Brownian Motion and Characterization of Domains

Problem 01-003, *by* Lucio R. Berrone (Universidad Nacional de Rosario, Rosario, Argentina).

(i) Let Ω be a plane domain limited by a regular Jordan curve Γ. For every Lebesgue measurable subset E of Γ and every point $z \in \Omega$, consider the probability $P(E;z)$ that a Brownian particle starting its motion at z hits the boundary Γ (for the first time) at a point belonging to E. Now let C be a constant such that $0 < C < |\Gamma|$ and set the following optimization problem:

$$\sup\{P(E;z) : |E| = C\}, \tag{1}$$

where $|\cdot|$ denotes the Lebesgue measure on the boundary Γ. The problem asks for a characterization of the domains Ω such that *single arcs* of the boundary are optimal subsets for (1) for every $z \in \Omega$ and every $0 < C < |\Gamma|$. In other words, naming a "C-window" of the domain Ω to a measurable subset E of Γ of length C, we look for the domains Ω such that the probability $P(E;z)$ is maximized by "single" C-windows for arbitrary $z \in \Omega$ and $0 < C < |\Gamma|$.

(ii) The same as (i) but for the case of an n-dimensional domain Ω.

Status. The proposer has solved (i), but (ii) is open.