Preface

The topics developed here have been standard knowledge for at least forty years. Our aim is only to provide newcomers with a basic knowledge of some tools in real analysis: the Hardy–Littlewood maximal operator, Calderón–Zygmund theory, Riesz transforms, Littlewood–Paley theory, Fourier multipliers, H^1 and BMO spaces, and interpolation of operators (real and complex methods). So, it would be in vain to search this book for the present trends of research in this field. We focus on singular convolution operators and develop the Calderón–Zygmund theory in this framework, although more general kernels are mentioned in passing. All this material is of common use in various parts of analysis: harmonic analysis, partial differential equations, signal processing, etc. But this book does not provide such developments in order to respect reader's motivations. This book should be thought as a mere introduction to the study of more complex monographs.

We seek concision, but this text is self-sufficient: there is no assertion without a proof, although some readers may find some of them a bit laconic. This means that some work is expected from the reader: he should read this book with pen in hand. At the end of each chapter a few exercises are proposed with sufficient hints so that a careful reader can solve them himself. Some exercises are just tests for comprehension, while others are important complements, the inclusion of which in the main text would have blurred the exposition.

The prerequisites are a basic knowledge of functional analysis, some acquaintance with measure and integration theory, and a certain familiarity with the Fourier transform in Euclidean spaces. Nevertheless some background material is provided in the appendices. This mainly deals with the use of distribution functions, which, of course, probabilists know very well, but which is not common knowledge among undergraduate students in mathematics.

A few subjects have been selected in this teeming field. As said above, we tried to go directly to the essentials. This is why some proofs, designed for this purpose, may not be straightforwardly transposable to a broader context. In the same spirit we lazily defined the H^1 space by means of atoms. This is not the initial definition, but it is very convenient to establish its duality with BMO. We preferred to develop the Littlewood–Paley theory by using vector singular integrals rather than all the apparatuses of systems of conjugate harmonic functions. Our approach is shorter but less powerful.

A chapter is devoted to Fourier series. Another chapter deals with singular integrals on other groups so that, in particular, it allows one to handle $\mathbb{Q}_p$, the field of p-adic numbers, as well as $\mathbb{F}_q((1/x))$, the field of Laurent series on a finite field.

The last chapter deals with examples of interpolation theorems.

This book grew out of lectures held at Université Paris-Sud at Orsay and at Tsinghua University at Beijing.

I warmly thank Professor Yao Jia-Yan, from Tsinghua University, not only for his careful reading of the manuscript and a wealth of valuable suggestions, but also for undertaking the translation of this book into Chinese. I also extend my thanks to Professor Arnaud Durand, from Université Paris-Sud, who used a preliminary version of this book as a textbook while teaching at Tsinghua University. His remarks contributed to improving the manuscript. Last, I would like to express my gratitude to Professor Wen Zhi-Ying, from Tsinghua University, who gave me the opportunity to stay and teach in prestigious Chinese universities, and to conduct research with many Chinese colleagues.