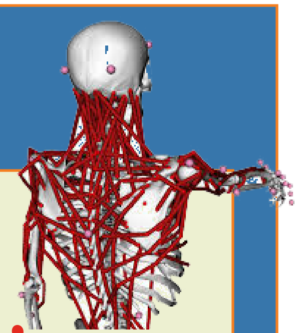
Math Matters

Apply It.



The math behind...
Bio-mechanical Simulations

Technical terms used:
Numerical modeling, continuum mechanics, finite element method, fluid-structure interaction, parallel computing

Uses and applications:
Fluid flows, tissue deformation, and interactions between fluids and tissues are ubiquitous in human bodies. Understanding how certain pathologies affect the flow dynamics in physiology and how these changes in return affect pathology development is very helpful to designs of medical detection methods and surgical procedures.

How it works:
Fluids' and solids' mechanical behaviors can be generally described by some governing partial differential equations, which are derived by applying Newton's second law to an infinitesimal block, as studied in continuum mechanics. Solids are commonly studied under a Lagrangian frame of reference, where the observer follows points on the material to look at variables of interest, usually displacements. The resulting differential equations can be nonlinear due to material properties or significant deformation. On the other hand, fluids can be conveniently studied under an Eulerian frame of reference, where the observer looks at variables of interest (pressure, velocity, density, etc.) at specific locations in space through which fluids flow over time; this results in nonlinear differential equations such as the Navier-Stokes equations.

Simulations of bio-mechanical systems generally focus on a specific region of interest (a section of blood vessel with partial vascular blockage, for instance). On the boundaries of this region, a set of constraints are enforced. They can be a force applied on a cross section of a bone, or a pulsating blood flow into an artery; these constraints are called boundary conditions. Combined with the differential equations, they pose boundary value problems, whose approximate solutions can be found numerically; a widely used numerical technique is the finite element method.

The finite element method divides the problem domain into smaller and simpler interconnecting parts (called finite elements, as the name of the method suggests) in what is called a meshing process. Each of these elements satisfies the aforementioned differential equations, and the corresponding boundary conditions if an element happens to be on the domain boundary, yielding a set of element equations. The finite element method then assembles these equations into a global system of equations to solve, with thousands to millions of unknowns depending on the number and the type of elements used. The computational results can then be visualized or used for analysis.

Numerous schemes have been developed to account for fluid-structure interaction; among them is the immersed finite element method, which handles the fluid-structure interaction efficiently without the time-consuming re-meshing process that other methods need. A few simulations modeling heart and blood vessels are performed to study the human cardiovascular system.

Interesting fact:
The nonlinear equations need to be linearized and then solved iteratively. This adds to the already heavy compu-tational burden for computers solving large-scale problems. As a result, parallel computing is frequently employed, and up to thousands of CPUs could be used to solve a dynamic bio-mechanical problem.

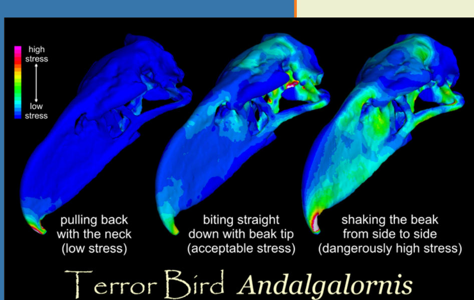
References:

Y. Fung, A First Course in Continuum Mechanics (Prentice-Hall, Englewood Cliffs, NJ, 1977).

L. Zhang, A. Gerstenberger, X. Wang, and W. Liu, Immersed finite element method. Comp. Methods in Appl. Mech. Engr. 193, 2051-2067 (2004).

W. Liu et al., Immersed finite element method and its applications to biological systems. Comp. Methods in Appl. Mech. Engr. 195, 1722-1749 (2006).

Submitted by Jubiao Yang, Rensselaer Polytechnic Institute, Math Matters, Apply it! Contest, February 2015.



Society for Industrial and Applied Mathematics
www.siam.org
www.siam.org/careers/matters.php