	278
7.2	Uniform Contraction Mapping Principle	281
7.3	Newton's Method	285
7.4	The Implicit and Inverse Function Theorems	293
7.5	Conditioning	301
	Exercises	310
III	Nonlinear Analysis II	317
8	Integration I	319
8.1	Multivariable Integration	320
8.2	Overview of Daniell–Lebesgue Integration	326
8.3	Measure Zero and Measurability	331

8.4	Monotone Convergence and Integration on Unbounded Domains	335
8.5	Fatou's Lemma and the Dominated Convergence Theorem	340
8.6	Fubini's Theorem and Leibniz's Integral Rule	344
8.7	Change of Variables	349
	Exercises	356
9	* Integration II	361
9.1	Every Normed Space Has a Unique Completion	361
9.2	More about Measure Zero	364
9.3	Lebesgue-Integrable Functions	367
9.4	Proof of Fubini's Theorem	372
9.5	Proof of the Change of Variables Theorem	374
	Exercises	378
10	Calculus on Manifolds	381
10.1	Curves and Arclength	381
10.2	Line Integrals	386
10.3	Parametrized Manifolds	389
10.4	* Integration on Manifolds	393
10.5	Green's Theorem	396
	Exercises	403
11	Complex Analysis	407
11.1	Holomorphic Functions	407
11.2	Properties and Examples	411
11.3	Contour Integrals	416
11.4	Cauchy's Integral Formula	424
11.5	Consequences of Cauchy's Integral Formula	429
11.6	Power Series and Laurent Series	433
11.7	The Residue Theorem	438
11.8	* The Argument Principle and Its Consequences	445
	Exercises	451
IV	Linear Analysis II	457
12	Spectral Calculus	459
12.1	Projections	460
12.2	Generalized Eigenvectors	465
12.3	The Resolvent	470
12.4	Spectral Resolution	475
12.5	Spectral Decomposition I	480
12.6	Spectral Decomposition II	483
12.7	Spectral Mapping Theorem	489
12.8	The Perron–Frobenius Theorem	494
12.9	The Drazin Inverse	500
12.10	* Jordan Canonical Form	506
	Exercises	511

13	Iterative Methods	519
13.1	Methods for Linear Systems	520
13.2	Minimal Polynomials and Krylov Subspaces	526
13.3	The Arnoldi Iteration and GMRES Methods	530
13.4	* Computing Eigenvalues I	538
13.5	* Computing Eigenvalues II	543
	Exercises	548
14	Spectra and Pseudospectra	553
14.1	The Pseudospectrum	554
14.2	Asymptotic and Transient Behavior	561
14.3	* Proof of the Kreiss Matrix Theorem	566
	Exercises	570
15	Rings and Polynomials	573
15.1	Definition and Examples	574
15.2	Euclidean Domains	583
15.3	The Fundamental Theorem of Arithmetic	588
15.4	Homomorphisms	592
15.5	Quotients and the First Isomorphism Theorem	598
15.6	The Chinese Remainder Theorem	601
15.7	Polynomial Interpolation and Spectral Decomposition	610
	Exercises	618
V	Appendices	625
A	Foundations of Abstract Mathematics	627
A.1	Sets and Relations	627
A.2	Functions	635
A.3	Orderings	643
A.4	Zorn's Lemma, the Axiom of Choice, and Well Ordering	647
A.5	Cardinality	648
B	The Complex Numbers and Other Fields	653
B.1	Complex Numbers	653
B.2	Fields	659
C	Topics in Matrix Analysis	663
C.1	Matrix Algebra	663
C.2	Block Matrices	665
C.3	Cross Products	667
D	The Greek Alphabet	669
	Bibliography	671
	Index	679