

A Double Recurrence

Problem 01-004, by A. M. DAWES (University of Western Ontario, London, ON, Canada).

The doubly indexed sequence $d_{m,n}$ is defined by initial conditions

$$d_{1,n} = \frac{1}{2^{n-1}} \quad (n \geq 1) \quad \text{and} \quad d_{m,1} = \frac{2^m - 1}{2^{m-1}} \quad (m \geq 1)$$

and recurrence

$$d_{m,n} = \frac{d_{m-1,n} + d_{m,n-1}}{2} \quad \text{for } m, n > 1.$$

- (a) Find an expression for $d_{m,n}$.
- (b) In particular, find the value of $d_{m,m}$.
- (c) Find the sum of row m in the matrix $(d_{m,n})$; i.e., find $\sum_{j=1}^{\infty} d_{m,j}$.

Status. This proposer has a solution. Others are invited.