

Apply It.



The math behind... A Guitar

Technical terms used:

Luthier, timbre, rule of 18, chromatic scale, octave, frets, scale length, Pythagorean principle.

Uses and applications:

Before the Greek Philosopher, Pythagoras (570 - 475 BC), musical instruments were limited. His principle of creating musical notes in fifths and octaves, eventually resulted in the twelve-pitch chromatic guitar scale.

How it works:

The guitar produces very sophisticated music by producing different sound frequencies. Guitars designers/builders are called luthiers.

Stretched tightly between two points, a string when plucked, makes a specific tonal pitch. Pythagoras discovered that if the length of this string were to be cut in half, the pitch made would be equal in tone but at twice the frequency; an octave. The length of the string, tension, and its gauge (thickness) determine the pitch. On a guitar, the string length is controlled by the placement of frets on the neck.

Pleasant to the Western ear, the guitar neck is designed on a 12-pitch chromatic scale based on his principle. The 12th fret is the octave of that string. A formula, developed by Vincenzo Galilei in the 16th Century, mathematically determined the calculation of the distance between the tones initially uncovered by Pythagoras.

There are several different scale lengths on various guitars; determined by the instrument's designer, the player's personal preferences, gauge of the strings and length of the neck. Divide any scale's open string length by 17.817, and the result is the distance from the front edge of the nut to the first fret. For several centuries, this number was rounded to 18 and referred to as the "rule of 18."

For example, a typical guitar scale length (Fender Telecaster) is 25.5 inches (648 mm). The 25.5" scale length, divided by 17.817 is 1.4312173" (36.482011mm), rounded down to 1.431" (36.48mm). This is the placement for the first fret; the second note in the chromatic scale (the first note being the open string). A digital fret calculator is available on the Stewart-MacDonald website.

Formula for Calculating Fret Spacing:*

$$D_n = [(L - D_{n-1}) \div 17.817] + D_{n-1}$$

Calculating the formula down the remaining scale length one locates the positions of the remaining frets on down the neck. As the string gets shorter, the distance between frets shrinks proportionately. The shorter scale length creates a warmer tone. Check out different brands and models to see for yourself.

Interesting Fact: An instrument's timbre results from the unique tones created by varying scale lengths, tone woods, electronic pickups, guitar body sizes, etc. The variety of sounds emitting from guitars is virtually unlimited. The face of a guitar is the soundboard. Harder woods in an acoustic guitar's soundboard make for brighter notes with less sustain. Softer woods (like spruce) make warmer tones with more sustain.

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Reference:

National STEM Guitar Building Project: *<http://www.guitarbuilding.org>

Mathematics, Music, and the Guitar, David Hornbeck, <http://jwilson.coe.uga.edu/EMAT6450/Class%20Projects/Hornbeck/Math,%20Music,%20&%20Guitar.pdf>

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