A Problem Concerning Negative Squares

Problem 02-007, *by* Torben Maack Bisgaard.

The *number of negative squares* of a hermitian matrix is its number of negative eigenvalues counted with multiplicity. For every positive integer k let $M(k)$ be the statement (which may be true or false) that if A is a symmetric real matrix of order km of the block form

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1k} \\ A_{21} & A_{22} & \cdots & A_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ A_{k1} & A_{k2} & \cdots & A_{kk} \end{pmatrix},$$

where the A_{ij} are symmetric matrices of order m and if A has exactly κ negative squares, then the "trace matrix" $A_{11} + A_{22} + \cdots + A_{kk}$ has at most κ negative squares. Prove $M(8)$.

Status. The proposer has a solution.