A Complex Arithmetic-Geometric Mean Inequality

Problem 03-003, *by* JONATHAN BORWEIN (Simon Fraser University, Burnaby, BC, Canada).

Show that the complex extension of the arithmetic-geometric mean inequality

$$\left|\frac{a+b}{2}\right| \geq \sqrt{|ab|}$$

holds for complex a, b "outside" a cardioid. What can be found by way of complex extension of $(a_1 + a_2 + \cdots + a_n)/n \geq (a_1 a_2 \cdots a_n)^{1/n}$, which holds for positive $a_1, a_2, \ldots, a_n$?

Status. This problem appears in *Mathematics by Experiment* (A. K. Peters Ltd., 2003), which is coauthored by the proposer. It is easily solved, but the issue of reasonable generalizations remains entirely open.