Unsolved Problems in Common Waveform Analysis

Problem 01-002, *by* Yuchuan Wei (Beijing University of Aeronautics and Astronautics).

In addition to the sine-cosine functions, sawtooth wave, square wave, triangular wave, and trapezoidal wave are also common waveforms (periodic functions) in electronics. For the development of electronic techniques based on common waveforms, we should generalize Fourier analysis based on the sine-cosine function to frequency analysis based on common waveforms, shortened to common waveform analysis.

Suppose that $X(x)$ and $Y(x)$ are general even and odd periodic functions of period 2π. The central problem we pose is as follows: what are the necessary and sufficient conditions for the function system

$$1, X(x), Y(x), X(2x), Y(2x), \ldots, X(nx), Y(nx), \ldots \tag{1}$$

to be a complete system, a basis, or an unconditional basis of $L^2[-\pi, \pi]$?

An important example is the square wave system, in which $X(x) = \frac{\pi}{4}\text{sgn}(\cos x)$ and $Y(x) = \frac{\pi}{4}\text{sgn}(\sin x)$, where sgn is the sign function. Is the square wave system a basis of $L^2[-\pi, \pi]$?

If the function system (1) is a basis of $L^2[-\pi, \pi]$, we may ask further whether the expansion of each function $f(x) \in L^2[-\pi, \pi]$ in the basis, namely

$$f(x) = C_0 + \sum_{n=1}^{\infty} C(n)X(nx) + D(n)Y(nx)$$

converges almost everywhere in $[-\pi, \pi]$. In fact this is a generalized version of Luzin's problem on Fourier series.

Some partial results and other related problems can be seen in [1] or [2].

REFERENCES

[1] Y. Wei and Q. Zhang, *Common Waveform Analysis*, Kluwer, Boston, 2000.

[2] Y. Wei, *Frequency analysis based on general periodic functions*, J. Math. Phys., 40 (1999), pp. 3654–3684.

Status. This problem is unsolved.