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Preconditioning and the Conjugate Gradient Method in the Context of Solving PDEs. *By Josef Málek and Zdeněk Strakoš.* SIAM, Philadelphia, 2015. $39.00, x+104 pp., softcover. ISBN 978-1-611973-83-9.

This is an unusual and refreshing book. Two authors with different expertise have collaborated to write a short book that neither of them separately could have written. It is an exploration of the solution of self-adjoint elliptic PDEs by numerical methods and covers the functional analysis setting, finite element approximation and preconditioned conjugate gradient (CG) solution of the derived linear systems of equations; in particular it is an exploration of the interaction of these different aspects in the overall numerical solution. As the authors summarise in the final chapter: "The road from PDEs through functional analysis and discretization to iterative methods is long and not without danger.... Deep expertise within one part is not sufficient and cannot compensate for deficiencies at other parts".

Following an introduction to the general perspective of the book, the first five very carefully written chapters lead us through linear elliptic PDEs, elements of functional analysis, Riesz maps and operator preconditioning, the CG method in Hilbert space, finite-dimensional Hilbert spaces and the matrix form of CG; this is material which can be found in other places, but which is rarely put together in such a coherent and consistent fashion. The remaining seven very short chapters (as short as one and a half pages and typically three or four) build on the foundation and comment on Galerkin discretization, preconditioning as basis transformation, consistency, stability and convergence, local and global information content, limits of condition number based convergence descriptions, inexact computation, a posteriori error analysis and iterative stopping criteria before the final summary and outlook which gives a retrospective and overview on the described mathematical landscape.

This is a different and thought-provoking read for experts and a piece of real mathematical scholarship for beginners to chew on, though I think there are easier (not more accurate nor more worthwhile) descriptions for those wishing to understand this material for the first time. Two books would of course be in that case required; one on functional analysis and finite element approximation (such as Brenner and Scott [1]) and one on iterative linear algebra (such as Greenbaum [2]). I wonder how many (how few!) have actually read both of these "standard" texts!

As those of us who know the authors might have expected, there is a thorough bibliography with a broad coverage; numerous quotes in the text have been extracted to reinforce key ideas and viewpoints.

In this time when journals have generally become more specialized, there are many narrowly focused conferences—one hears colleagues claim that they no longer go to general conferences such as the SIAM Annual Meetings because there is too wide a coverage,... this is a breath of fresh air. In general with the growth of applicable mathematics individuals can be led to focus more keenly on the small part of the subject in which they can have some chance to make a contribution. But thank goodness that there are leaders in our field who still seek for synthesis and holistic understanding of the broad collection of ideas required to *solve* a problem. Here is a book that expounds in precisely that spirit; it was a very stimulating read for this reviewer!

REFERENCES

[1] S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, 3rd ed., Springer, New York, 2008.

[2] A. Greenbaum, *Iterative Methods for Solving Linear Systems*, SIAM, Philadelphia, 1997.

Andy Wathen
Oxford University