Math Matters

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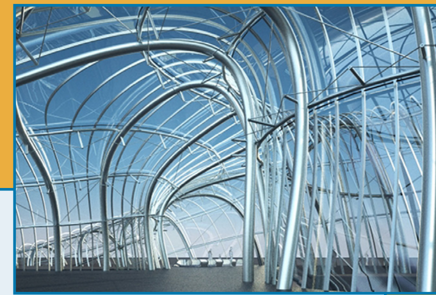

The math behind... Freeform Architecture

Some Technical terms used:

Discrete differential geometry, polyhedral surface, planar quadrilateral mesh, multilayer construction, conical mesh, discrete-isothermic, discrete minimal surface

Uses and applications:

Many constructions in various countries, for example: Yas Island Marina Hotel (Abu Dhabi), Graz Art Museum (Austria), Museum of Islamic Arts at Louvre Museum (Paris, France), The Blob at Admirant (Eindhoven, Netherlands), KREOD (London, United Kingdom).

How it works:

In the last years, freeform architecture became more and more popular. A variety of complex geometric shapes are used for the facades of buildings. Because of the large scale of architecture, it is necessary to decompose the surface into small panels. Here mathematics comes into play: In discrete differential geometry, presently emerging on the border between differential and discrete geometry, one aims to discretize smooth surfaces and the methods of differential geometry. This knowledge helps to efficiently design and manufacture freeform surfaces.

A natural discretization is to cover the freeform surface by planar panel elements, leading to meshes with planar faces. It turns out that triangular meshes cause problems in the architectural realization when a support structure consisting of beams is needed. These beams cross each other in nodes, which are difficult to manufacture if the nodes have torsion, i.e. if the symmetry planes of the beams do not intersect in a common axis. In general, nodes of triangle meshes have torsion, in contrast to preferred planar quadrilateral meshes. Also, quadrilateral meshes do allow multilayer constructions often used in steel-glass constructions. Mathematically, multilayer constructions are described by edge-parallel meshes. A class of meshes with torsion-free nodes and equally-spaced parallel meshes are conical meshes. Equivalently, the faces adjacent to a node are tangent to a right circular cone. Any such mesh is parallel to a mesh whose faces are tangent to a sphere. If and only if the polar dual of the latter is discrete-isothermic, the conical mesh has equilibrium forces. For this, one imagines the external forces acting in the nodes, and a system of internal forces along the edges compensating these forces. Discrete-isothermic surfaces give rise to an appropriate notion of discrete minimal surfaces. The latter surfaces nicely discretize soap films and lead to a rich theory of discrete analogs of smooth minimal surfaces.

Interesting facts:

In 2012 and 2013, the 1st floor of the Eiffel Tower underwent redevelopment. The paneling of the freeform front facades of the pavilions by cylindrical glass panels was computed using modern methods of discrete differential geometry, which led to a reduction of production costs by approximately 70% compared to actual industry standards.

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[3] A.I. Bobenko, J.M. Sullivan, P. Schröder, G.M. Ziegler (Eds.): Discrete Differential Geometry. Oberwolfach Seminars 38, Birkhäuser (2008)

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